2005 International Conference on Wireless Networks, Communications and Mobile Computing

Optimal Power Control for Multiple Access Channel with Peak and Average Power Constraints

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Abstract— **In this paper, we study optimal power control for multiple access channel with peak and average power constraints. Perfect channel information is assumed to be available at both the transmitter and the receiver. We characterize the structures of the optimal power control policy and show that the optimal policy allows multiple users to transmit at peak power and at most one user to transmit at an intermediate power level. This is different from the optimal TDMA policy (at most one user is allowed to transmit at any given time) for the case of average power constraint only. We find closed-form expressions for computing the optimal power control policy and the maximal sum capacity. Numerical results indicate that under the peak power constraint, the optimal power control policy still achieves very close to the sum capacity of the multiple access channel with average power constraint only.**

I. Introduction

Due to the time-varying nature of wireless channels, it is shown in [1] that for a single user system, when the channel information is available at both the transmitter and the receiver, the wireless user should exploit time-diversity to adapt transmission power according to the channel state and transmit at higher rates when the channel is good. In a multi-user system, optimal power control policies ([2], [3]) are developed to exploit both multi-user diversity and time-diversity with the goal of maximizing the sum capacity of multiuser networks. For an uplink cellular system, [2] shows that time division multiple access (TDMA) schemes maximize the sum capacity. The optimal power control policy depends on the channel conditions of all users and it has a structure that, at any given time, only one user with the best channel condition transmits. However, since each user transmits only when his channel is the best among all users, the amount of time that each user occupies the channel decreases as the number of users increases. Under the average power constraint, this implies that each user would transmit at a higher power level when it is his turn to transmit. Consequently, as the system size grows, each users' signal transmission becomes increasingly bursty and large peak-to-average power ratios occur (more details are included in Section III).

In this work, we study optimal power control under both peak and average power constraint. The peak power constraint is independent of the number of users in the system, therefore in our system we have a fixed peak to average power ratio. We characterize the structures of the optimal power control policy and provide close-form expressions for computing the optimal sum capacity. Under the peak power constraint, we show that the optimal policy is no longer a TDMA policy. Instead, it allows multiple users to transmit simultaneously, possibly all with the peak power. Our results show that the peak power constraint does not impose much capacity penalties against the case of average power constraint only. In addition, the number of simultaneous transmissions can be kept small.

In other related work, optimal power control with peak power constraint is studied in [4] for a single user system. In this work, we consider a multi-user system. Optimal power control for multiple antenna channels are studied in [5] under the average power constraint.

II. Uplink optimal power control

Consider an uplink cellular network with K users. Let $\gamma_i, i = 1, \cdots, K$ be the channel gain between user i and the base station. Under Rayleigh fading, we assume that $\sqrt{\gamma_i}$ has a Rayleigh distribution and γ_i has an exponential distribution with parameter 1. Let $\gamma = (\gamma_1, \cdots, \gamma_K)$ denote the channel gain vector of all users and $u_i(\gamma)$ be the power allocation to user i when the channel gain vector is γ. Under the power control policy $u_i(\gamma)$, the received signal at the base station can be written as

$$y = \sum_{i=1}^{K} \sqrt{u_i(\gamma)\gamma_i}\ x_i + n, \tag{1}$$

where x_i is the transmitted signal from user i which satisfies $E(|x_i|^2) = 1$, n is white Gaussian noise with zero mean and unit variance.

For a fixed γ, equation (1) represents a Gaussian multiple access channel with a sum capacity of $\frac{1}{2}\log\left(1 + \sum_{i=1}^{K} u_i(\gamma)\gamma_i\right)$ [6]. Assume that channel fading information is available at both the transmitter and the receiver, we can average over all possible fading realizations to obtain $\frac{1}{2}E\left[\log\left(1 + \sum_{i=1}^{K} u_i(\gamma)\gamma_i\right)\right]$ as the sum capacity for the fading case. Our objective is to find the optimal power control policy $u_i(\gamma), i = 1, \cdots, K$ to maximize the sum capacity under both peak and average power constraints. Let $p(\gamma)$ be the probability density function of γ. We formulate the following optimization problem.

$$\max \ \iint\cdots\int \tfrac{1}{2}\log\left(1 + \textstyle\sum_{i=1}^{K} u_i(\gamma)\gamma_i\right) p(\gamma)d\,\gamma \tag{2}$$

$$\text{subject to} \ \iint\cdots\int u_i(\gamma)p(\gamma)d\gamma \le P_i, \quad i = 1, \cdots, K \tag{3}$$

$$\text{and} \qquad u_i(\gamma) \le \hat{P}, \quad i = 1, \cdots, K. \tag{4}$$

Note that we consider both the average power constraint (3) and the peak power constraint (4), while in [1], [2], [3] only the average power constraint is considered.

A. Structures of the optimal power control policy

In order to characterize the structures of the optimal power control policy, we first introduce the Lagrange multipliers, λ_i, corresponding to each average power constraint in (3). The Lagrangian function is expressed as

$$L(\{u_i\},\{\lambda_i\}) = \iint \cdots \int \log\left(1+\sum_{i=1}^{K} u_i(\gamma)\gamma_i\right) p(\gamma)d\,\gamma - \sum_{i=1}^{K}\left(\lambda_i \iint \cdots \int u_i(\gamma)p(\gamma)d\,\gamma\right). \quad (5)$$

Due to the convexity of the logarithm, the Karush-Kuhn-Tucker (KKT) conditions [7] are necessary and sufficient for optimality. Taking the derivative with respect to u_i, we obtain a set of KKT conditions for every $i = 1, \cdots, K$ and γ.

$$\frac{\gamma_i}{\lambda_i} \ \geq \ 1+\sum_{j=1}^{K} u_j(\gamma)\gamma_j \qquad \text{if } u_i(\gamma) = \hat{P} \quad (6)$$

$$\frac{\gamma_i}{\lambda_i} \ = \ 1+\sum_{j=1}^{K} u_j(\gamma)\gamma_j \qquad \text{if } 0 < u_i(\gamma) < \hat{P} \quad (7)$$

$$\frac{\gamma_i}{\lambda_i} \ \leq \ 1+\sum_{j=1}^{K} u_j(\gamma)\gamma_j \qquad \text{if } u_i(\gamma) = 0 \quad (8)$$

Since the summation terms in (6)-(8) are the same for all $i = 1, \cdots, K$, we make the following observation:

(1) If $\frac{\gamma_{i_1}}{\lambda_{i_1}} \geq \frac{\gamma_{i_2}}{\lambda_{i_2}}$, then we must have $u_{i_1}(\gamma) \geq u_{i_2}(\gamma)$.

(2) At most one user can be allocated an intermediate power level $0 < u_i(\gamma) < \hat{P}$. This is true because the probability that the left hand side of (7) is the same for two different users equals 0. Hence, when there is no peak power constraint (or $\hat{P} = \infty$), the optimal power control policy reduces to the TDMA policy in [2].

(3) Let us reorder the sequence $\{\gamma_i/\lambda_i, i = 1, \cdots K\}$ such that

$$\frac{\gamma_{i_1}}{\lambda_{i_1}} \geq \frac{\gamma_{i_2}}{\lambda_{i_2}} \geq \cdots \frac{\gamma_{i_K}}{\lambda_{i_K}}.$$

It then follows from (1) and (2) that, the optimal power control policy satisfies the following properties: (a) there exists an integer N (which depends on γ) such that the best N users $i_1, \cdots, i_N$ are allocated full power $\hat{P}$. (b) User i_{N+1} either uses an intermediate power level or is inactive (zero power allocation). (c) Users $i_{N+2}, \cdots, i_K$ are all inactive.

Next, we will show how to determine the value of N and the optimal power allocation for user i_{N+1}. For notation simplicity, we assume that

$$\frac{\gamma_1}{\lambda_1} \geq \frac{\gamma_2}{\lambda_2} \geq \cdots \frac{\gamma_K}{\lambda_K}.$$

For every $i = 1, \cdots, K$, let $a_i = \frac{\gamma_i}{\lambda_i}$ and $b_i = 1 + \hat{P}\sum_{j=1}^{i}\gamma_j$. Note that b_i equals the summation term $1 + \sum_{j=1}^{K} u_j(\gamma)\gamma_j$ in (6)-(8) if the first i users are allocated peak power and the remaining users are inactive.

Since $\{a_i\}$ is an non-increasing sequence and b_i is a non-decreasing sequence, we can find an integer N such that $a_N \geq b_N$ and $a_{N+1} < b_{N+1}$. There are two cases shown in Figure 1, in which we let $N = 4$ as an example.

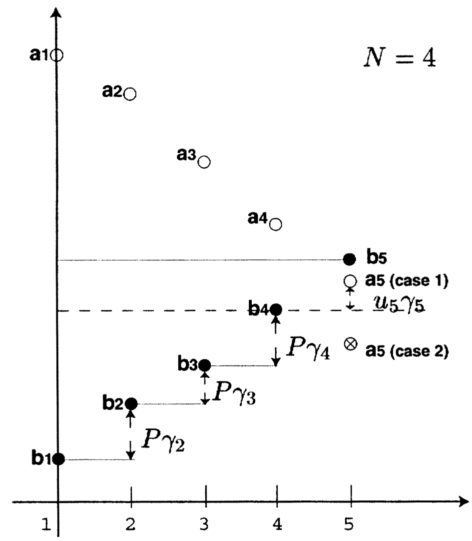

Fig. 1. Optimal power control policy with peak power constraint.

Case 1 Assume $b_N < a_{N+1} < b_{N+1}$. In this case, user $N+1$ should not be allocated peak power $\hat{P}$. Otherwise the KKT condition (6) will be violated. Since $a_{N+1} > b_N$, we can allocate an intermediate power level to user $N+1$ to ensure that the equality in the KKT condition (7) holds. The optimal power allocation becomes

$$u_i(\gamma) = \begin{cases} \hat{P}, & \text{if } i \leq N \\ \frac{a_{N+1}-b_N}{\gamma_{N+1}} & \text{if } i = N+1 \\ 0 & \text{else.} \end{cases} \quad (9)$$

Using this power allocation, one can see that the summation term in (6)-(8) equals a_{N+1}. Due to the monotone property of the sequence $\{a_i\}$, the KKT conditions are clearly satisfied for power control policies of the form (9).

Case 2 Assume $a_{N+1} \leq b_N$. In this case, user $N+1$ is inactive. The optimal power allocation becomes

$$u_i(\gamma) = \begin{cases} \hat{P}, & \text{if } i \leq N \\ 0 & \text{else.} \end{cases} \quad (10)$$

The corresponding summation term in (6)-(8) equals b_N which also ensures that the KKT conditions are satisfied.

As seen from (9) and (10), we note a useful fact that the optimal power allocation for user i only depends on $(\gamma_1, \cdots, \gamma_i)$.

B. Computation of the optimal Lagrange multiplier

Given the structures of the optimal power control policy (9) and (10), the optimal Lagrange multiplier can be found by choosing $\{\lambda_i\}$ such that the average power constraint (3) is satisfied for each user i. This, however, involves the evaluation of K-dimensional integrals. In the following, we consider the scenario when the average power constraint $P_i = P_a$ is the same for all users. By symmetry, all λ_i must be equal and we denote the common value by λ. In order to find λ that satisfies (3), we need to simplify (3) as much as possible and reduce the dimension of integration.

Let $1_{\{.\}}$ denote the indicator function. Since the users are symmetric, we have

$$\begin{aligned} E[u_i(\gamma)] &= \sum_{m=1}^{K} E\left[u_i(\gamma)1_{\{\text{user } i \text{ has the m-th best channel}\}}\right] \\ &= \sum_{m=1}^{K} E\left[u_m(\gamma)1_{\{\text{user } m \text{ has the m-th best channel}\}}\right]. \end{aligned} \tag{11}$$

The case when $m = 1$ needs to be handled separately. For every $m \geq 2$, we have

$$\begin{aligned} &E\left[u_m(\gamma)1_{\{\text{user } m \text{ has the m-th best channel}\}}\right] \\ &= \binom{K-1}{m-1}E\left[u_m(\gamma)1_{\{\min(\gamma_1,\cdots,\gamma_{m-1})\geq\gamma_m,\max(\gamma_{m+1},\cdots,\gamma_K)\leq\gamma_m\}}\right] \\ &= \binom{K-1}{m-1}\int_0^\infty\left[\left[\int_{\gamma_m}^\infty\cdots\int_{\gamma_m}^\infty u_m(\gamma)e^{(-\sum_{j=1}^{m-1}\gamma_j)}d\gamma_1\cdots d\gamma_{m-1}\right.\right. \\ &\qquad \left.\cdot\int_0^{\gamma_m}\cdots\int_0^{\gamma_m} e^{(-\sum_{j=m+1}^{K}\gamma_j)}d\gamma_{m+1}\cdots d\gamma_K\right]e^{-\gamma_m}d\gamma_m \\ &= \binom{K-1}{m-1}\int_0^\infty\left[\int_{\gamma_m}^\infty\cdots\int_{\gamma_m}^\infty u_m(\gamma_1,\cdots,\gamma_m)\ e^{(-\sum_{j=1}^{m-1}\gamma_j)}\right. \\ &\qquad \left. d\gamma_1\cdots d\gamma_{m-1}\right](1-e^{-\gamma_m})^{K-m}e^{-\gamma_m}d\gamma_m \end{aligned} \tag{12}$$

In order to compute the integral in (12), we consider two cases with $u_m(\gamma_1,\cdots,\gamma_m) = \hat{P}$ or $0 < u_m(\gamma_1,\cdots,\gamma_m) < \hat{P}$.

(1) If $u_m(\gamma_1,\cdots,\gamma_m) = \hat{P}$, we must have $a_m \geq b_m$ and therefore

$$\sum_{j=1}^{m-1}\gamma_j \leq \frac{\frac{\gamma_m}{\lambda}-1}{\hat{P}} - \gamma_m. \tag{13}$$

(2) If $0 < u_m(\gamma_1,\cdots,\gamma_m) < \hat{P}$, then we have $b_{m-1} \leq a_m \leq b_m$ and therefore

$$\frac{\frac{\gamma_m}{\lambda}-1}{\hat{P}} - \gamma_m \leq \sum_{j=1}^{m-1}\gamma_j \leq \frac{\frac{\gamma_m}{\lambda}-1}{\hat{P}}. \tag{14}$$

It follows from (9) that the optimal power allocation is

$$u_m(\gamma_1,\cdots,\gamma_m) = \frac{1}{\lambda} - \frac{1+\hat{P}\sum_{j=1}^{m-1}\gamma_j}{\gamma_m}. \tag{15}$$

(Note that if we let $m = 1$, the optimal power allocation (15) reduces to the optimal water-filling power control policy in [2] with only average power constraint.)

Hence, the integral in (12) equals

$$\begin{aligned} &\int_0^\infty\left[\int_{\gamma_m}^\infty\cdots\int_{\gamma_m}^\infty\left(\hat{P}\cdot 1_{\left\{\sum_{j=1}^{m-1}\gamma_j\leq\frac{\frac{\gamma_m}{\lambda}-1}{\hat{P}}-\gamma_m\right\}}\right.\right. \\ &+\left(\frac{1}{\lambda}-\frac{1+\hat{P}\sum_{j=1}^{m-1}\gamma_j}{\gamma_m}\right)\cdot 1_{\left\{\frac{\frac{\gamma_m}{\lambda}-1}{\hat{P}}-\gamma_m\leq\sum_{j=1}^{m-1}\gamma_j\leq\frac{\frac{\gamma_m}{\lambda}-1}{\hat{P}}\right\}}\Bigg) \\ &\left.\cdot e^{(-\sum_{j=1}^{m-1}\gamma_j)}d\gamma_1\cdots d\gamma_{m-1}\right](1-e^{-\gamma_m})^{K-m}e^{-\gamma_m}d\gamma_m \end{aligned} \tag{16}$$

Let $\tilde{\gamma}_j = \gamma_j - \gamma_m, j = 1,\cdots,m-1$ and $x = \sum_{j=1}^{m-1}\tilde{\gamma}_j$. Then the probability density function of x has a gamma distribution $g_{m-1}(x) = x^{m-2}e^{-x}/(m-2)!$. The integral in (16) can be simplified as

$$\begin{aligned} &\int_0^\infty\left[\hat{P}\int_0^{r_{m,\lambda}(\gamma_m)} g_{m-1}(x)\ dx\right. \\ &\left.+\int_{r_{m,\lambda}(\gamma_m)}^{r_{m-1,\lambda}(\gamma_m)} f_{m-1,\lambda}(\gamma_m,x)\ g_{m-1}(x)\ dx\right]q_m(\gamma_m)d\gamma_m \end{aligned} \tag{17}$$

where

$$\begin{aligned} q_m(\gamma_m) &= [1-\exp(-\gamma_m)]^{K-m}[\exp(-\gamma_m)]^m \\ f_{m-1,\lambda}(\gamma_m,x) &= 1/\lambda - [1+\hat{P}(x+(m-1)\gamma_m)]/\gamma_m \\ r_{m,\lambda}(\gamma_m) &= \max\left(0,(\gamma_m/\lambda-1)/\hat{P}-m\gamma_m\right) \end{aligned}$$

The formula for computing λ is therefore given by

$$\begin{aligned} P_a = &\hat{P}\int_{\frac{1}{1/\lambda-\hat{P}}}^\infty q_1(\gamma_1)d\gamma_1 + \int_\lambda^{\frac{1}{1/\lambda-\hat{P}}} q_1(\gamma_1)\left(\frac{1}{\lambda}-\frac{1}{\gamma_1}\right)d\gamma_1 \\ &+\sum_{m=2}^{K}\binom{K-1}{m-1}\int_0^\infty\left[\hat{P}\int_0^{r_{m,\lambda}(\gamma_m)} g_{m-1}(x)\ dx\right. \\ &\left.+\int_{r_{m,\lambda}(\gamma_m)}^{r_{m-1,\lambda}(\gamma_m)} f_{m-1,\lambda}(\gamma_m,x)\ g_{m-1}(x)\ dx\right]q_m(\gamma_m)d\gamma_m, \end{aligned} \tag{18}$$

where the first two terms correspond to $m = 1$.

C. Computation of the optimal sum capacity

Once the optimal λ is found by solving (18), the optimal power control policy is uniquely determined. In this section, we derive analytical expressions for computing the optimal sum capacity.

Let $c(\gamma) = 1 + \sum_{j=1}^{K} u_j(\gamma)\gamma_j$, we have

$$\begin{aligned} C_{\text{opt}} &= \frac{1}{2}E[\log c(\gamma)] \\ &= \frac{1}{2}\sum_{m=1}^{K} E\left[\log(c(\gamma))\cdot 1_{\{\text{ there are exactly m active users }\}}\right] \\ &= \frac{1}{2}K\sum_{m=1}^{K} E\left[\log(c(\gamma))\cdot 1_{\{\text{m active users, user m has m-th best channel}\}}\right] \\ &= \frac{1}{2}K\sum_{m=1}^{K}\binom{K-1}{m-1}E\left[\log(c(\gamma))\cdot 1_{\{\text{m active users}\}}\right. \\ &\qquad\left.\cdot 1_{\{\min(\gamma_1,\cdots,\gamma_{m-1})\geq\gamma_m,\max(\gamma_{m+1},\cdots,\gamma_K)\leq\gamma_m\}}\right] \\ &= \frac{1}{2}K\sum_{m=1}^{K}\binom{K-1}{m-1}E\left[\log(c(\gamma))\cdot 1_{\{\text{m active users}\}}\right. \\ &\qquad\cdot 1_{\{\min(\gamma_1,\cdots,\gamma_{m-1})\geq\gamma_m,\max(\gamma_{m+1},\cdots,\gamma_K)\leq\gamma_m\}} \\ &\qquad\left.\cdot\left(1_{\{u_m(\gamma)=\hat{P}\}}+1_{\{0<u_m(\gamma)<\hat{P}\}}\right)\right] \end{aligned} \tag{19}$$

We again consider two cases.

(1) Assume $u_m(\gamma) = \hat{P}$. To ensure that there are exactly m active users, we require that $a_m \geq b_m \geq a_{m+1}$. Hence, the following conditions must be satisfied:

$$\gamma_{m+1} \leq \lambda\Big(1+\hat{P}\sum_{j=1}^{m}\gamma_j\Big) \quad (20)$$

$$\sum_{j=1}^{m}\gamma_j \leq \frac{\frac{\gamma_m}{\lambda}-1}{\hat{P}} \quad (21)$$

$$c(\gamma) = b_m = 1+\hat{P}\sum_{j=1}^{m}\gamma_j \quad (22)$$

Note that (21) and (20) imply that $\gamma_{m+1} \leq \gamma_m$. It follows that the integral term in (19) that involves $1_{\{u_m(\gamma)=\hat{P}\}}$ equals

$$\begin{aligned}
&= \int_0^\infty \Bigg[\int_{\gamma_m}^\infty\cdots\int_{\gamma_m}^\infty \log\Big(1+\hat{P}\sum_{j=1}^{m}\gamma_j\Big)\cdot 1_{\{\sum_{j=1}^{m}\gamma_j \leq \frac{\gamma_m/\lambda-1}{\hat{P}}\}} \\
&\cdot \int_0^{\lambda(1+\hat{P}\sum_{j=1}^{m}\gamma_j)}\cdots\int_0^{\lambda(1+\hat{P}\sum_{j=1}^{m}\gamma_j)} e^{(-\sum_{j=m+1}^{K}\gamma_j)} d\gamma_{m+1}\cdots d\gamma_K \\
&\cdot e^{(-\sum_{j=1}^{m-1}\gamma_j)} d\gamma_1\cdots d\gamma_{m-1}\Bigg] e^{-\gamma_m} d\gamma_m \\
&= \int_0^\infty \Bigg[\int_{\gamma_m}^\infty\cdots\int_{\gamma_m}^\infty \log\Big(1+\hat{P}\sum_{j=1}^{m}\gamma_j\Big)\cdot 1_{\{\sum_{j=1}^{m}\gamma_j \leq \frac{\gamma_m/\lambda-1}{\hat{P}}\}} \\
&\cdot \Big(1-e^{-\lambda(1+\hat{P}\sum_{j=1}^{m}\gamma_j)}\Big)^{(K-m)} e^{-(\sum_{j=1}^{m-1}\gamma_j)} d\gamma_1\cdots d\gamma_{m-1}\Bigg] e^{-\gamma_m} d\gamma_m
\end{aligned} \quad (23)$$

(2) Assume $0 < u_m(\gamma) < \hat{P}$. We have $b_{m-1} \leq a_m \leq b_m$ which guarantees that there are exactly m active users. Hence, we must have

$$\frac{\frac{\gamma_m}{\lambda}-1}{\hat{P}} - \gamma_m \leq \sum_{j=1}^{m-1}\gamma_j \leq \frac{\frac{\gamma_m}{\lambda}-1}{\hat{P}}. \quad (24)$$

$$c(\gamma) = a_m = \frac{\gamma_m}{\lambda}. \quad (25)$$

It follows that the integral term in (19) that involves $1_{\{0<u_m(\gamma)<\hat{P}\}}$ equals

$$\begin{aligned}
&\int_0^\infty \Bigg[\int_{\gamma_m}^\infty\cdots\int_{\gamma_m}^\infty \Big(\log\frac{\gamma_m}{\lambda}\Big)\cdot 1_{\Big\{\frac{\frac{\gamma_m}{\lambda}-1}{\hat{P}}-\gamma_m \leq \sum_{j=1}^{m-1}\gamma_j \leq \frac{\frac{\gamma_m}{\lambda}-1}{\hat{P}}\Big\}} \\
&\cdot e^{(-\sum_{j=1}^{m-1}\gamma_j)} d\gamma_1\cdots d\gamma_{m-1}\Bigg] (1-e^{-\gamma_m})^{K-m} e^{-\gamma_m} d\gamma_m
\end{aligned} \quad (26)$$

Finally, we use the transformation $\tilde{\gamma}_j = \gamma_j - \gamma_m$ and $x = \sum_{j=1}^{m-1}\tilde{\gamma}_j$ to arrive at a Gamma distribution and hence reduce the dimensions of the integral. After simplifications, the optimal sum capacity can be written as

$$\begin{aligned}
C_{\text{opt}} &= \frac{K}{2}\Bigg[\int_{\frac{1}{1/\lambda-\hat{P}}}^{\infty} s_1(\gamma_1,0)\, d\gamma_1 + \int_{\lambda}^{\frac{1}{1/\lambda-\hat{P}}} q_1(\gamma_1)\log\Big(\frac{\gamma_1}{\lambda}\Big)\, d\gamma_1\Bigg] \\
&+ \sum_{m=2}^{K}\frac{K}{2}\binom{K-1}{m-1}\Bigg[\int_0^\infty\int_0^{r_{m,\lambda}(\gamma_m)} s_m(\gamma_m,x) g_{m-1}(x)\, dx\, d\gamma_m \\
&+ \int_0^\infty \log\Big(\frac{\gamma_m}{\lambda}\Big)\int_{r_{m,\lambda}(\gamma_m)}^{r_{m-1,\lambda}(\gamma_m)} g_{m-1}(x)\, dx\, q_m(\gamma_m) d\gamma_m\Bigg],
\end{aligned} \quad (27)$$

where

$$\begin{aligned}
s_m(\gamma_m,x) &= \log\big(1+\hat{P}(m\gamma_m+x)\big)\cdot(e^{-\gamma_m})^m \\
&\cdot\big[1-e^{-\lambda(1+\hat{P}(m\gamma_m+x))}\big]^{K-m}.
\end{aligned} \quad (28)$$

and the first two terms in (27) correspond to $m = 1$.

III. Numerical Results

In this section, we present numerical results based on the close-form expressions derived in Section II.

First, we study the case with only average power constraint to establish our performance benchmark. We assume an average transmission power of $P_a = 10$ such that the received signal-to-noise power ratio $10\log_{10} P_a$ equals 10 dB. Given a total of K users in the system, the optimal power control policy (average power constraint only) allows a maximum transmission power $P_{\max}(K) = 1/\lambda$. As shown in Figure 2, the maximum to average power ratio $P_{\max}(K)/P_a$ increases linearly with respect to K due to the fact that as K increases, the amount of time each user transmits decreases. When $K = 16$, we have $P_{\max}/P_a = 16.03$ and the optimal sum capacity equals 4.4947 bits.

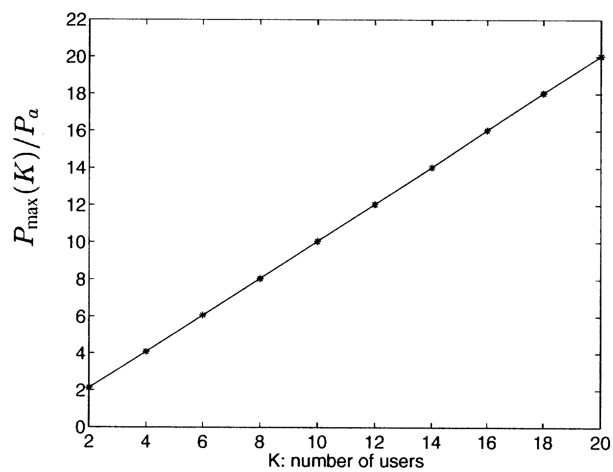

Fig. 2. Maximum to average power ratio versus the total number of users in a system with average power constraint only.

Next, we examine the sum capacity under both peak and average power constraints. Compared to the average power constraint only scenario where the maximum transmission power $P_{\max}(K)$ increases linearly with K, here we require that at any time each users' transmission power is no more than $\hat{P}$, which is independent of K. In Figure 3, we plot the optimal sum capacity as a function of the peak-to-average power ratio

$\hat{P}/P_a$. For each $\hat{P}/P_a$, we first solve (18) numerically to find the Lagrange multiplier λ. Then we apply (27) to compute the sum capacity corresponding to the optimal power control policy given by λ. Figure 3 shows that as $\hat{P}$ increases, the sum capacity also increases. When $\hat{P}/P_a = 16$, the capacity with peak power constraint equals the capacity with average power constraint only (4.4947 bits). When $\hat{P}/P_a = 8$, the sum capacity with peak power constraint attains about $4.3968/4.4947 = 97.82\%$ of the sum capacity with average power constraint only. Note that by imposing the peak power constraint, we reduce the maximum to average power ratio from $P_{\max}(16)/P_a \approx 16$ to $\hat{P}/P_a = 8$, which is a 3 dB reduction, with only a 2% loss in sum capacity. When $\hat{P}/P_a = 2$, which corresponds to a 9 dB reduction from $P_{\max}(16)/P_a \approx 16$, the sum capacity achieved is about 89.3% of the sum capacity with average power constraint only. However, as shown in Figure 4, in this case we would require many users to transmit simultaneously with large probabilities.

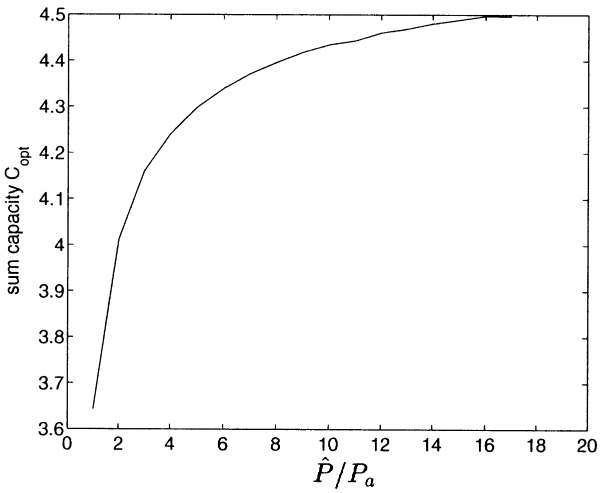

Fig. 3. Optimal sum capacity versus peak to average power ratio.

In Figure 4, we study the probability distribution of the number of active users under the peak power constraint. Let m denote the number of active users. Given $\hat{P}$, the probability that there are exactly m active users can be computed by evaluating the integral terms in (27) that correspond to m active users, and replacing $s_m(\gamma_m, x)$ and $\log\left(\frac{\gamma_m}{\lambda}\right)$ by 1. In Figure 4, multiple points are plotted for each $\hat{P}/P_a$. The number to the right of each point represents m, the number of active users. The y-coordinate of each point represents the probability that there are exactly m active users. The highest point for a given $\hat{P}/P_a$ represents the most likely (with the largest probability) number of active users. For instance, three points are plotted for $\hat{P}/P_a = 8$. From top to bottom, we see that with probabilities of 0.61, 0.36, and 0.03, there are two, three, and one active users, respectively. The probability that there is no active user is close to 0, hence not shown in the figure. We also observe that when $\hat{P}/P_a$ is small, it is likely to have a large number of active users. For instance, when $\hat{P}/P_a = 2$, with a probability of 0.31, there are 8 active users. However, as $\hat{P}/P_a$ increases, the number of active users reduces. When $\hat{P}/P_a = 8$, there are at most three active users. When $\hat{P}/P_a = 16$, with a probability of 0.987 there is only one active user, hence the resulting optimal power control policy is almost TDMA as in the case of average power constraint only.

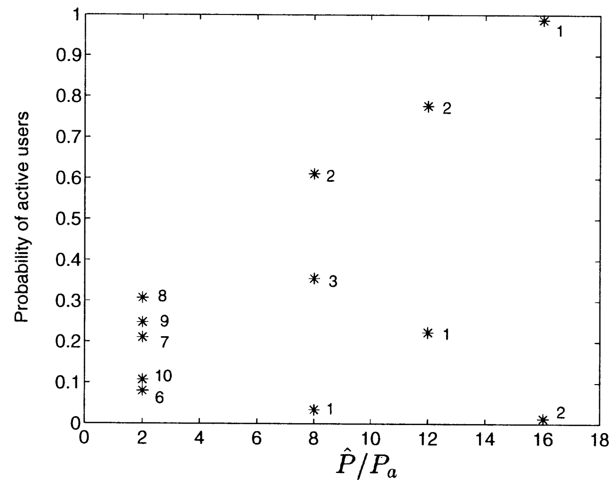

Fig. 4. Probability distribution of the number of active users

IV. Conclusion

In this paper, we study optimal power control for multiple access channel with peak and average power constraints. We obtain a complete characterization of the optimal power control policy which generalizes previous results on the optimality of TDMA type policies for multiple access channels with only average power constraint. Analytical expressions are derived for computing the optimal policy, the optimal sum capacity, and the probability distribution of the number of active users, assuming that users in the network are symmetric. By allowing more than one user to transmit at a given time, the proposed power control policies under the peak power constraint have the advantage of reducing the maximum to average power ratio and shortening the transmission delay due to opportunistic scheduling in the multi-user network. In addition, the sum capacity achieved by the proposed policies are close to that achieved with only the average power constraint.

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