Design of Nonbinary LDPC Codes for Multiple-Antenna Transmission

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Outline

- Motivation
- Two nonbinary LDPC coded system
  - Non-iterative system
  - Iterative system
- Code design for iterative system
- Simulation results
- Conclusion

Motivation

- MIMO system has been widely used to increase system capacity
- LDPC codes can be employed to approach the MIMO channel capacity
  - S. ten Brink, G. Kramer, and A. Ashikhmin study the binary LDPC code design for MIMO [1].
- Nonbinary LDPC code design has been studied for AWGN and shows better performance than binary codes [2].

[1] S. ten Brink, G. Kramer, and A. Ashikhmin, "Design of low-density parity-check codes for modulation and detection," *IEEE Trans. Commun.*,vol. 52, pp. 670–678, Apr. 2004.

[2] A. Bennatan and D. Burshtein, "Design and analysis of nonbinary LDPC codes for arbitrary discrete-memoryless channels," *IEEE Trans. Inform. Theory*, vol. 52, pp. 549–583, Feb. 2006.

Motivation

- Our contribution
  - The first work to apply irregular nonbinary LDPC to MIMO channel
  - Propose two nonbinary LDPC coded MIMO systems.
  - Extend EXIT chart based code design methods to nonbinary iterative systems
  - Provide comparison with optimal binary LDPC coded systems

Introduction of binary LDPC

- A subclass of linear block codes
- Specified by a parity check matrix $(n\text{-}k) \times n$

n: code length k: length of information sequence

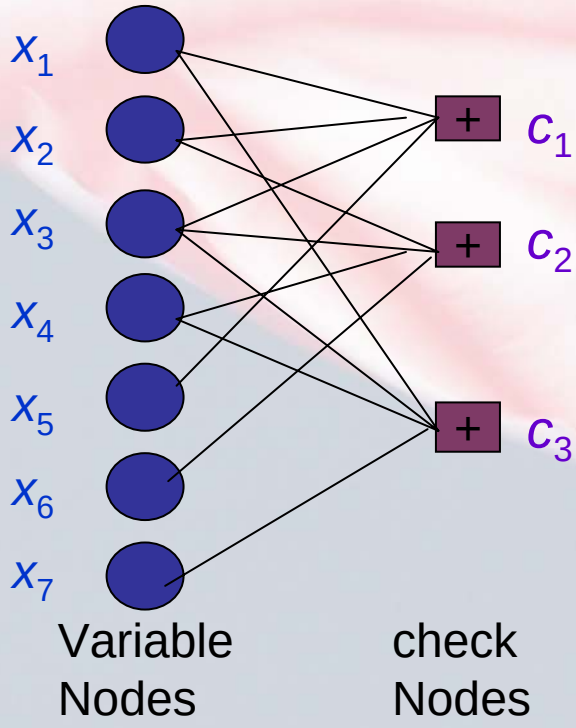
$$\mathbf{H} = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & & \cdots & & x_7 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\mathbf{H}\mathbf{x}^T = \mathbf{0}$$

$$c_1 : x_1 + x_2 + x_3 + x_5 = 0$$

$$c_2 : x_2 + x_3 + x_4 + x_6 = 0$$

$$c_3 : x_1 + x_3 + x_4 + x_7 = 0$$



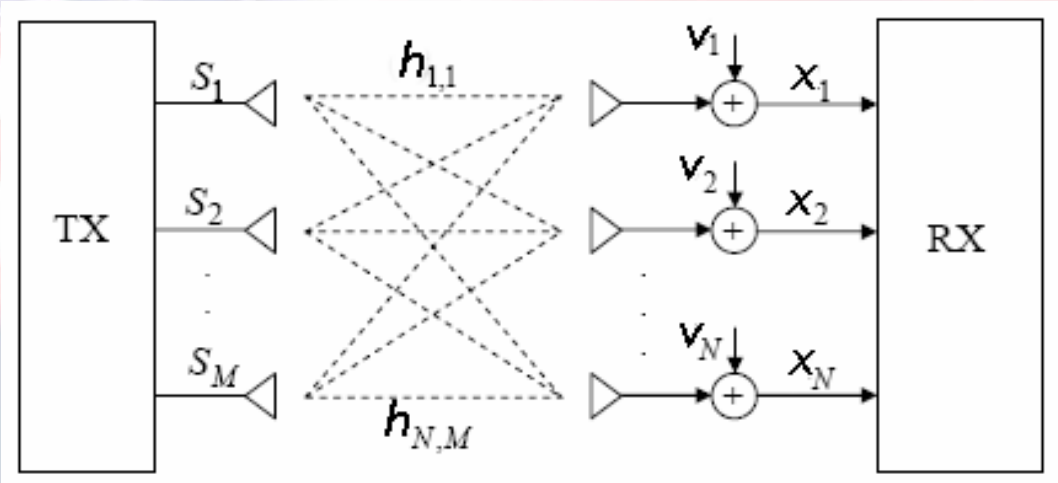
Definition of nonbinary LDPC

- For nonbinary codes, the ones in parity check matrix are replaced by nonzero elements in GF(q)

$$\mathbf{H} = \begin{bmatrix} 3 & 7 & 1 & 0 & 3 & 0 & 0 \\ 0 & 2 & 5 & 6 & 0 & 3 & 0 \\ 4 & 0 & 2 & 7 & 0 & 0 & 5 \end{bmatrix}$$

Application to MIMO

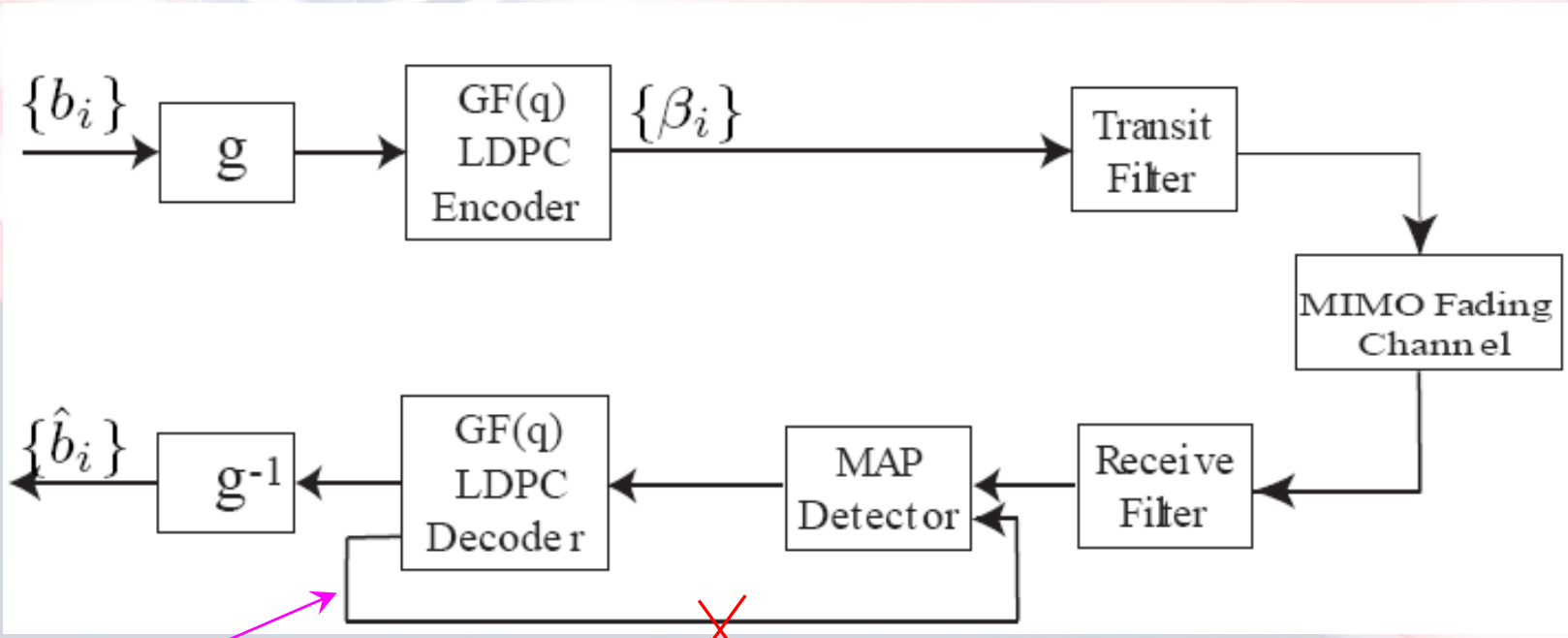
- Channel model



$$\mathbf{X} = \sqrt{\frac{\rho}{M}}\mathbf{HS} + \mathbf{V}$$

Assume each entry of channel matrix is independent, follows Rayleigh fading, and is known by receiver

System block diagram



Iterative system: Soft messages are exchanged between detector and decoder iteratively.

Non-iterative system: the detection is performed only once.

R.-H. Peng and R.-R. Chen, “Application of nonbinary ldpc codes for communication over fading channels using higher modulations,” to appear: *Proc. IEEE Globecom’06* .

Code design for iterative system

- Irregular codes can achieve much better performance than regular codes
- Density evolution and EXIT chart are two widely used methods to design binary irregular codes.
- So far, design of nonbinary codes is limited to AWGN channel
- We focus on the code design for iterative MIMO system based on EXIT chart method
- The idea of EXIT chart is tracking the soft message changed during iterations using mutual information

.

Binary LDPC code design

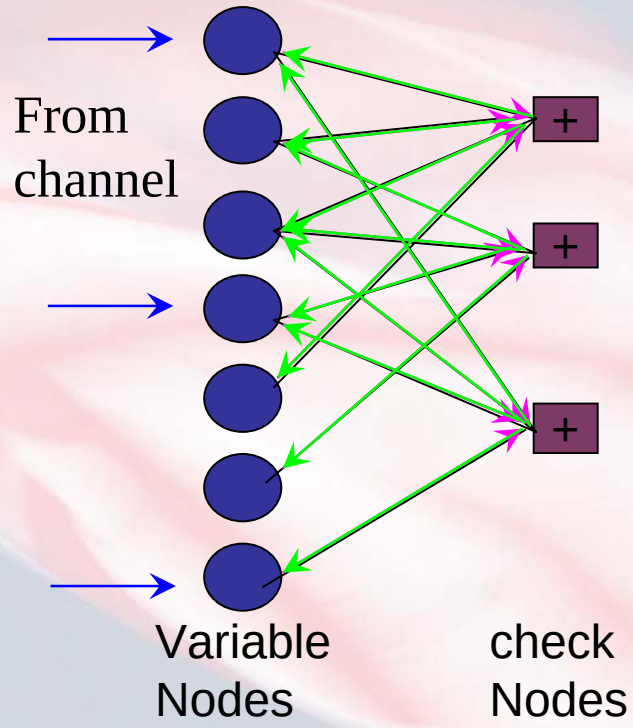
Three types of soft messages

- Channel message
- Variable to Check message
- Check to Variable message

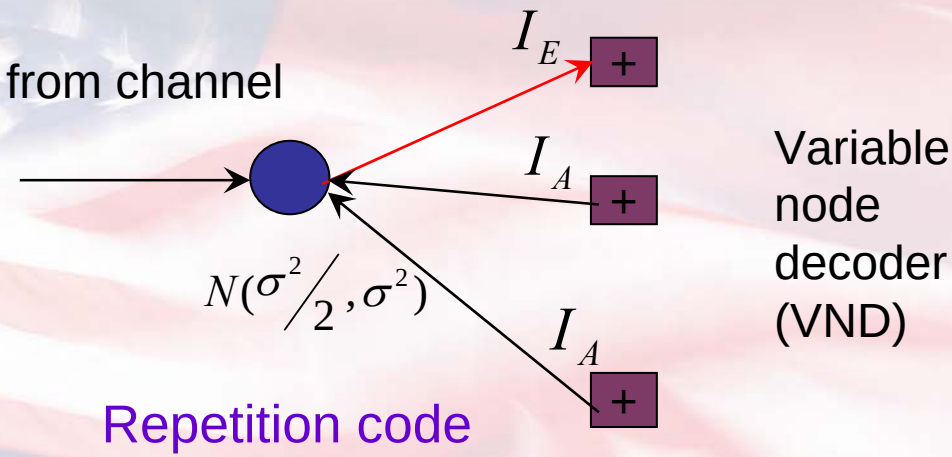
- Assumption: All messages are Gaussian distributed

$$N(\sigma^2/2, \sigma^2)$$

- EXIT function describe the relation of input message and output message in terms of mutual information. $I_{out}(I_{in})$



Variable to Check Message



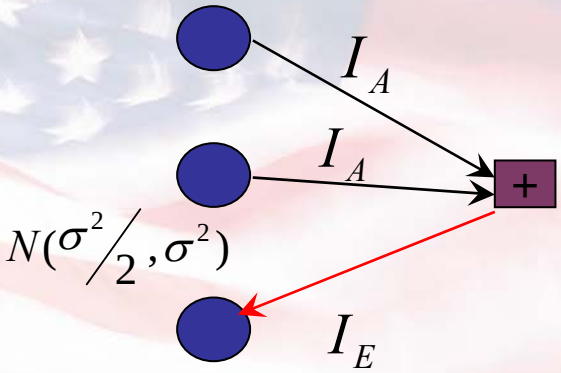
VND EXIT function:

$$I_{E,VND}(I_A, d_v, SNR) = J(\sqrt{(d_v - 1)[J^{-1}(I_A)]^2 + \sigma_{ch}^2})$$

$J(\sigma)$ is the mutual information of AWGN channel with variance σ^2

d_v is the degree of the variable node

Check to Variable Message



Check node decoder (CND)

Single parity check code

CND EXIT function:

$$I_{E,CND}(I_A, d_c) \approx 1 - J(\sqrt{(d_c - 1) \cdot J^{-1}(1 - I_A)})$$

d_c is the degree of check node

Code design using EXIT chart

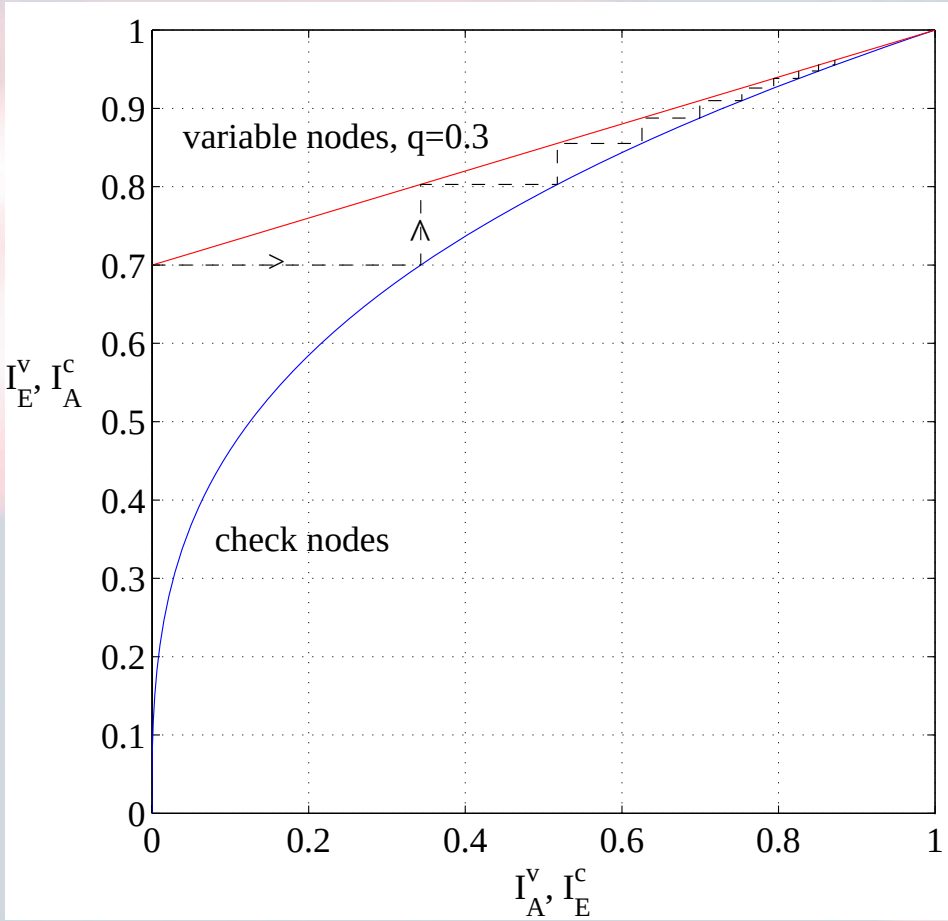
- EXIT function for MIMO Detector can be evaluated by Monte Carlo simulation
- Construct combined EXIT function of detector and VND

$$I_{E,VND/DET}(I_{A,VND/DET}, d_v, SNR)$$

- The EXIT function of irregular codes is a linear combination of the EXIT function of regular codes

Code design using EXIT chart

- To ensure successful decoding, VND/DET EXIT curve should lie above CND EXIT curve
- To approach capacity, VND/DET EXIT curve should match CND EXIT curve
- Code design can be done by curve matching



A. Ashikhmin,G. Kramer and S. ten Brink “Extrinsic Information Transfer Functions: Model and Erasure Channel Properties," *IEEE Trans. Inform. Theory*., vol.50, Nov. 2004.

Challenges of nonbinary code design

- Soft message in binary system is LLR.

$$\ln \frac{\mathrm{p}(b=0)}{\mathrm{p}(b=1)}$$

- Soft message in nonbinary system is a vector-LLRV denote the log-likelihood ratio of being one element in GF(q).

$$\mathbf{z} = \{z_0, z_1, \cdots, z_{q-1}\}$$

$$\text{where } z_i = \ln \frac{\mathrm{p}(\beta=0)}{\mathrm{p}(\beta=i)}$$

$$i \in \{0,1,\cdots,q-1\}$$

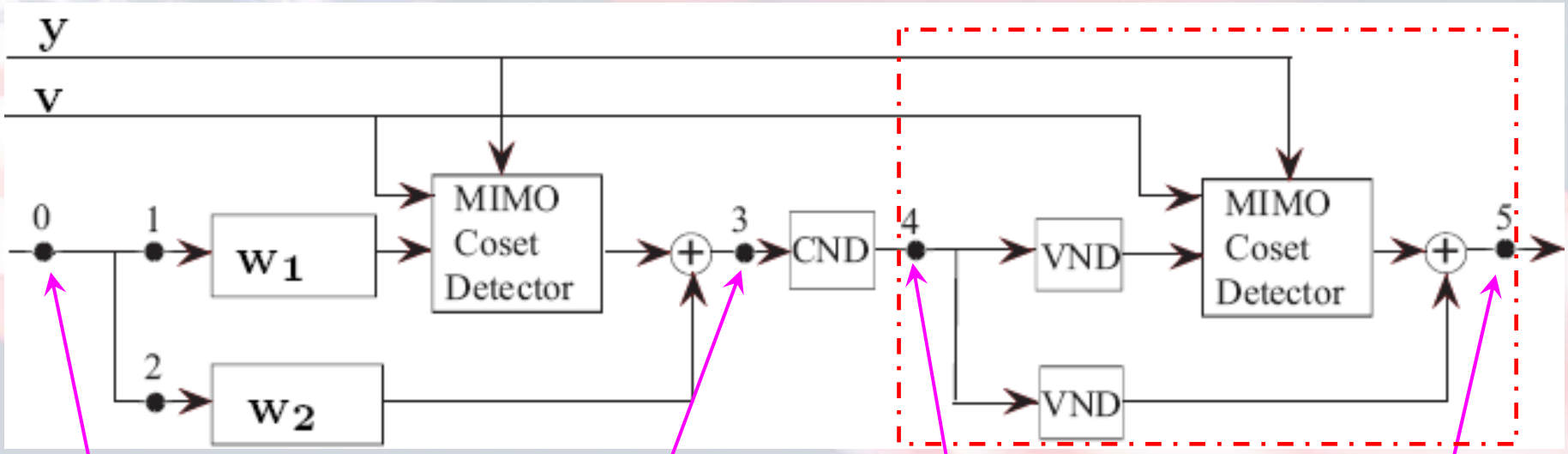
Challenges of nonbinary code design

- It is more complex to evaluate the mutual information because of the multi-dimensional integration of the soft message
- Only sum of check to variable messages can be modeled as Gaussian vector with mean ***m*** and variance **Σ**

$$\mathbf{m} = \begin{bmatrix} \sigma^2/2 \\ \sigma^2/2 \\ \vdots \\ \sigma^2/2 \end{bmatrix} \quad \boldsymbol{\Sigma} = \begin{bmatrix} \sigma^2 & & & \sigma^2/2 \\ & \sigma^2 & & \\ & & \ddots & \\ \sigma^2/2 & & & \sigma^2 \end{bmatrix}$$

- Other messages may be evaluated by Monte Carlo simulation

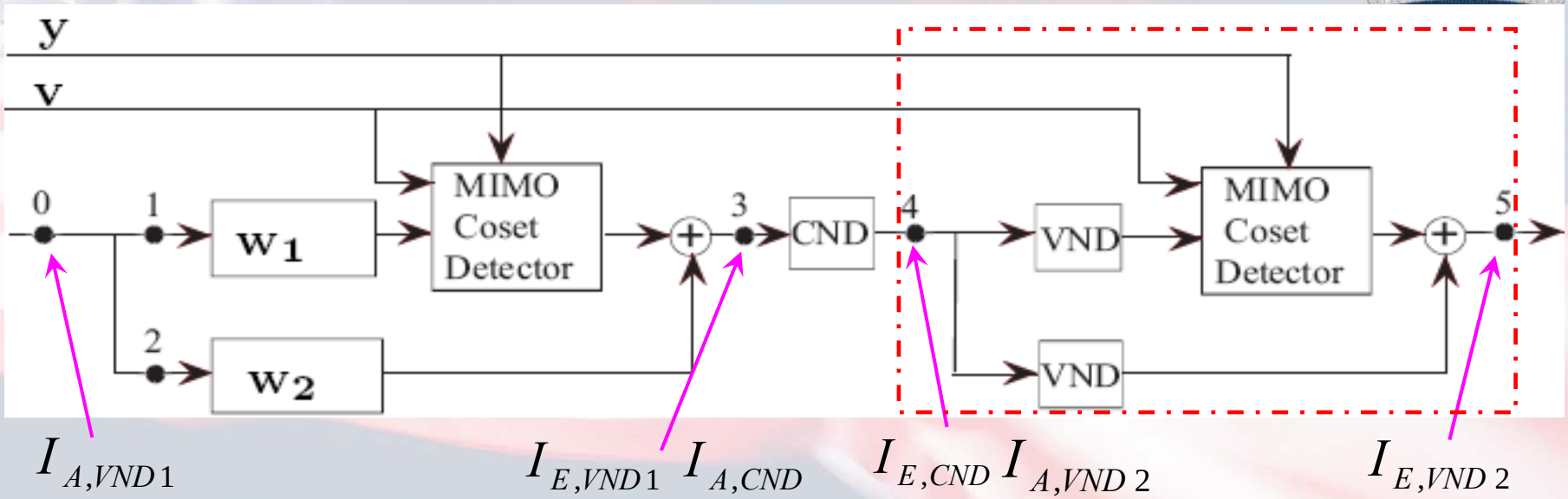
Generating of the EXIT curvrs using open-loop system



Input : d_v, d_c, σ^2, received vector $\mathbf{y}$, coset vector $\mathbf{v}$

$\mathbf{w}_1$ and $\mathbf{w}_2$ are Gaussian vector with $\sigma_1^2 = \sigma^2, \sigma_2^2 = \sigma^2 \dfrac{d_v}{d_v - 1}$. Model the sum of check to variable message

Generating of the EXIT curvrs using open-loop system



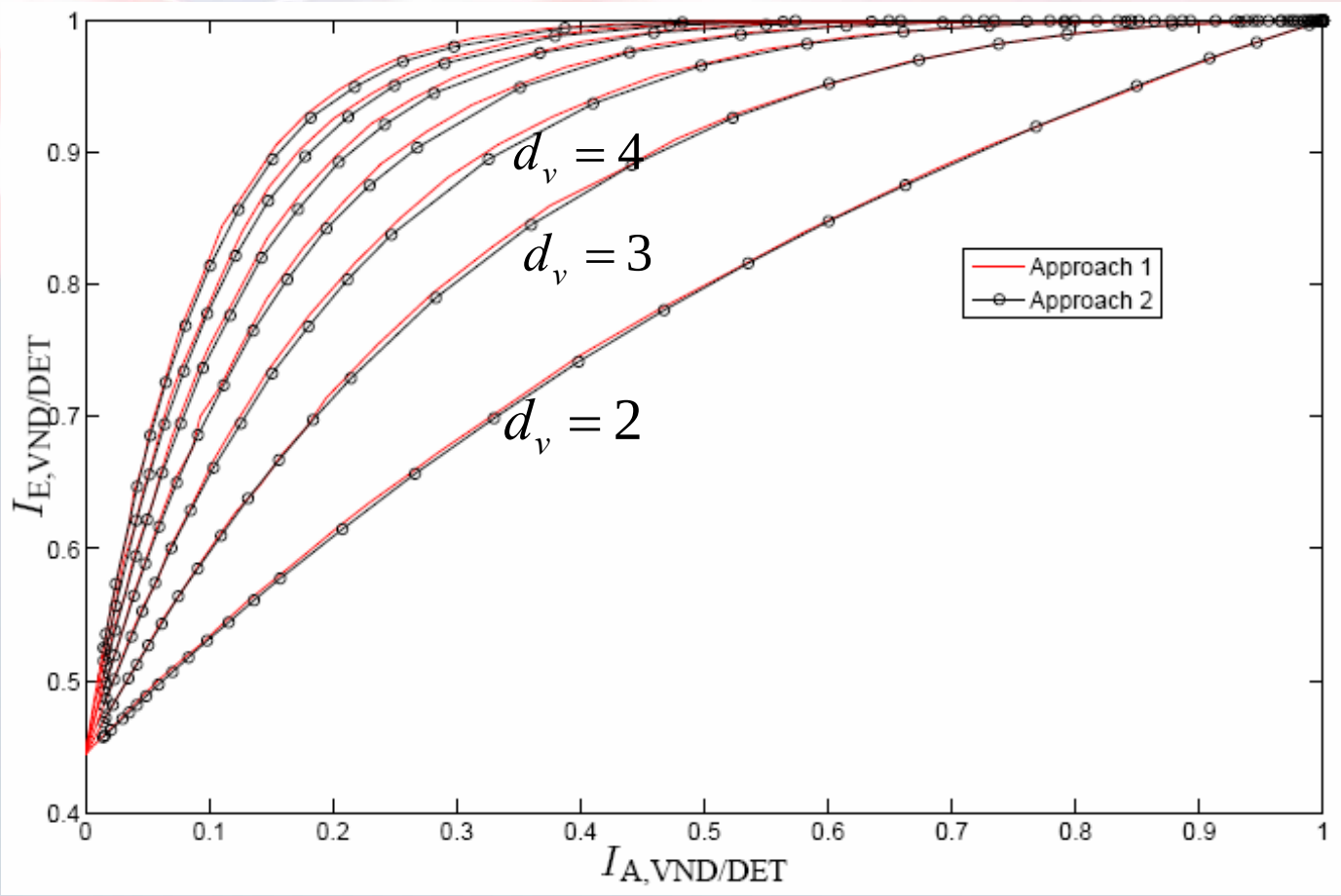
- For each (d_v, d_c) We compute:

$$I_{E,VND/DET1}(I_{A,VND/DET1}, d_v, d_c)$$
$$I_{E,VND/DET2}(I_{A,VND/DET2}, d_v, d_c)$$
$$I_{E,CND}(I_A, d_v, d_c)$$

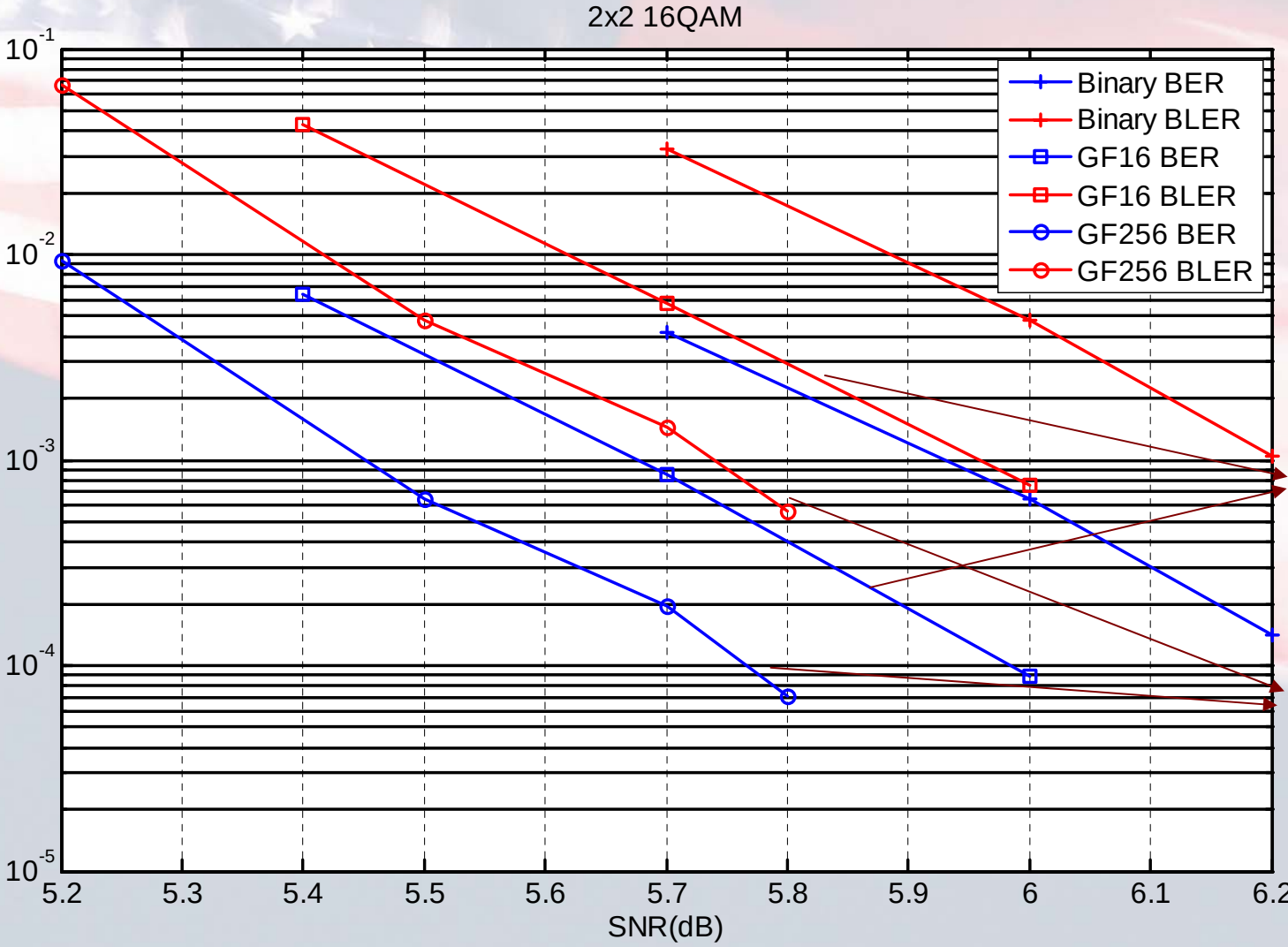
- $I_{E,VND/DET2}(I_{A,VND/DET2}, d_v, d_c)$ is more accurate, since no Guassian assuption is made.

Generating of the EXIT curvrs using open-loop system

- Comparison of two VND/DET EXIT curves



Simulation results



GF(16) lowest complexity, better performance, than GF(2)

GF(256) best performance, close complexity with GF(2)

Conclusion

- Application of nonbinary LDPC codes for iterative and noniterative MIMO system are studied
- Code design for nonbinary LDPC coded iterative system is proposed
- Nonbinary LDPC codes achieve better performance and lower complexity than the optimal binary LDPC codes and therefore are good candidate of MIMO system.
- Future work
  - Low complexity decoding algorithm
  - Code construction with linear encoding

Thanks !