Indirect Radio Interferometric Localization via Pairwise Distances

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Abstract

The Radio Interferometric Positioning System (RIPS), introduced by Maroti et. al. [1], provides a means for very accurate sensor localization with very minimal device hardware requirements. To avoid stopping in a significant number of locally optimal location solutions, RIPS employs a genetic optimization localization algorithm. This paper proposes an indirect localization algorithm which first estimates pairwise distances, and then uses them to estimate coordinates via distributed weighted multidimensional scaling (dwMDS). The pairwise distances are iteratively improved and coordinates re-estimated. While suboptimal, simulations show the proposed method can achieve 50 cm RMS location errors. The algorithm provides a tradeoff between computational complexity and accuracy, and may enable distributed RIM-based localization.

1. Introduction

Radio interferometric measurements (RIMs) have the potential to enable very accurate localization in networks of very simple wireless sensors. Maroti et. al. in [1] presented analysis, algorithms, and experimental verification for the radio interferometric positioning system (RIPS), and were able to demonstrate an average localization error of 3 cm in an experimental system. Such results are more suggestive of the accuracies expected from ultra-wideband (UWB) localization systems. In contrast, RIMs can be made with narrowband transceivers, are not subject to the strict (regulatory) transmit power limitations of UWB, and require only low-speed sampling and signal processing.

Localization algorithms which use RIMs are more complicated than those which use pairwise distance estimates for two related reasons. First, each RIM is a function of

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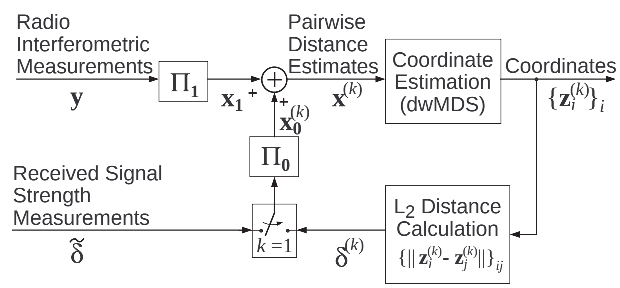

Figure 1. Flow chart of localization algorithm.

the coordinates of four sensors (two transmitters and two receivers), compared with two sensors for each pairwise distance measurement. Second, iterative optimization algorithms based on RIM data must avoid many local optima. In [1], a genetic algorithm is used to find a globally optimal solution even in the presence of these multiple local optima. While well suited for this purpose, genetic algorithms can be undesirable in terms of convergence speed.

This paper presents an alternative cooperative localization algorithm which solves indirectly for the coordinates, as shown in Fig. 1. As suggested by [2], pairwise distances are used as an intermediary in the localization process. However, pairwise distance estimation solely from RIM data is an underdetermined problem. Thus, pairwise distance space is separated into a signal space and a null space, as described in Section 2, and RIMs are used to estimate the pairwise distances in the signal space.

For the pairwise distance contribution from the null space, we propose an iterative algorithm which initializes using measurements of received signal strength (RSS). Since both RSS and RIMs can be implemented with very simple devices, requiring both does not add to device cost or complexity. After this initialization with RSS measurements, coordinates are estimated, and the null space pairwise distance contribution is re-estimated.

This iterative algorithm is described in Section 3, after Section 2 introduces the measurements. Next, the algorithm is simulated in Section 4. Section 5 concludes and discusses the significant work-in-progress.

2. Problem Formulation

This paper considers cooperative localization, *i.e.*, the estimation of the unknown coordinates of sensors $\{\mathbf{z}_i\}_{i=1}^{n}$, where $\mathbf{z}_i$ is a length-D actual coordinate vector. Reference sensors $\{\mathbf{z}_i\}_{i=n+1}^{n+m}$ have perfect *a priori* coordinate knowledge, and the total number of sensors $N = n + m$. We assume multiple radio interferometric measurements and pairwise distance estimates (obtained from RSS) are available as described in this section.

2.1. Radio Interferometric Measurements

RIMs require two sensors, i and j, transmitting simultaneously at slightly different frequencies, f_i and f_j. The combined signal exhibits a beat frequency, $|f_i - f_j|$, with a phase that is a function of receiver location. The difference in this phase at two different receiving sensors k and l is a function of the coordinates of the two transmitters and two receivers. In the absence of noise and multipath, the difference in phase measured at k and l, $\Phi_{i,j,k,l}$, is [1]

$$\Phi_{i,j,k,l} = \frac{2\pi}{\lambda}\left(d_{i,l} - d_{j,l} + d_{j,k} - d_{i,k}\right) (\mathrm{mod}\, 2\pi)$$

where $\lambda = c/f$, c the speed of light, $f = (f_i + f_j)/2$, and $d_{a,b} = \|\mathbf{z}_a - \mathbf{z}_b\|_2$ denotes Euclidean distance. It is shown in [1] that the phase ambiguity (due to the $\mathrm{mod}\, 2\pi$) can be disambiguated by performing several measurements (on the order of 10) with the same four devices at different transmit frequency pairs f_i and f_j. After this measurement and estimation process, the disambiguated phase is multiplied by $\lambda/(2\pi)$ and referred to as the four-wise distance $d_{i,j,k,l}$. Including the effects of noise,

$$d_{i,j,k,l} = d_{i,l} - d_{j,l} + d_{j,k} - d_{i,k} + \epsilon_{i,j,k,l}$$

where $\epsilon_{i,j,k,l}$ is the measurement error. Note that

$$d_{i,j,k,l} = [1, -1, 1, -1] \cdot [d_{i,l}, d_{j,l}, d_{j,k}, d_{i,k}]^T + \epsilon_{i,j,k,l}$$

2.1.1. Range Limits on Measurements: Not all sensors are 'in range', because energy limited sensors use low transmit powers. Thus measurements cannot be made between all sensors. We assume M different RIMs are made

$$\mathbf{r} = [d_{i_1,j_1,k_1,l_1}, \ldots, d_{i_M,j_M,k_M,l_M}]^T,$$

where i_p is the first node involved in the pth measurement, j_p is the second node involved in the pth measurement, etc., and that (i_p, l_p), (j_p, l_p), (j_p, k_p), (i_p, k_p) are pairs of sensors which can make measurements. Next, we denote $\mathcal{H}$ to be the set of all pairs (a, b) of sensors which are involved in the M RIMs. Finally, we denote $\mathbf{x}$ to be the vector listing all the unique 'in-range' pair-wise distances,

$$\mathbf{x} = [d_{a_1,b_1}, \ldots, d_{a_P,b_P}]^T, \tag{1}$$

where $(a_1, b_1), \ldots, (a_P, b_P)$ is an ordering of set $\mathcal{H}$, and $P = |\mathcal{H}| \leq \frac{N(N-1)}{2}$, with equality only if all nodes are fully connected.

2.1.2. Linear RIM Model: Matrix A relates $\mathbf{x}$ and $\mathbf{r}$,

$$\mathbf{r} = A\mathbf{x} + \boldsymbol{\epsilon}_r. \tag{2}$$

where A is an $M \times P$ matrix, and $\boldsymbol{\epsilon}_r$ is an error vector. Row p of matrix A is zero except for four positions: it is equal to 1 in the positions corresponding to pairwise distances d_{i_p,l_p} and d_{j_p,k_p}, and it is equal to -1 in the positions corresponding to pairwise distances d_{j_p,l_p} and d_{i_p,k_p}.

2.1.3. Number of Independent RIMs: In [4], it was proven that for a fully connected network, $\mathcal{R}\left(A^T\right) = \frac{N(N-3)}{2}$. This would be N less than P (the number of columns of A), since $P = \frac{N(N-1)}{2}$ in a fully connected network. While proofs are not available for the non-fully-connected networks, in our range-limited connectivity graphs, simulations show that $\mathcal{R}\left(A^T\right) = P - N$. Because matrix A^T is low-rank, we cannot simply estimate pairwise distances $\mathbf{x}$ directly from $\mathbf{r}$ without other constraints. For example, it can be seen from (2) that if $\mathbf{x}^{(0)}$ is a solution to (2), then $\mathbf{x}^{(0)} + c\mathbf{1}_P$, $c \in \Re$ is also a solution, where $\mathbf{1}_P$ is the P-length vector of ones.

2.1.4. Singular Value Decomposition of A: In particular, assume the singular value decomposition (SVD) of A is given by

$$A = U\Lambda V^T = \begin{bmatrix} U_1 & U_0 \end{bmatrix} \begin{bmatrix} \Lambda_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_0^T \end{bmatrix} \tag{3}$$

where U is the $M \times M$ matrix of the left singular vectors, Λ is the $M \times P$ singular value matrix, and V is the $P \times P$ matrix of the right singular vectors. We assume that A has $P - N$ non-zero singular values and N zero singular values. Assuming that singular vectors are sorted from highest to lowest singular value magnitude, U, Λ, and V are partitioned as in the right-most side of (3). Thus V_1 are the $P-N$ non-zero-valued and V_0 are the N zero-valued right singular vectors, Λ_1 is a diagonal matrix of the $P - N$ non-zero singular values, and U_1 are their corresponding left singular vectors.

2.1.5. Signal and Null Spaces: The columns of V form a basis for $\Re^P$, *i.e.*, the space of pairwise distances. In particular, V_1 spans the row (or RIM 'signal') space of A, and V_0 spans its null space. In other words, if we denote $\mathbf{x}_1^{(k)}$ to be a solution to (2), then $\mathbf{x}_1^{(k)} + V_0\boldsymbol{\alpha}_0$, for any $\boldsymbol{\alpha}_0 \in \Re^N$ is also a solution. To be more explicit, write a pairwise distance estimate $\mathbf{x}^{(k)} \in \Re^P$,

$$\mathbf{x}^{(k)} = \mathbf{x}_1 + \mathbf{x}_0^{(k)} = V_1\boldsymbol{\alpha}_1 + V_0\boldsymbol{\alpha}_0^{(k)} \tag{4}$$

where $\boldsymbol{\alpha}_0^{(k)} \in \Re^N$, $\boldsymbol{\alpha}_1 \in \Re^{P-N}$, $\mathbf{x}_1 = V_1\boldsymbol{\alpha}_1$, $\mathbf{x}_0^{(k)} = V_0\boldsymbol{\alpha}_0^{(k)}$, and the superscript (k) identifies an iteration number, to be introduced in Section 3.3.

2.1.6. Estimation of $\mathbf{x}_1$: The vector $\mathbf{x}_1$ can be well-determined from RIMs. Consider estimating $\boldsymbol{\alpha}_1 \in \Re^{P-N}$ by minimizing the squared-error cost function,

$$S_1 = \|\mathbf{r} - AV_1\boldsymbol{\alpha}_1\|^2. \tag{5}$$

Cost S_1 has a minimum at $\boldsymbol{\alpha}_1 = S_1^{-1}U_1^T\mathbf{r}$. This solution exists since $AV_1 = U_1\Lambda_1$ is full rank. As a result,

$$\mathbf{x}_1 = \Pi_1\mathbf{r}, \quad \text{where} \quad \Pi_1 = V_1S_1^{-1}U_1^T. \tag{6}$$

2.2. Received Signal Strength Measurements

Measured RSS, $P_{i,j}$, can be measured on very simple wireless devices. We assume in this paper that a model for RSS as a function of distance is known, *i.e.*, that the path loss exponent for the environment of interest is known. As a result, we talk about the distances $\delta_{i,j}$ that are estimated from $P_{i,j}$, as described in detail in [3].

Distance estimates $\delta_{i,j}$ are assumed to have very significant errors. In fact, the motivation for the proposed method assumes that RSS measurements are very noisy. We note that for the channel measured in [3], the standard deviation of $\delta_{i,j}$ is 53% of the actual distance between i and j.

We assume that RSS measurements and the associated pairwise distance estimates are available from each pair in $\mathcal{H}$. Thus the full RSS distance estimate vector is

$$\boldsymbol{\delta} = [\delta_{a_1,b_1}, \ldots, \delta_{a_P,b_P}]^T.$$

In the proposed method, $\boldsymbol{\delta}$ is used solely for the estimation of $\mathbf{x}_0^{(0)}$. A simple initial null-space distance estimate $\mathbf{x}_0^{(0)}$ can be formed by projecting $\boldsymbol{\delta}$ onto the null space of A,

$$\mathbf{x}_0^{(0)} = \Pi_0\boldsymbol{\delta}, \quad \text{where} \quad \Pi_0 = V_0V_0^T \tag{7}$$

This projection can also be interpreted as the least-squared error estimate of $\alpha_0^{(0)}$ in (4) from the RSS measurements $\boldsymbol{\delta}$.

3. Localization Algorithm

The proposed algorithm (Fig. 1) has three stages:

1. Forming an initial pairwise distance estimate $\mathbf{x}^{(0)}$ as described in Section 3.1.
2. Estimating coordinates $\{\mathbf{z}_i^k\}$ for sensors $i = 1 \ldots n$ given $\mathbf{x}^{(k)}$, as described in Section 3.2.
3. Calculating an updated pairwise distance estimate $\mathbf{x}^{(k+1)} = \Pi_1\mathbf{r} + \Pi_0\boldsymbol{\delta}^{(k+1)}$ (Section 3.3) and returning to Step 2, until the stopping criteria is met.

3.1. Initialization

A simple scheme for initialization would be to set $\mathbf{x}^{(0)} = \Pi_1\mathbf{r} + \Pi_0\boldsymbol{\delta}$ for iteration $k = 0$, as suggested by (6) and (7). However, RSS distance estimates δ tend to be biased significantly lower than the actual distance due to the biasing effect of neighbor selection [5]. In order to provide a less biased initialization point, the proposed algorithm sets $\mathbf{x}^{(0)}$ as

$$\begin{aligned} \mathbf{x}^{(0)} &= \Pi_1\mathbf{r} + \Pi_0\tilde{\boldsymbol{\delta}} \\ \tilde{\delta} &= \max(\Pi_1\mathbf{r} + \Pi_0\boldsymbol{\delta}, \boldsymbol{\delta}) \end{aligned} \tag{8}$$

where $\max(\cdot,\cdot)$ is an element-wise maximum of its two vector arguments. Equation (8) is further motivated by simulation in Section 4.3.

3.2. Estimation of Coordinates

From pairwise distance estimates $\mathbf{x}^{(k)}$, any distributed location estimator might be used to estimate coordinates. In this paper, we apply the dwMDS algorithm, a distributed sensor coordinate estimation algorithm which minimizes a non-linear weighted least-squared error cost function via majorization [5, 6]. Each sensor participates in a round by updating its own coordinate to minimize a local cost function, and communicating its new estimate to its neighbors. Each round of the algorithm is guaranteed to decrease the cost function. The complexity of the algorithm is $\mathcal{O}(NKL)$, where K is the average number of neighbors, and L is the number of iterations.

3.3. Iterative Update of Distances

An iterative update scheme is a means to improve the null space contribution $\mathbf{x}_0^{(k)}$ to the pairwise distance vector $\mathbf{x}^{(k)}$. Since estimation of coordinates from pairwise distances is overdetermined, the coordinate estimate can provide information about which distance estimates were most reliable. In the proposed method, the updated pairwise distance vector is given as

$$\begin{aligned} \mathbf{x}^{(k+1)} &= \Pi_1\mathbf{r} + \Pi_0\boldsymbol{\delta}^{(k+1)} \\ \text{where} \quad \boldsymbol{\delta}^{(k+1)} &= [\delta_{a_1,b_1}^{(k+1)}, \ldots, \delta_{a_P,b_P}^{(k+1)}]^T \\ \text{and} \quad \delta_{i,j}^{(k+1)} &= \|\mathbf{z}_i^{(k)} - \mathbf{z}_j^{(k)}\|. \end{aligned} \tag{9}$$

3.3.1. Stopping Condition: The iteration stops when either $k \geq k_{max}$ or $\mathbf{x}^{(k)}$ no longer shows increasing agreement with the RIM data. Specifically, defining the squared-error cost $S^{(k)} = \|\mathbf{r} - A\mathbf{x}^{(k)}\|^2$, stop at iteration $k-1$ if $k \geq 1$ and $S^{(k)} > S^{(k-1)}$. In the simulations in Section 4, we use a maximum number of iterations of $k_{max} = 10$.

4. Simulation Results

We simulate in Matlab the proposed localization algorithm. For each trial, we first generate for RIMs with the model that ϵ_r are independent, zero-mean Gaussian with variance $\sigma_r^2 = (0.20\text{m})^2$. Note this model is known to be unrealistic, as noise in [1] was shown to be multi-modal in distribution, and future research must evaluate the effects of non-Gaussian RIM noise. We also generate $P_{i,j}$ assuming the log-normal RSS model with channel parameter $\sigma_{dB}/n_p = 1.7$ (as measured in [3]). We assume a pair (i, j) is *connected* if $P_{i,j} > P_0$, the receiver threshold. Here, P_0 is chosen such that the nominal range is 6 meters, *i.e.*, the probability of being 'in range' at a distance of 6 meters is 50% [6]. This receiver connectedness determines which RIMs and RSS measurements can physically be measured in the sensor network, as described in Sections 2.1 and 2.2.

We consider a 10m by 10m square deployment area. In all geometries, we use $N = 25$ total sensors, and assume that $m = 4$ sensors closest to each corner of the square deployment area are reference devices. We note that execution time for one trial is about 1.6 seconds on a AMD Turion 64-based laptop running Windows XP.

4.1. Two Example Trial Runs

First, for a 5 by 5 grid of sensors, we plot a single trial in Fig. 2(a). Second, for a random deployment of $N = 25$ sensors (each was independently selected from a uniform distribution on $[0, 10]^2$), we plot a single trial in Fig. 2(b). In both figures, coordinate estimates $\mathbf{z}_i^{(k)}$ are plotted for $k = 0 \ldots 9$ and for each unknown-location sensor $i = 1 \ldots n$. Each figure shows the improvement gained through the iterative update, although errors remain in the final solution. Fig. 2(a) demonstrates something often seen in the simulation trials, that a sensor's coordinate errors are very correlated with those of its neighbors. Fig. 2(b) shows two or three sensors with large errors; other trials confirm that this problem is most likely for sensors outside of the convex hull of the reference sensors.

4.2. Estimator Mean and Covariance

Next, in the same two sensor deployments (from Section 4.1), 100 trials are run to determine the estimator mean and covariance, which are shown in Figs. 3(a) and (b). As a summary, let the mean bias $\bar{b}$ and the RMS standard deviation $\bar{\sigma}$ of the estimator be defined as $\bar{b} = \frac{1}{n}\sum_{i=1}^{n} \|\bar{\mathbf{z}}_i - \mathbf{z}_i\|$, and $\bar{\sigma} = [\text{tr}\, \mathbf{C}/n]^{1/2}$, where $\bar{\mathbf{z}}_i$ is the mean of the final estimates of sensor i over all trials, $\mathbf{z}_i$ is the actual location of sensor i, and $\mathbf{C}$ is the covariance of the coordinate estimates over all trials. For the grid geometry in Fig. 3(a), $\bar{b} = 0.10$ and $\bar{\sigma} = 0.51$. For the random deployment in Fig. 3(b), $\bar{b} = 0.15$ and $\bar{\sigma} = 0.32$. Note that the sensors near or outside of the convex hull of the reference sensors, contribute the bias and standard deviation of location error.

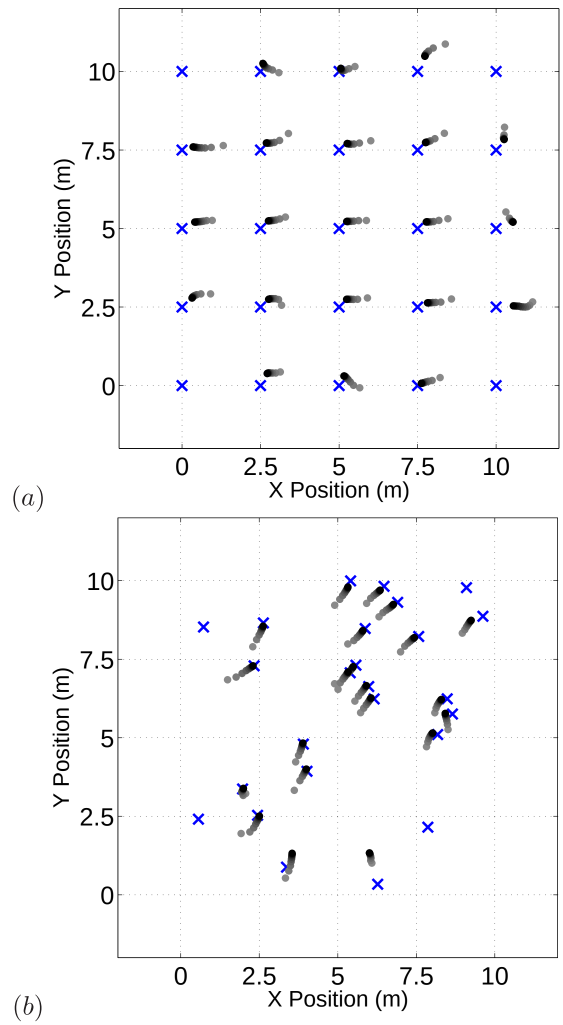

Figure 2. Actual coordinates (×) and coordinate estimates $\mathbf{z}_i^{(k)}$ (•, darker for higher k), for (a) 5×5 grid, and (b) random deployment.

4.3. Remaining Local Minima Problem

To help judge the severity of the local optima that hamper the performance of the iterative update scheme, we test two schemes to initialize $\mathbf{x}^{(0)}$ instead of using (8):

1. *Oracle*: In this clearly unrealistic case, use the actual distance vector $\mathbf{x}$, and set $\mathbf{x}^{(0)} = \Pi_1\mathbf{r} + \Pi_0\mathbf{x}$.
2. *LS Initialization*: Simply use the RSS distance vector $\boldsymbol{\delta}$ (rather than $\tilde{\boldsymbol{\delta}}$), and set $\mathbf{x}^{(0)} = \Pi_1\mathbf{r} + \Pi_0\boldsymbol{\delta}$.

The oracle initialization provides a lower bound on the best that could be done with a RIMs-based indirect coordinate optimization method. The LS initialization shows how much worse our iterative update method would perform if initialized with less accurate pairwise distance estimates.

Both alternate initialization methods are simulated in the 5 by 5 grid geometry (with other settings unchanged) for 100 trials. For the oracle initialization, $\bar{b} = 0.004$ and $\bar{\sigma} = 0.065$. For the LS initialization, $\bar{b} = 0.34$ and $\bar{\sigma} = 0.60$. Since initialization clearly has a significant impact, these results show that local minima are an issue for the proposed method. The results from LS initialization motivate the use of (8) as an initialization method. Also, better initialization may be a fruitful source of future accuracy increases.

5. Conclusion and Future Work

The proposed method achieves in simulations RMS localization errors of about half of a meter and lower. Indirect coordinate optimization, by using pairwise distances as an interim step, offers a useful tradeoff between computational complexity and accuracy; and may in the future enable a fully distributed RIM-based localization estimator.

Outlier elimination may be a key strategy for improved performance. From simulation experiences, large RSS errors can 'push' a sensor far away from its actual coordinate, and the iterative update may not be able to recover. The dwMDS algorithm includes an adaptive mechanism in which pairwise distance estimates that appear too low or too high are ignored [5]. Possibly, the coordinate estimation algorithm could provide side information on which distances are least reliable so that their effects can be negated.

Full distribution of the algorithm will require significant future work. The dwMDS is already fully distributed. The vector projections required by the subspace projection operations can be computed quickly by randomized gossiping [7]. However, fully distributed calculation of the projection matrices will be required. These may be formulated as least-squares problems involving matrix A, which has significant local structure – A is zero for pairs involving non-neighbors. Distributed calculation will be less expensive than for randomly structured sparse matrices. For large N, a hybrid approach might be used in which subsets of sensors estimate their maps separately and then fuse them together.

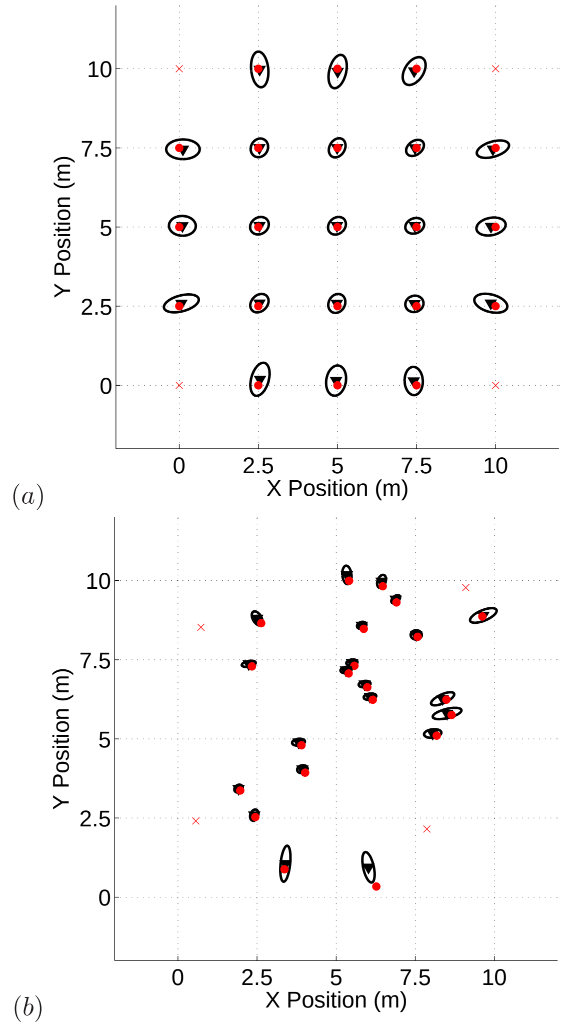

Figure 3. Estimator mean (▼) and 1-σ uncertainty ellipses (◯) compared to actual locations (•) and reference devices (×), for (a) 5×5 grid, and (b) random deployment.

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