Fractional-Order Signal Processing using a Polymer-Electrolyte Transistor

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Abstract— **Fractional-order systems have applications in the areas of Flight Control, Robotics, Missile Guidance, Control of Structural Vibrations of Space Platforms and Sensor Technology. Fractional-order transfer functions can characterize complex nonlinear dynamics with many fewer parameters than integer-order functions. This paper addresses the use of a polymer-electrolyte transistor (PET) for use in implementating fractional-order algorithms for signal processing. The PET's advantage over the conventional RC and RL circuits is that it can be both functionally scaled and varied for dynamic fractional-order parameter controllability.**

Keywords: Fractional-Order, Signal Processing, Polymer-electrolyte transistor, acid/base chemistry, diffusion, electrokinetic flow.

1. INTRODUCTION

Calculus students become quickly familiar with the various higher-order derivatives of a function *f*: $f', f'', f'''\ldots$ where $f^{(n)}$ denotes the n^{th}-order derivative. Must *n* always be an integer? Surprisingly, no! Certain functions can be considered to have *fractional*-order derivatives [1]. Consider sin(*x*). Repeated differentiation gives this general formula for the n^{th} derivative: $D^n \sin(x) = \sin(x + n\pi/2)$. Here one can freely substitute non-integer values for *n* to achieve a derivative of any fractional order. Conversely, one can also speak of fractional integrals.

This obscure branch of mathematics is indeed useful. Transfer functions based on fractional calculus have proven ideal for characterizing complex nonlinear dynamics [2,11]. Podlubny et al. have built fractional order controllers using RC and LC networks [2]. However, their operation depends on near-zero-loss dielectrics, which are difficult to achieve.

Rather than seeking extremely low-loss dielectrics, we propose instead to seek materials showing the desired fractional-derivative response. For example, a material with loss modulus nearly equal to the storage modulus over a large frequency range would yield a device exhibiting a ½-order derivative [3].

We are investigating novel, ionic/organic transistors that, not only have good potential for fractional order behavior, but also enjoy other advantages over conventional electronics.

Theses devices use charge carriers such as electrolytes to conduct a signal. In contrast, digital electronics employs only one signal carrier, *i.e.*, the electron. Current and voltage signals are simply different views of electron flow. Even the positive "hole" flow found in semiconductors consists simply of electrons moving in reverse, and may be modeled as such. Ionic devices, however, will employ multiple signal types. In addition to electric and light signals, there are ions and molecules that serve as signals by participating in chemical reactions. Such species have unique behavior that varies with chemical context. Unlike electrons, they must maintain their unique identities when modeled.

Organic devices are less robust than semiconductors, able to endure fewer environmental extremes. This fragility can translate into less reliability, and necessitate redundancy, dimensional scalability and error-correction techniques on a scale greater than in silicon processing.

This paper focuses on examining the fractional dynamic response of the LSL polymer-electrolyte transistor (PET) [5], utilizing the modeling program COMSOL Multiphysics.

II. Fractional-Order Signal Processing

Fractional transfer functions can characterize complex nonlinear dynamics with only a few parameters, in contrast to integer expressions, that even with ten times as many parameters, still do not fully model the physical phenomena.

A. Equivalent Half-Order Models. Many fractional algorithms and processes are modeled using LC and RC circuits. In the proton exchange membrane fuel cell (PEMFC) [3], fractional controllers will be used to control these complex systems. The dynamic model of PEMFC, taken from Iftikhar [3], uses non-integer derivatives to model diffusion phenomena. The fractional order model has the advantage of having least number of parameters while being valid on a

wide frequency range. This allows simulating an accurate dynamic response of complex diffusion systems [6].

B. Fractional-order Modeling. We have realized that half-order derivatives and integrals can lead to 3-D physical geometric designs that can be modeled using the COMSOL Multiphysics program. Once we determine the "best-fit" half-order expressions from simulations, future designs will require only a parametric gain constant variation between rectangular and spherical coordinates. These expressions will be more functional and less complex than Fick's laws of diffusion [7]. There are several reasons for the 3-D modeling, and hence the complexity of the calculations. The first is in the attempt to increase the accuracy and complexity of the representations of the physical conservation laws. For example, in chemical models, increasing the number of chemical species increases the number of dependent variables. The obvious need for increased independent variables comes from the need to represent sensor phenomena in two and three spatial dimensions.

However, even higher dimensional problems arise when the independent variables are not the spatial coordinates but are various state descriptions; such as higher-dimensional problems common in physics and chemistry. Unfortunately, numerical methods that work well in one or two dimensions often are unusable in three dimensions. Therefore, increasing the degrees of freedom related to 3-D modeling requires non-integer derivation, where it has already been used to correctly model the diffusion phenomenon of magnetic field in electrical machines [2]. The resulting parameters of such non-integer order models have a close link with the physical characteristics of the system and are precise, having fewer parameters and being valid on a wide frequency range.

III. PET Model Definition [5]

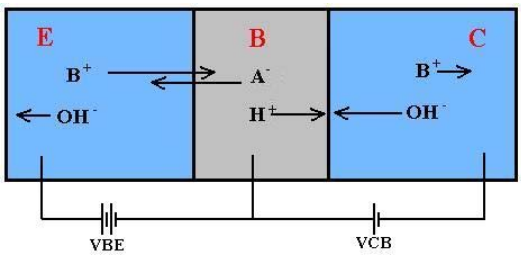

Figure 1. A basic PET schematic showing electrolyte migration through the emitter "E", base "B", and collector "C" under the effect of a voltage potential[5].

The polymer-electrolyte transistor has a physical design similar to a basic bipolar junction transistor. In the PET, three separate source regions of ions are connected by a polymer gel. This gel acts as a medium through which the ions are allowed to diffuse, but there is no direct convective path between the three source regions [8]-[9].

Fig. 1 depicts the basic function of the PET, showing the device biased such that VE>VB>VC. The ionic species present are the alkaline BOH and the acidic HA, where B^+ is a cation and A^- is an anion. With the voltage potentials as depicted, the EB region is forward biased and the BC region is reverse biased. In a forward-biased junction, the anion and cation flow into the middle region, and allow for ionic current to flow. A reverse-biased junction is one where the H^+ and OH^- ions flow into the junction and recombine, leaving very few ions to conduct a signal [8]. With the cation flow from the emitter, though, there will be a cation concentration in the BC junction that will contribute to the total current.

A. Governing Equations. Ionic diffusion in the PET is described by Fick's first law of diffusion for each ionic species[7]. This, combined with the flow of ions under an electric field, produces the electrokinetic flow equation, where the first term governs diffusive flux, and the second term governs ionic flux under an electric field:

$$\nabla \cdot (-D\nabla c - z\mu_m Fc\nabla V) = R \qquad (1)$$

Where D is the diffusion coefficient (m^2/s), c is the concentration of the given ionic species (mol/m^3), z is the charge number (unitless), μ_m is the ionic mobility ($mol/m \cdot s$), F is Faraday's constant (C/mol), V is the electric potential (V), and R is the reaction, or source, term ($mol/m^3 \cdot s$).

When initially determining the system of equations to govern this system, the equation for current flow due to electrons in a semiconductor material was investigated. In a semiconductor device, flux due to diffusion and drift is shown below:

$$J = qD_n\nabla n + q\mu nE \qquad (2)$$

Where J is the flux of electrons (A/m^2), q is the elementary charge (C), n the density of electrons ($1/cm^3$), E the electric field (V/m), and all other units the same as in Eqn. 1 [8]. Interestingly, manipulation of this equation to apply to electrolytes moving under an electric field, where the units of flux are $mol/m^2 s$, simplifies to the equation system of equation (1).

To set up the voltage potentials in the system, COMSOL's conductive media DC application is used. The governing equation for this system, shown below, is simply Ohm's law in differential equation form:

$$-\nabla \cdot d(\sigma\nabla V) = dQ_j, \qquad (3)$$

where d is the thickness (m), σ is the conductivity (S/m), V is the voltage (V), and Q_j is the current (A/m^3).

B. Fractional Control. The PET has fractional-order characteristics that are controlled by concentration, In this case we are using the alkaline source at the emitter electrolyte, BOH, as the fractional controller.

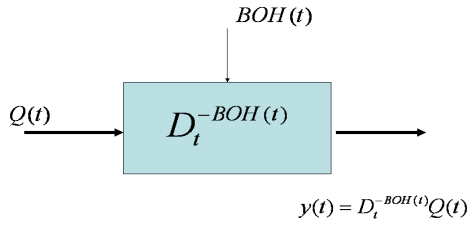

Figure 2. Block diagram for transfer functions of variable hybrid fractional PET structure integral.

Consider the fractional integral equation[1]

$$y(t) = D_t^{-BOH(t)}Q(t) \qquad (4)$$

and the inferred differential equation

$$Q(t) = D_t^{BOH(t)}y(t) \qquad (5)$$

The concentration *BOH* in the fractional calculus can take on any real (or complex) value. The question is asked, *What is a desirable definition for the fractional integral when BOH is allowed to vary either with t* or *y*? More specifically, *What is an appropriate model definition for Equation 6?*

$$D_t^{-BOH(t)}Q(t) \qquad (6)$$

From Equation 6, fractional analysis provides the key for telling us how nonlinear systems respond to prescribed changes. The non-integer derivative of the physical quantity $Q(t)$ can be manipulated algebraically and the results can be interpreted to provide information about the physical processes involved in the flux equation (Eqn. 2). The fundamental quest of fractional analysis is this: *If physical phenomena are dimensionally homogenous, the flux equation can be reduced to a relationship among non-integer products of system variables and fractional parameters.* The fractional products should be graphically interpreted from the 2-D multi-physics modeling. The term $D_t^{-BOH(t)}Q(t)$ does not depend on the fundamental units of measurement, but on differential non-integer rate-of-change, somewhat similar to the hybrid-pi models parameters used in electronics. Fig. 3 shows the envisioned hybrid PET device as a fractional-order feedback device for an Op-Amp. The differential equations involving fractional-order derivatives provides the $(1/n)^{th}$ fractional derivative, and can be designed and fabricated to cover the frequency range and functional type required by a specific application [10,11].

[1] A integral is simply a negative-order derivative.

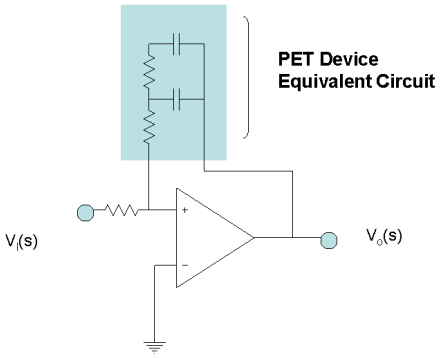

Figure 3 Half order derivative that will be implemented by the PET for variable hybrid control

This poses the following questions of: What fractional-order forms a complete model set for the PET device equivalent circuit model of Fig. 3? How is the fractional-order controlled? These questions will be answered by looking at the following PET simulation example [5].

C. PET 2-D Model. The 2-D model created in COMSOL Multiphysics consists of three source regions of electrolytes, separated by a polymer, shown in Figure 4. As modeled in 2D, the device is 4.5 mm long in the x-direction, by 2.3 mm wide in the y-direction.

D. Varied Electrolyte in the Base Region. Injection of charge carriers into the base region of the PET results in a noticeable change in the steady state currents in Fig. 4. This was accomplished by adding additional salt electrolyte concentration to the base region, and observing its effect on the total current change from initial to steady state. Voltage values in the device were held constant at VE = 3.5 V, VB = 2.0 V, and VC = 0V.

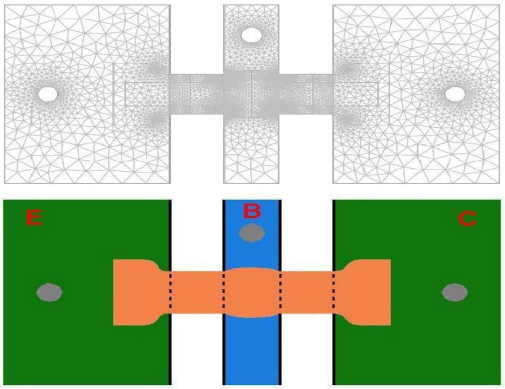

Figure 4. PET 2-D Model showing the mesh in COMSOL (top) and the different regions colored for clarity (bottom).

From Fig. 4, the currents through the BC region of the device grow more positive with additional salt, while the currents in the EB region become more negative. This property of the

PET to add additional charge carriers to specific regions can be exploited to modify the fractional characteristics of the device, and is relatively simple to accomplish.

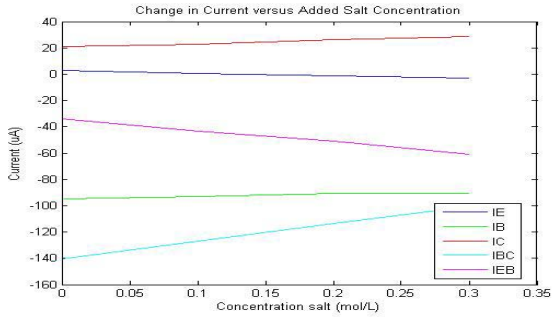

Figure 5. Steady-state current plot for the different regions of the device versus an increase in the concentration of salt electrolyte added to the base region of the PET.

E. Emitter Region "Doping". The concentration of the alkaline source in the emitter of the PET was varied from 0.1 M to 0.4 M, and the transient current response of the device was plotted for the BC and the EB channel. From Figs. 5 and 6, the currents show an inverse relationship to the increased emitter doping. For increasing doping values, the BC channel current becomes more negative, while the EB channel current grows more positive.

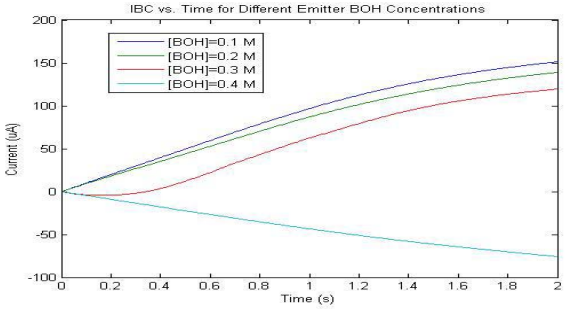

Figure 6. Plot of current in the BC channel versus time for different concentration doping of the alkaline source at the emitter.

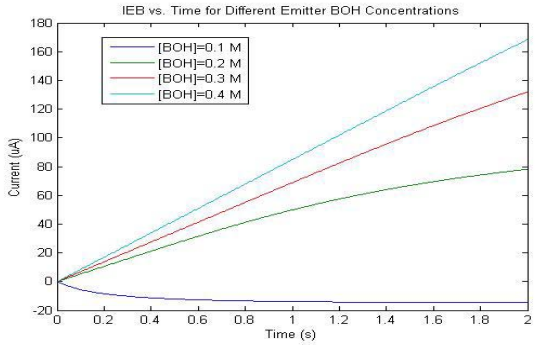

Figure 6. Plot of current in the EB channel versus time

IV. Summary

The three dimensional diffusion equation was simulated for the ionic Polymer-Electrolyte Transistor (PET) to determine fractional-order model characteristics. Fractional-order analog capacitive element circuits have been addressed by many researchers, but the development of a variable controlled non-integer fractional-order device, such as the PET, was not reported in the literature before this article. The PET material parameters depend on thermal, ionic, electric and magnetic fields, where the multi-physical equations are nonlinear, allowing its use as a "variable" fractional-order feedback controller element for hybrid signal processing. Determination of the fractional-order parameters for the PET can be determined by the use of the COMSOL simulation package.

V. References

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