Linear Assignment Problem

Given n positions on which to place m modules $(n \geq m)$.

$$x_{ij} = \begin{cases} 1 & \text{if module } i \text{ is assigned to position } j \\ 0 & \text{otherwise} \end{cases}$$

A_{ij} = affinity of module i to position j

Optimization problem:

$$Minimize \quad \sum_{i,j} A_{ij} x_{ij}$$

subject to:

$$\begin{cases} \Sigma_j \, x_{ij} = 1, & i = 1, ..., m \\ \Sigma_i \, x_{ij} \leq 1, & i = 1, ..., n \\ x_{ij} = 0, 1 \end{cases}$$

Polynomial time solutions exist [$Hun80$]

Quadratic Assignment Problem

C_{ij} = connection count between modules i, j

D_{ij} = distance between positions i, j

If module i is assigned to position k and module j is assigned to position l, then the assignment cost is $C_{ij}D_{kl}$.

Optimization problem:

$$Minimize \quad \sum_{i,j} \sum_{k,l} C_{ij} D_{kl} x_{ik} x_{jl}$$

subject to:

$$\begin{cases} \Sigma_j \, x_{ij} = 1, & i = 1, ..., m \\ \Sigma_i \, x_{ij} \leq 1, & i = 1, ..., n \\ & x_{ij} = 0, 1 \end{cases}$$

The problem is NP-hard [$Gar72$].