HW #2 Solutions

\[ D = 1/0 = \text{fault-free} \]

(a) Test for \( x \) stuck at 0.

Fault excitation \( x = 1 \Rightarrow a = 1 \Rightarrow x = 1/0 = D \).

Propagate \( x \rightarrow p \rightarrow z \).

Side input non-controlling values. \( Q = 1, R = 1 \).

Justify. \( R = 1 \Rightarrow c = d = 1 \).

\[ a = 1 \Rightarrow y = 1 \]. for \( Q = 1 \), we need \( b = 0 \).

but \( b = 0 \Rightarrow p = 1 \). propagation is blocked at \( p \).

\( \Rightarrow \) No Test for \( x/0 \).

(b) Along the same lines. \( y/0 \) has no test.

\( y = 1 \Rightarrow a = 1 \Rightarrow b = 1 \Rightarrow p = D \Rightarrow z = 1 \)

Fault propagation is blocked at \( p \) & \( \overline{Q} \).

No test for single faults \( x/0, y/0 \).
(c) Test for multifault \( (x/0, y/0) \)

\[
\begin{align*}
\text{multipath } a \rightarrow x \rightarrow p \rightarrow z \\
a \rightarrow y \rightarrow q \rightarrow z \\
\end{align*}
\]

\( x/0, y/0 \) \Rightarrow q = 1 \text{ for excitation.} \\
x = y = f = D.

\( a = 1, b = 1 \) \quad P = D \quad \overline{q} = \overline{D} \\
\overline{c} = \overline{d} = 1 \quad R = 1 \\
\overline{D} = D \quad z = D.

\text{fault propagates.}

\[ T(x/0, y/0) = \{ a, b, c, d = 1111 \} \]

(c) Single faults are untestable \( \rightarrow \) redundancy. Multi-faults are testable. In the presence of a redundant fault, \( (x/0) \), another redundant \( (y/0) \) fault becomes testable. This invalidates our single-fault testability assumptions. Under a frequent-testing strategy, if any fault appears, we'll be able to test it.
Q2 (a) Test for \( b/1 \)

\[ \begin{align*}
A & \quad \text{require} \quad c = 0, \quad b = 0/1 = D \\
\text{Side-inputs} \quad A = B = 1. \quad \text{But} \quad P = 1 & \Rightarrow Z_2 = 1
\end{align*} \]

\text{Fault propagation blocked at } Z_2. \quad \text{No Test } b/1.

(b) Distinguish between (fault effects) \( a/0, c/0 \). 

\[ \begin{align*}
A = 1, \quad B = 1, \quad a = D, \quad p = D. \\
\text{P \rightarrow Z_1 \quad two outputs} \\
\text{P \rightarrow Z_2 \quad to observe fault effect} \\
c = 0 & \Rightarrow Z_1 = 1 \quad \text{blocks fault @ Z_1} \\
\text{but } c = 0 & \Rightarrow q = 0.
\end{align*} \]

\[ \begin{align*}
q = 0, \quad p = D & \Rightarrow Z_2 = D \\
\text{also} \quad c = 1, \quad A = 1, \quad B = 1 & \Rightarrow g = 1 \Rightarrow Z_2 = 1 \quad (\text{blocked}) \\
\text{But} \quad c = 1, \quad p = D & \Rightarrow Z_1 = D \quad (\text{observed})
\end{align*} \]
So \( T_9 \% : \) \( \{ \begin{align*} & \text{A}=1 \quad \text{B}=1 \quad \text{C}=0 : \text{Fault observed \( @ \text{Z}_2 = D \)} \quad \text{2 test vectors.} \\
& \text{A}=1 \quad \text{B}=1 \quad \text{C}=1 : \quad \text{1 \( @ \text{Z}_1 = D \) } 
\end{align*} \)

\[ T_{C9} \% : \]

\( C \) \quad \text{\( \Rightarrow \) \( \)} \quad \text{\( \text{Z}_1 \)}

\( A \quad \text{\( \Rightarrow \) \( \)} \quad \text{\( \text{C} \)}

\( A \quad \text{\( \Rightarrow \) \( \)} \quad \text{\( \text{Z}_2 \)}

\( \% \text{ can only be observed \( @ \text{Z}_1 \)} \)

\( \% \text{ \( \Rightarrow \) } \text{C}=\% \text{ } \Rightarrow \text{A}=\text{B}=1 \)

\( C=1, \text{C}=\% \text{ } \Rightarrow \text{Z}_1=\% \text{ = D. } \)

\( \text{A}=1 \text{ B}=1 \text{ C}=1 : \text{Fault effect observed \( @ \text{Z}_1 = D. \) } \)

\( \text{A}=1 \text{ B}=1 \text{ C}=1 : \text{Common test vector, for } \% \text{, } \% \).

\( \text{w/ } \% \text{ A}=\text{B}=\text{C}=1 \quad \text{w/ } \% \text{ A}=\text{B}=\text{C}=1 \)

\( \text{Z}_1=\text{D, Z}_2=1 \quad \text{Z}_1=\text{D, Z}=1 \)

\( \text{Same outputs under the same & only common test vector. } \)

\( \text{Cannot distinguish their effects. } \)
\[ a \]

\[ T \equiv \frac{a}{c}, \ \frac{b}{c} \]

\[ C = 0 \]

\[ A = 1 \]

\[ B = 1 \]

\[ \text{Excitation } A = 1 \]

\[ \text{Excitation } C = 0. \rightarrow Z_1 = 1 \]

Focus fault observation only at \( Z_2 \)

Fault propagation requires \( B = 1 \)

\[ \Rightarrow P = \frac{1}{2} = 0 \]

\[ Q = 0 \]

\[ Z_2 = D + D = 1 \]

No Test \( \frac{a}{c}, \frac{b}{c} \)
Finding all equivalent faults, pick only one from each equivalence class.

For gate $G_1$: \( \alpha'_1 = e'_1 = \bar{d}/0 \)  
\( G_2: f/0 = g/0 = k/1 \)  
\( G_3: j/1 = m/1 = p/1 \)  
\( G_4: n/0 = h/0 = q/0 \)  
\( G_5: q/0 = d/0 = r/1 \)  
\( G_6: p/0 = r/0 = z/1 \)  

From the first column, \( \alpha'_1, f/0, j/1, n/0, q/0, p/0 \) \( \rightarrow \) 6 faults

I'll keep faults from 1-6 sets, pick any one equivalent fault, disregard the rest.

If I test for \( p/0 \), I cannot distinguish it from \( n/0 \& z/1 \), so drop \( p/0, z/1 \).

B. Checkpoints of the ckt: PIs + Fanout branches

PIs: \( \alpha/0, a/1, b/0, b/1, c/0, c/1, d/0, d/1 \) = 8 faults.

FOB: \( e/0, e/1, f/0, f/1, g/0, g/1, h/0, h/1, m/0, m/1, n/0, n/1 \) = 12 faults.

Total ckt faults: \( 12 + 8 = 20 \)
Checkpoint theorem states that it is sufficient to test for only the checkpoint faults. 
No need to test for other faults.
All checkpoints tested \(\Rightarrow\) the whole CRT is tested.

But, it is not necessary to test all checkpoints faults. It might be possible to test for fewer than the checkpoint faults, in our case \(<20\) tests needed, as shown in (c).

(c) In (a), we have already found all equivalence relations. Now we find dominance relations:

\begin{align*}
G_1 & : j/1 > a/o, e/o \quad -7 \\
G_2 & : k/o > f/1, g/1 \quad -8 \\
G_3 & : b/o > j/o, m/o \quad -9 \\
G_4 & : q/1 > n/1, h/1 \quad -10 \\
G_5 & : y/o > q/1, d/1 \quad -11 \\
G_6 & : z/o > y/1, r/1 \quad -12
\end{align*}
Note. \( e/1 \) is a checkpoint fault.

But from (1) \( e/1 = 9/1 \).
\[ 9/1 \rightarrow 9/1. \] So if I test \( 9/1 \), I also tested \( e/1 \). So remove \( e/1 \) from the test set.

- 17 faults

Also combine equiv + domin. relations.

From (3) \( m/1 = 9/1 \).
\[ \text{From (7) } 9/1 \rightarrow 9/0. \]

So if I test \( 9/0 \), \( 9/1 \) is tested (due to dominance) and \( m/1 \) is also tested. (equiv. \( = 9/1 \))

So, remove \( m/1 \) from the checkpoint fault list.

- 18 faults

Also, from (5) \( d/0 = 9/0 \). 3. \( d = 9/1 \). If I test

\[ 1 \quad 4 \quad 9/0 = 9/0. \]

\( d/0 \), I also test \( 9/0 \).

- Remove \( 9/0 \) from cut branch fault from test set.

- 17 faults

But: \( 9/0 = h/0 \). So, remove \( h/0 \) too. = 16 faults

- \( f/0 = 9/0 \). 2. So remove \( 9/0 \). \( \rightarrow \) 15 faults.

Final list: \( a/0, a/1, b/0, b/1, 9/0, 9/1, d/0, d/1 \) minimal set of

- 15 tests
**Fig. 3:** The circuit diagram related to Checkpoint faults

\[ k = 0 \rightarrow k = 3 \]

**multipath:**

\[ k \rightarrow m \rightarrow p \rightarrow z \]

\[ k \rightarrow n \rightarrow q \rightarrow r \rightarrow z \]

\[ P = \overline{D}, \quad q = \overline{D}, \quad d = 1 \]

\[ \Rightarrow r = D \]

\[ z = 1 \]

Try single path: \[ k \rightarrow m \rightarrow p \rightarrow z \]

Put \( d = 0 \), \( r = 1 \) @ Gates

\[ k = \overline{D}, \quad b = e = 1, j = 0. \]

\[ m = \overline{D}, \quad p = \overline{D}, \quad z = D, \quad r = 1 \]

\[ q = 0 \text{ or } 1 \}

\[ b = 1 \]

\[ c = 1 \]

\[ d = 0. \]

\[ m = 0, \quad m = 0, \Rightarrow m = 0 \rightarrow D \]

\[ k = 0. \]

\[ f = 1, \quad g = 1 \Rightarrow b = 1, c = 1 \]

\[ d = 0. \]

\[ a = 0, \quad b = 1, c = 1, d = 0. \]

\[ z = D \]

Same as \( K/1 \).
\( n/1 \), \( n=0 \), \( \overline{m} = \overline{n} = 0 \)

Only single path possible.

\( n \rightarrow q \rightarrow r \rightarrow z \).

\[ h=1 = c=1 \rightarrow k=0, \overline{m}=0 \]

\( b=1 \)

\( m=0 \).

\( \downarrow \)

\( d=1 \)

\( b=1 \Rightarrow c=1 = j=0, \overline{m}=0 \).

\( \Rightarrow p=0 \rightarrow z=1 \)

No test.

\( n/1 = \text{redundancy} \).

Put \( n=1 \) & simplify.
Q5 PI = fanout free, but internal fanouts may exist.

Given test set $T$ that detects all $S/\Delta$ faults. Does it detect all $S/\Delta$ faults too? Yes, proof –

Any net is either:
1. Primary input = gate input, e.g., $a'$.
2. Fanout stem = gate output, e.g., net $x$.
3. Fanout branch = gate input, e.g., net $y$.
4. Non-fanout gate output, e.g., $f$.

Nand input $S/\Delta$ = output $S/\Delta$
If output $S/\Delta$ tested, gate input $S/\Delta$ tested.
$\Rightarrow$ all PI $S/\Delta$ tested, e.g., $a\% = b/\Delta$
and all fanout branches tested. $y/o = b/\Delta$

This takes care of 1 and 3.

Nand Gate output $S/\Delta$ dominates gate input $S/\Delta$
$\Rightarrow x/o > b/\Delta$ $T$ detects $b/\Delta$ (given)
So $T \uparrow x/o$ (given)

This takes care of 2 and 4.

All $S/\Delta$ also tested.
\[ \frac{\partial F}{\partial x} = F_x + F_x \]

Those conditions that allow changes in \( x \) to be visible at \( F \) \( \Rightarrow \) fault propagation condition.

So \( \frac{\partial F}{\partial x} \) - conditions that disallow changes in \( x \) to be visible at \( F \) \( \Rightarrow \) observability D.C.

\[ F = h + ac + bc \]

\[ F_h = F(h=1) = 1 + ac + bc = 1 \]

\[ F_{h'} = F(h=0) = ac + bc \]

\[ \frac{\partial F}{\partial h} = ac + bc = \text{observability D.C. of } h. \]

When \( ac = 1 \), changes in \( h \) are blocked at OR gate.

\( h = a \cdot b \) (case set). \( h_{DC} = ac + bc = D.C. \)

\[ \begin{array}{c|c|c|c|c}
   T \backslash \text{map} & 0 & 0 & 0 & x \\
   \hline
   0 & 0 & 0 & 0 & x \\
   1 & 0 & x & 1 & x \\
\end{array} \]

\( X \) overrides the on set. Put \( x = 0 \), \( h = 0 \)

\[ F = ac + bc \]

\( h = \text{redundant}. \)
87 was a free gift experiment.