

Chapter 8

Optimized Implementation of Logic Circuits

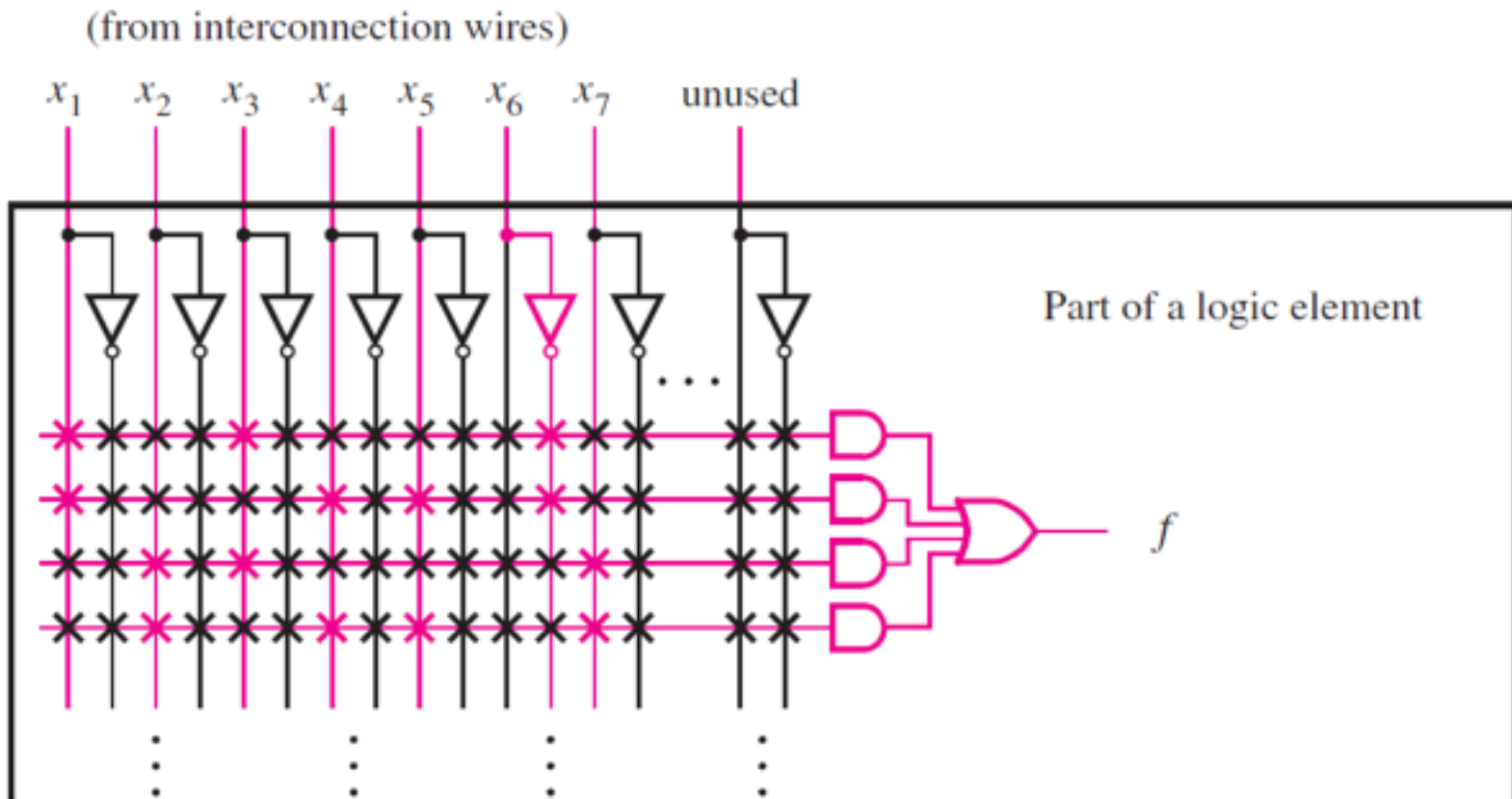


Figure 8.1. Implementation in a CPLD.

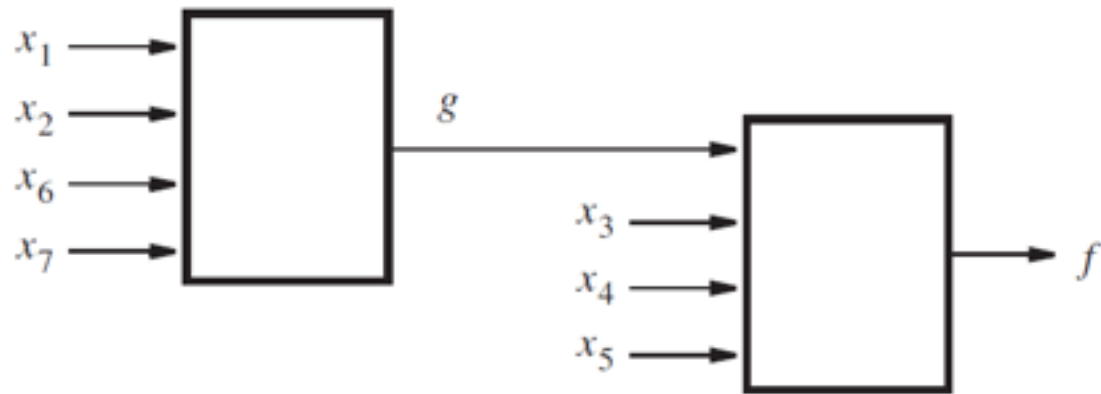


Figure 8.2. Using three LUTs after factoring.

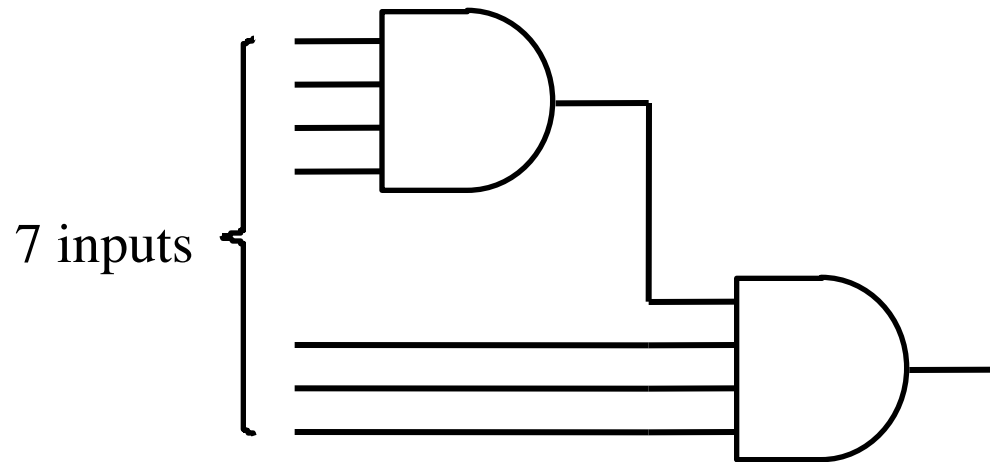


Figure 8.3. Using four-input AND gates to realize a seven-input product term.

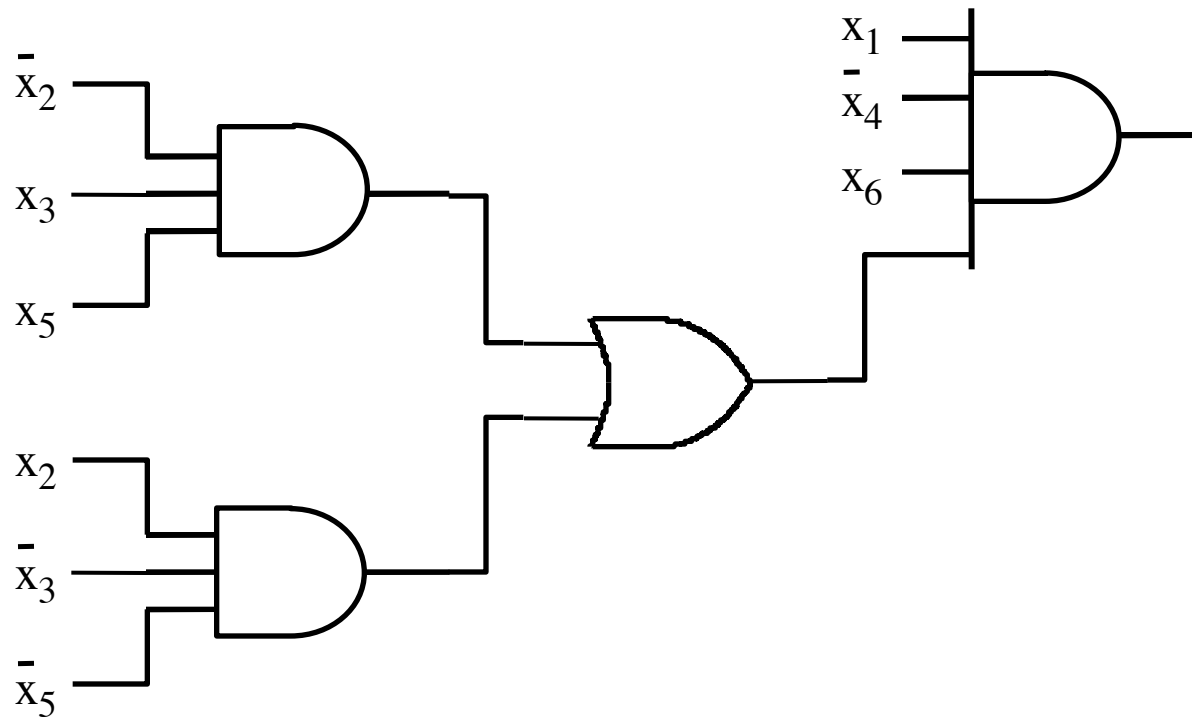


Figure 8.4. A factored circuit.

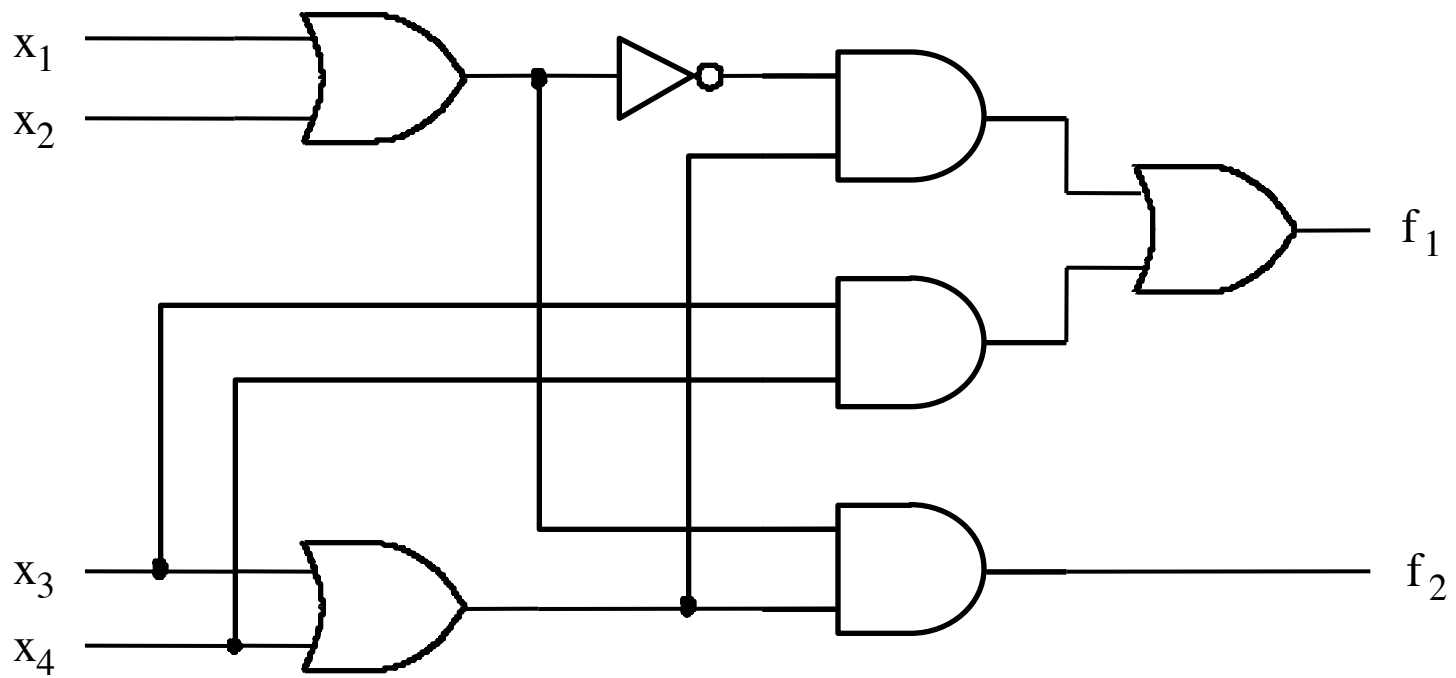
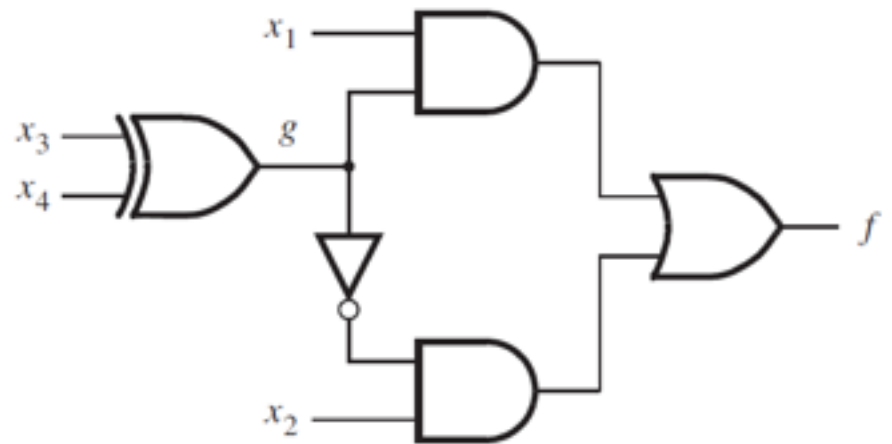


Figure 8.5. Circuit for Example 8.1.

x_1, x_2		x_3, x_4				
		00	01	11	10	
00	0	0	1	1	0	$x_2 \bar{x}_3 \bar{x}_4$
	1	0	0	1	1	$x_1 \bar{x}_3 x_4$
11	0	0	1	1	0	$x_2 x_3 x_4$
	1	0	0	1	1	$x_1 x_3 \bar{x}_4$

(a) Product terms

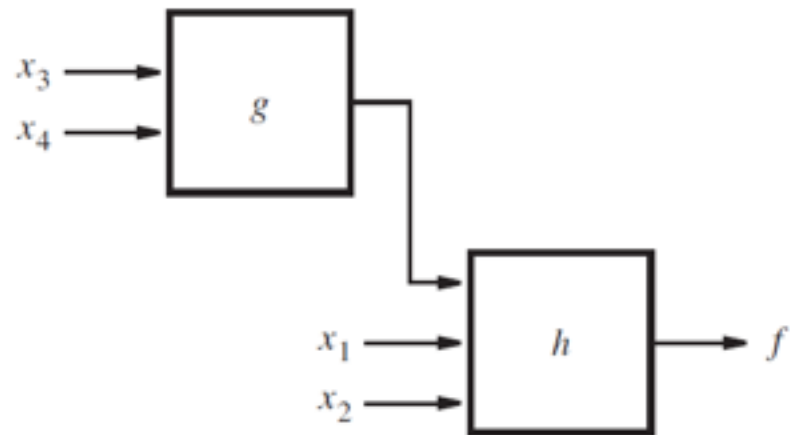


(b) Multilevel circuit

Figure 8.6. The function for Example 8.2.

x_1, x_2		\bar{g}	1	g	
		00	01	11	10
x_3, x_4	00	0	1	1	0
	01	0	0	1	1
	11	0	1	1	0
	10	0	0	1	1

(a) Subfunctions

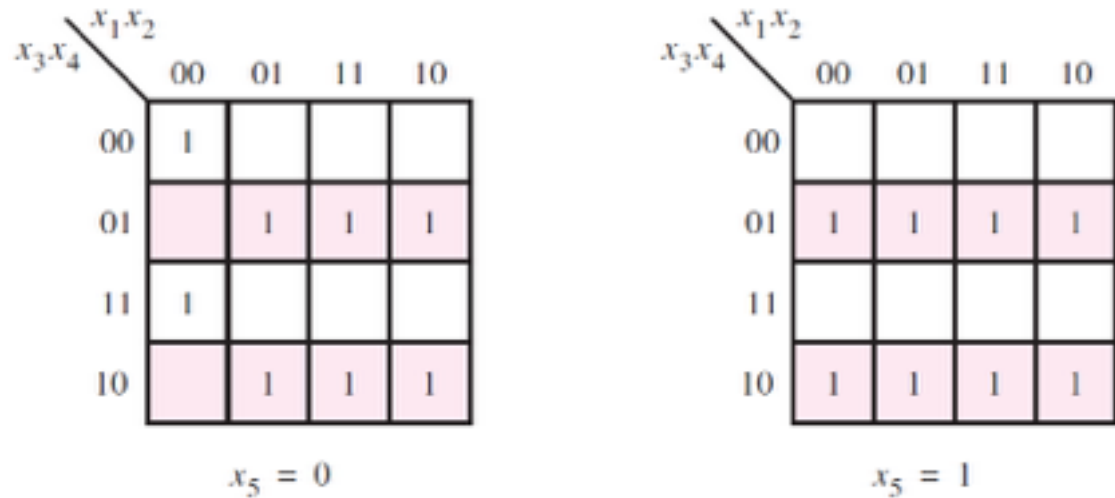


(b) The structure of decomposition

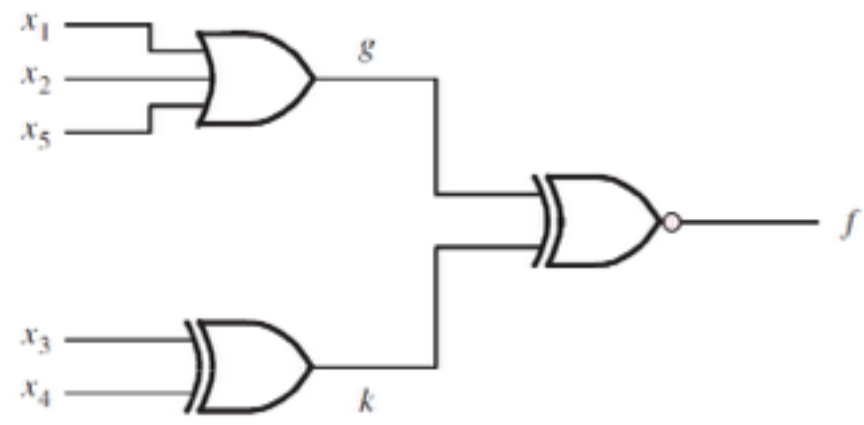
x_1, x_2		g			
		00	01	11	10
0	0	1	1	0	
1	0	0	1	1	

(c) Karnaugh map for $h(x_1, x_2, g)$

Figure 8.7. Subfunctions used in decomposition.



(a) Karnaugh map for the function f



(b) Circuit obtained using decomposition

Figure 8.8. Decomposition for Example 8.3.

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Figure 8.9. Implementation of XOR.

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Figure 8.10. Conversion to a NAND-gate circuit.

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Figure 8.11. Conversion to a NOR-gate circuit.

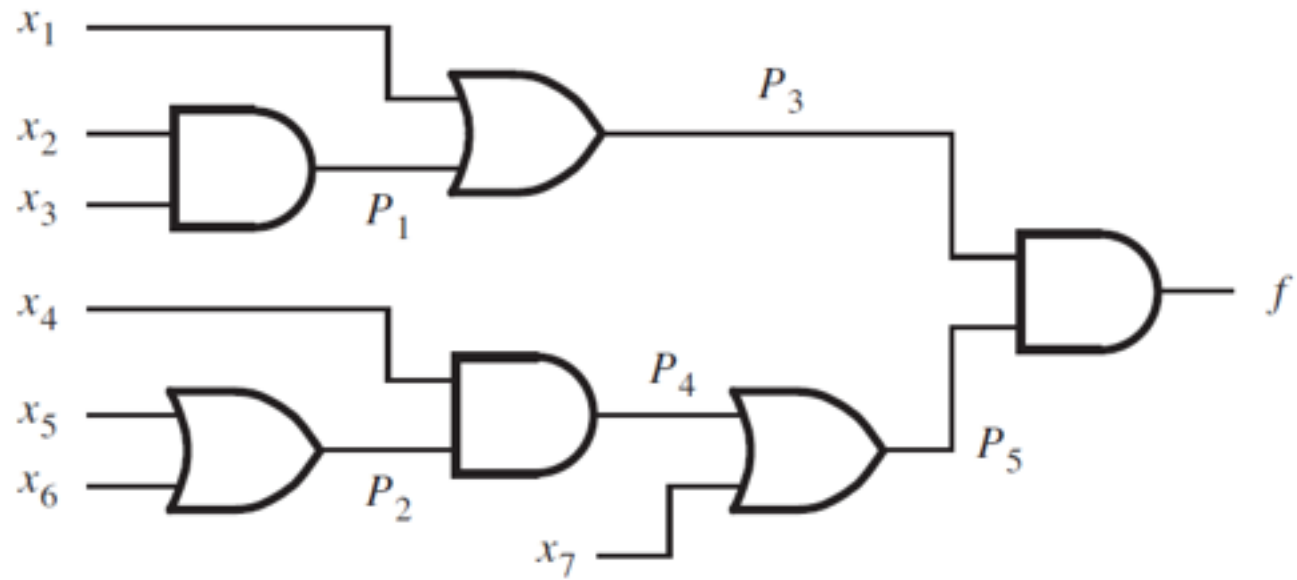


Figure 8.12. Circuit for Example 8.6.

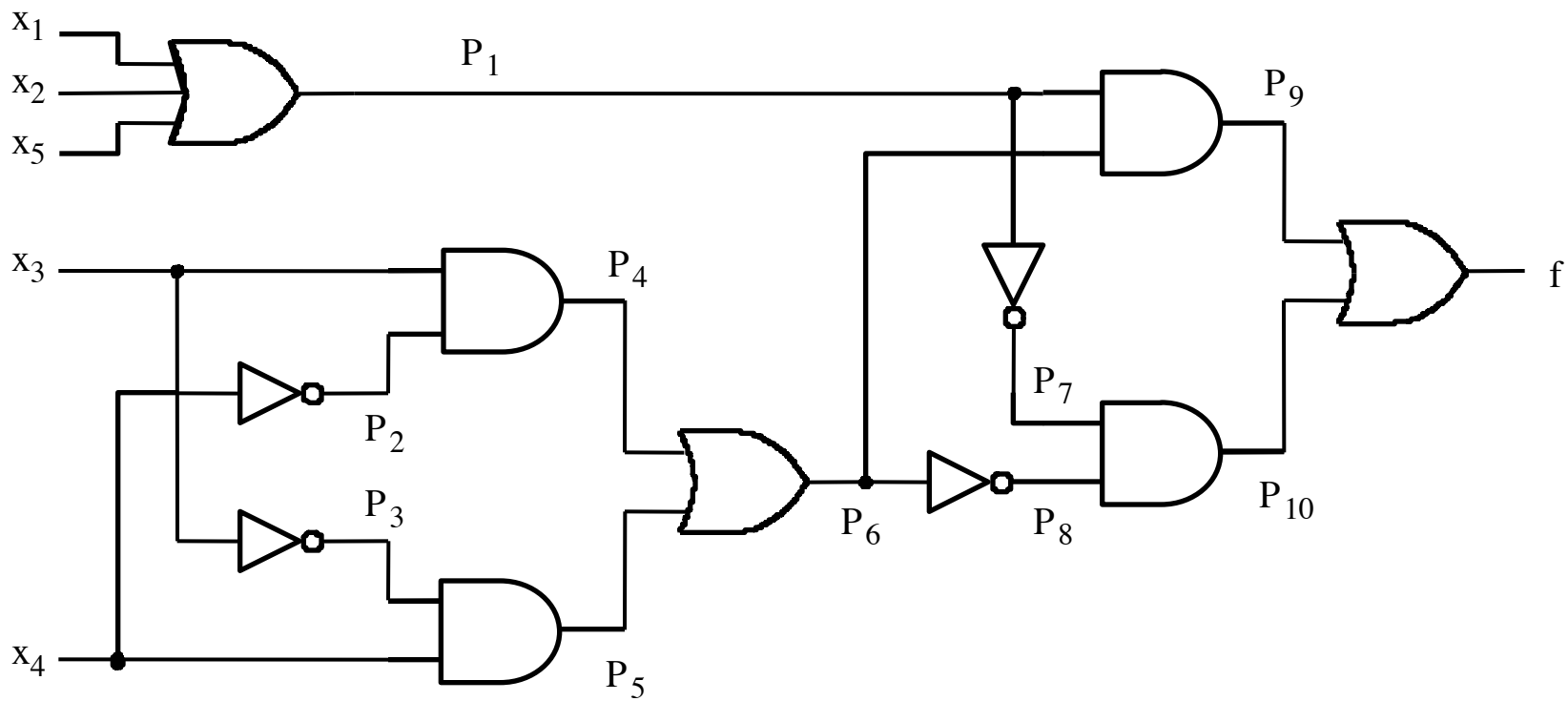


Figure 8.13. Circuit for Example 8.7.

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Figure 8.14. Circuit for Example 8.8.

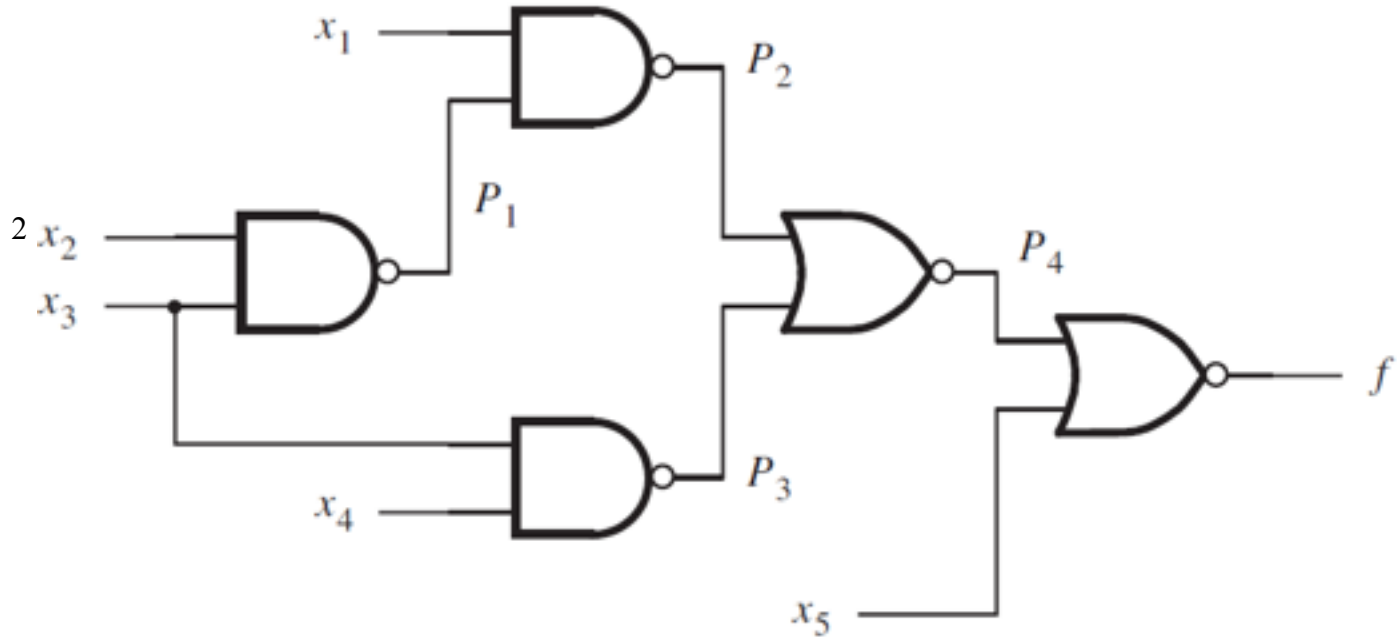


Figure 8.15. Circuit for Example 8.9.

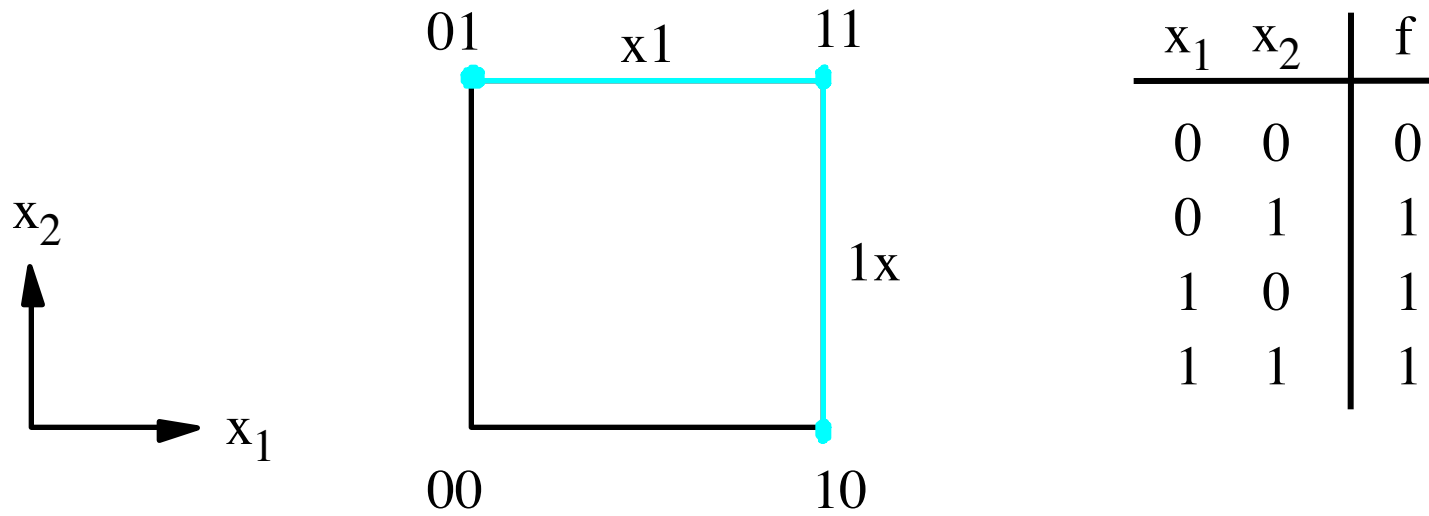


Figure 8.16. Representation of $f(x_1, x_2) = \sum m(1, 2, 3)$.

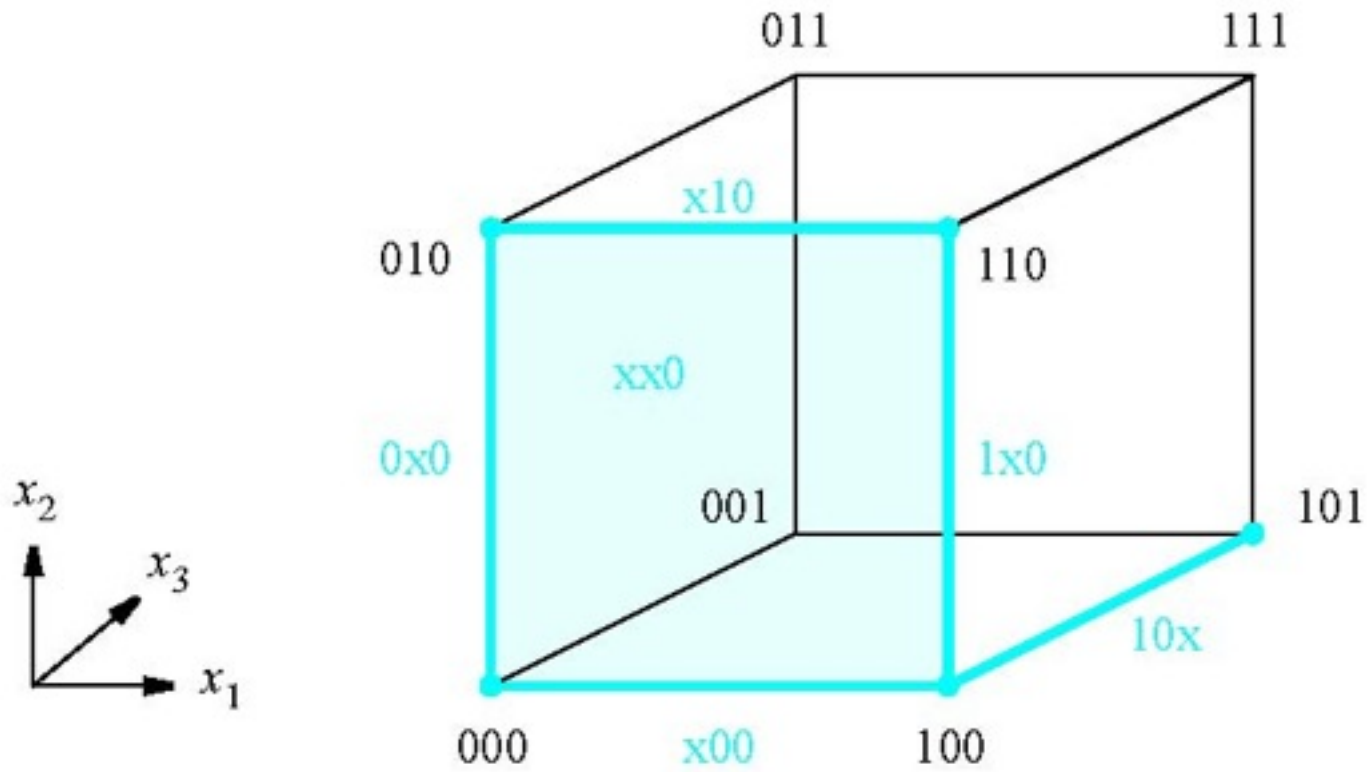


Figure 8.17. Representation of $f(x_1, x_2, x_3) = \sum m(0, 2, 4, 5, 6)$.

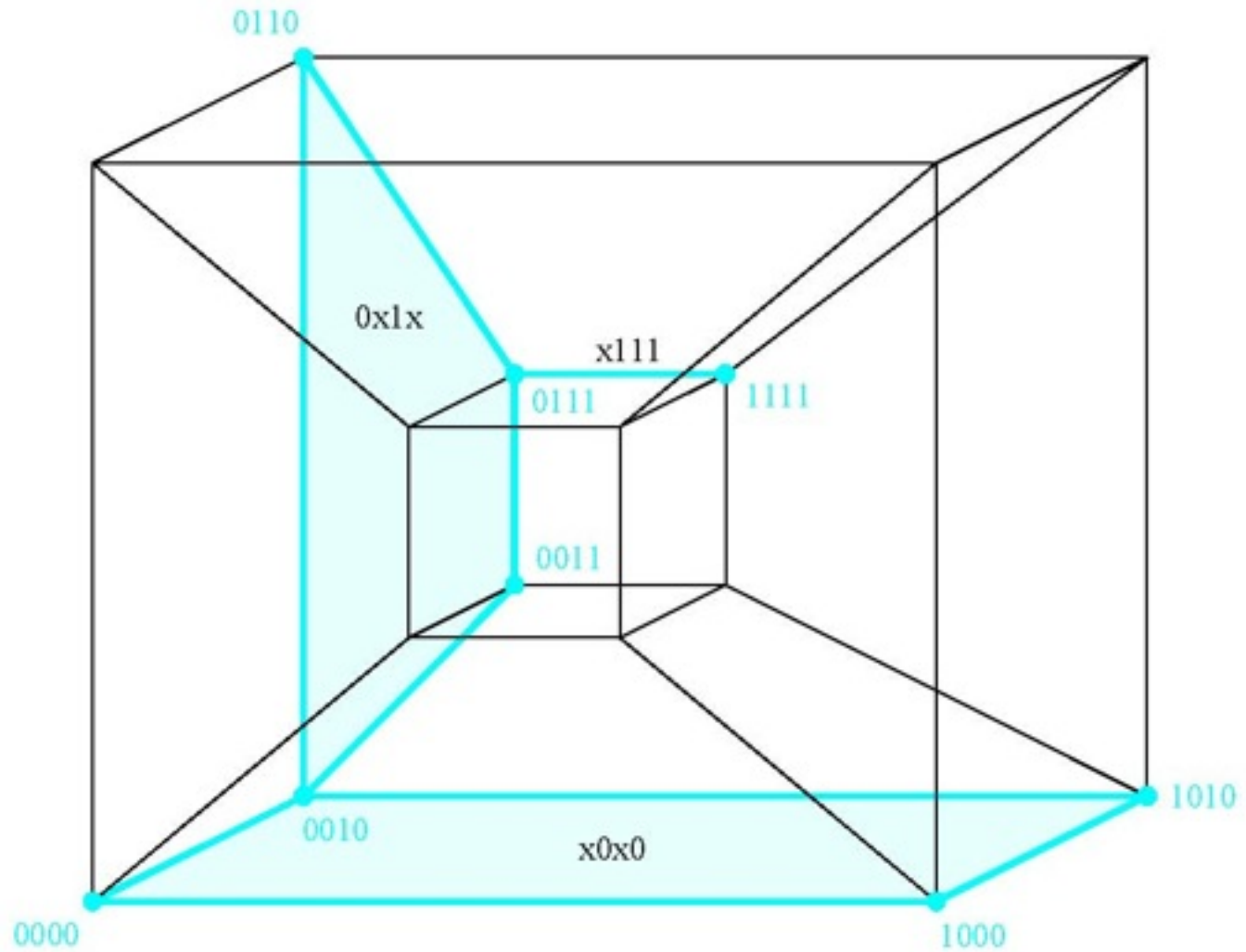
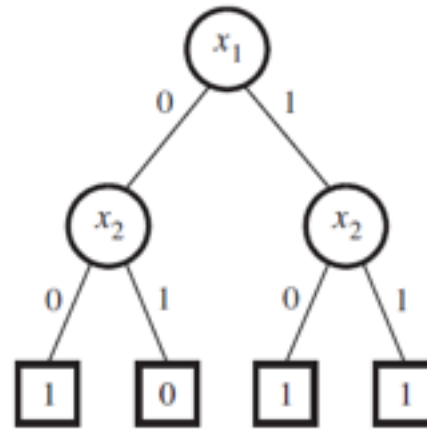


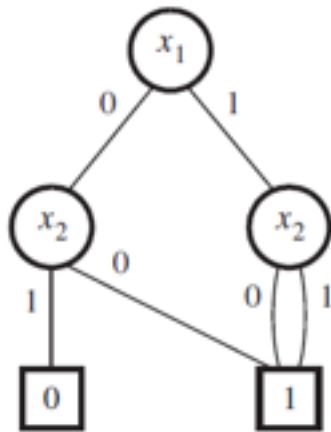
Figure 8.18. Representation of f_3 from Figure 2.54.

x_1	x_2	f
0	0	1
0	1	0
1	0	1
1	1	1

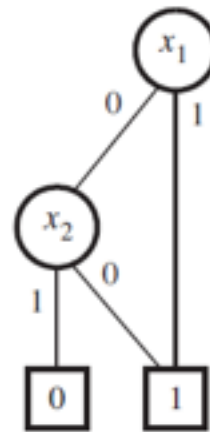
(a) Truth table



(b) Decision tree

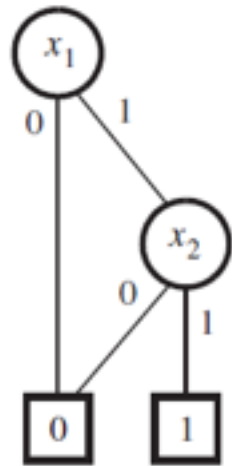


(c) Reducing nodes

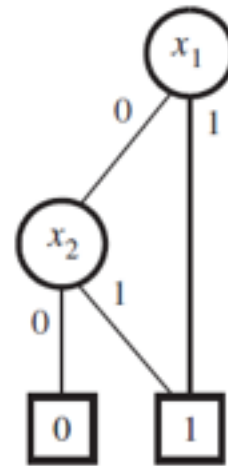


(d) BDD

Figure 8.19. Derivation of a binary decision diagram (BDD).

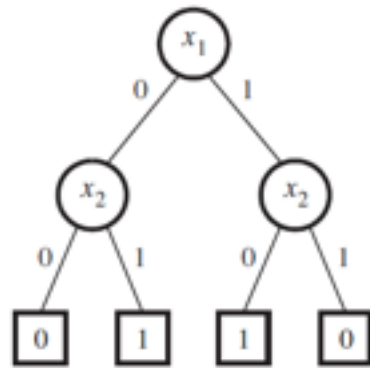


(a) AND

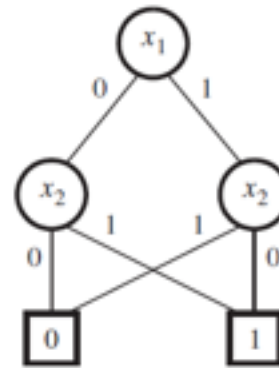


(b) OR

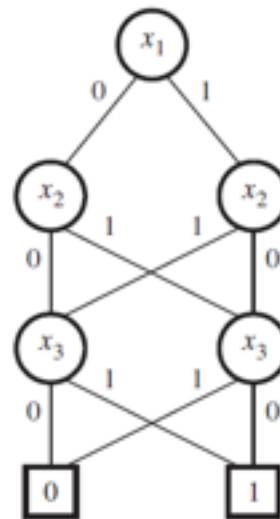
Figure 8.20. BDDs for the AND and OR functions.



(a) Decision tree

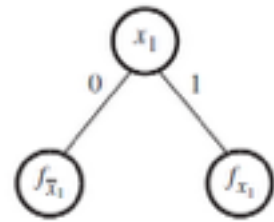


(b) BDD

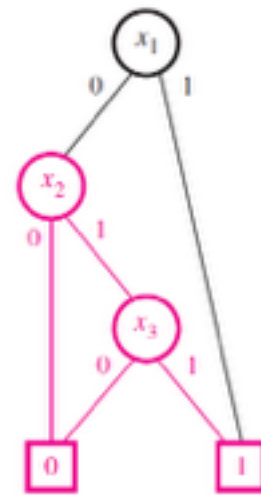


(c) 3-input BDD

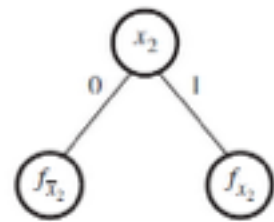
Figure 8.21. Derivation of BDDs for XOR functions.



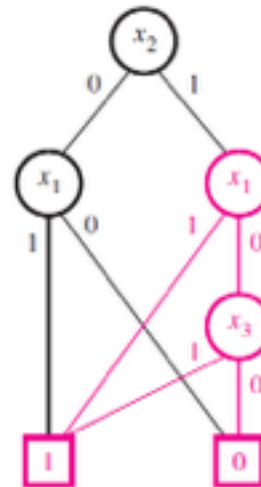
(a) Expansion using x_1



(b) BDD ordered x_1, x_2, x_3

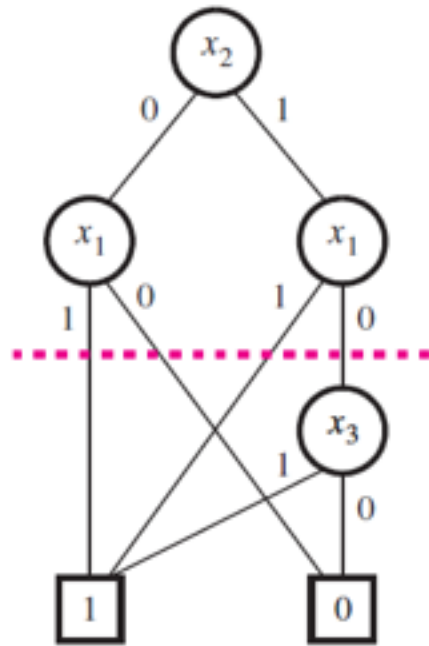


(c) Expansion using x_2



(d) BDD ordered x_2, x_1, x_3

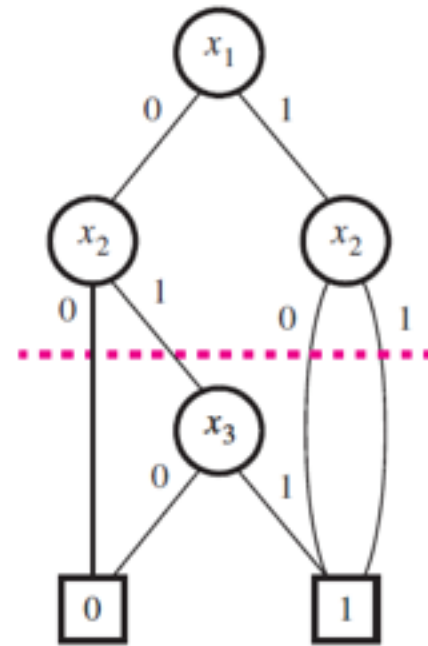
Figure 8.22. Derivation of BDDs for $f = x_1 + x_2 x_3$.



(a) BDD ordered x_2, x_1, x_3

x_1	x_2	Node
0	0	0
0	1	x_3
1	0	1
1	1	1

(b) Truth table



(c) Order x_1, x_2, x_3

Figure 8.23. Reordering the nodes in a BDD.

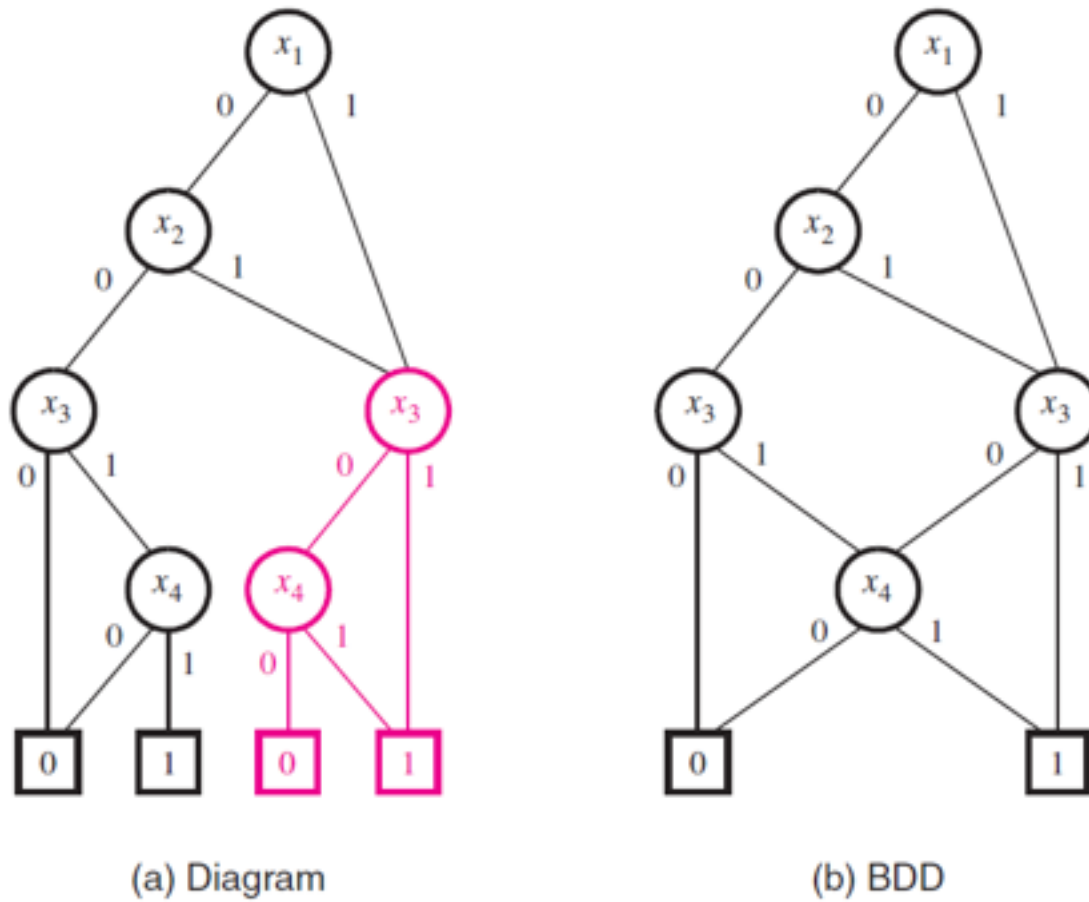


Figure 8.24. Derivation of a BDD for the function in Example 8.11.

List 1			List 2			List 3	
0	0 0 0 0	✓	0,4	0 x 0 0	✓	0,4,8,12 x x 0 0	
4	0 1 0 0	✓	0,8	x 0 0 0	✓		
8	1 0 0 0	✓	8,10	1 0 x 0			
10	1 0 1 0	✓	4,12	x 1 0 0	✓		
12	1 1 0 0	✓	8,12	1 x 0 0	✓		
11	1 0 1 1	✓	10,11	1 0 1 x			
13	1 1 0 1	✓	12,13	1 1 0 x			
15	1 1 1 1	✓	11,15	1 x 1 1			
			13,15	1 1 x 1			

Figure 8.25. Generation of prime implicants for the function in Figure 2.58.

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Figure 8.26. Selection of a cover for the function in Figure 2.58.

0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
6	0 1 1 0	✓
9	1 0 0 1	✓
12	1 1 0 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

0,1	0 0 0 x	✓
0,2	0 0 x 0	✓
0,8	x 0 0 0	✓
1,5	0 x 0 1	✓
2,6	0 x 1 0	✓
1,9	x 0 0 1	✓
8,9	1 0 0 x	✓
8,12	1 x 0 0	✓
5,7	0 1 x 1	✓
6,7	0 1 1 x	✓
5,13	x 1 0 1	✓
9,13	1 x 0 1	✓
12,13	1 1 0 x	✓
7,15	x 1 1 1	✓
13,15	1 1 x 1	✓

0,1,8,9	x 0 0 x
1,5,9,13	x x 0 1
8,9,12,13	1 x 0 x
5,7,13,15	x 1 x 1

Figure 8.27. Generation of prime implicants for the function in
 Example 8.12.

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Figure 8.28. Selection of a cover for the function in Example 8.12.

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Figure 8.29. Selection of a cover for the function in Example 8.13.

$A_i \backslash B_i$	0	1	x
0	0	\emptyset	0
1	\emptyset	1	1
x	0	1	x

$A_i * B_i$

Figure 8.30. The coordinate $*$ -operation.

$A_i \backslash B_i$	0	1	x
0	ε	\emptyset	ε
1	\emptyset	ε	ε
x	1	0	ε

$A_i \# B_i$

Figure 8.31. The coordinate #-operation.

		$x_1 x_2$			
		00	01	11	10
$x_3 x_4$	00	1	1	d	
	01		d	1	
	11				
	10	1		d	1

$$x_5 = 0$$

		$x_1 x_2$			
		00	01	11	10
$x_3 x_4$	00	1			
	01				
	11		1	1	1
	10	d	1		1

$$x_5 = 1$$

Figure 8.32. The function for Example 8.18.

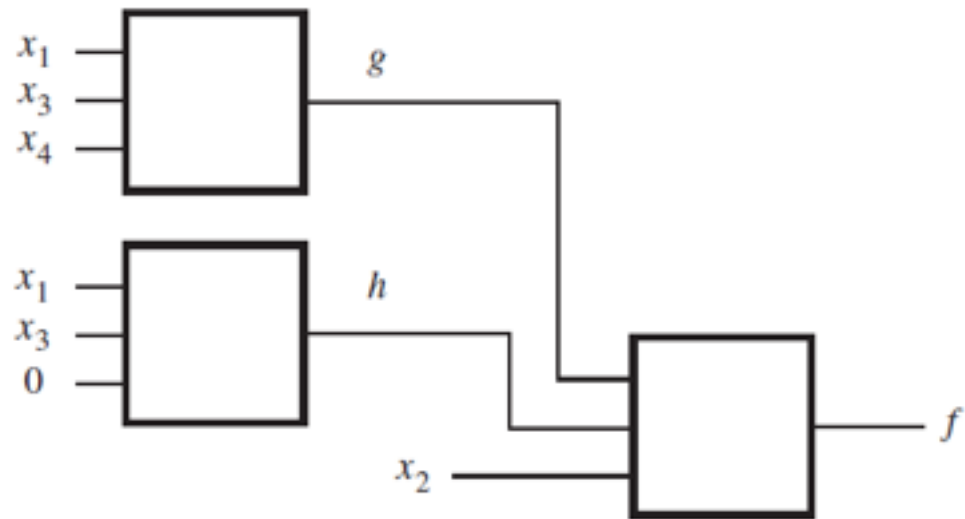


Figure 8.33. The function in Example 8.19.

$x_3x_4 \backslash x_1x_2$	h 00	01	11	g 10
00	0	0	0	0
01	0	1	1	1
11	1	1	1	1
10	0	1	1	1

Figure 8.34. The function for Example 8.20.

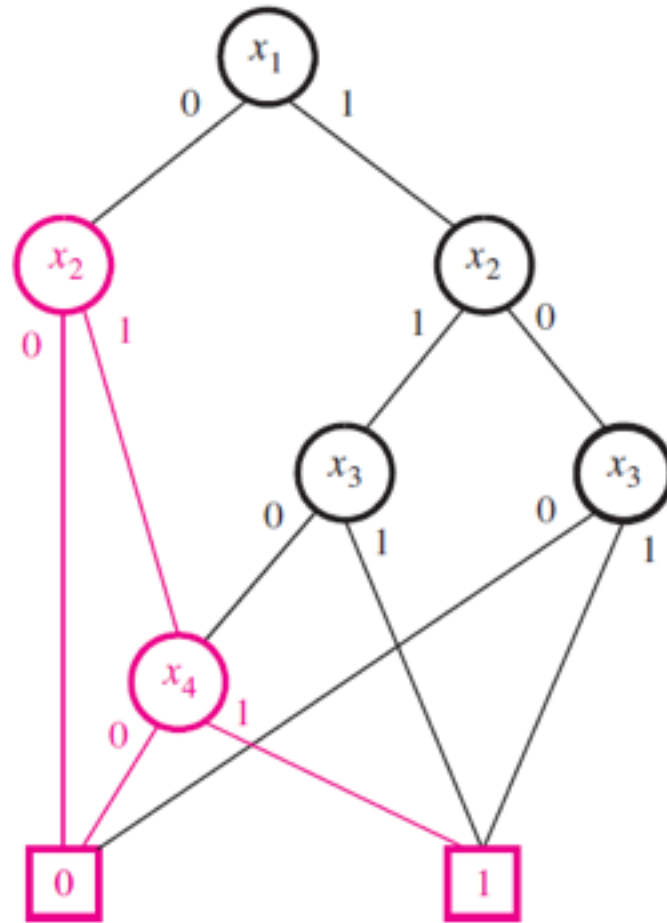
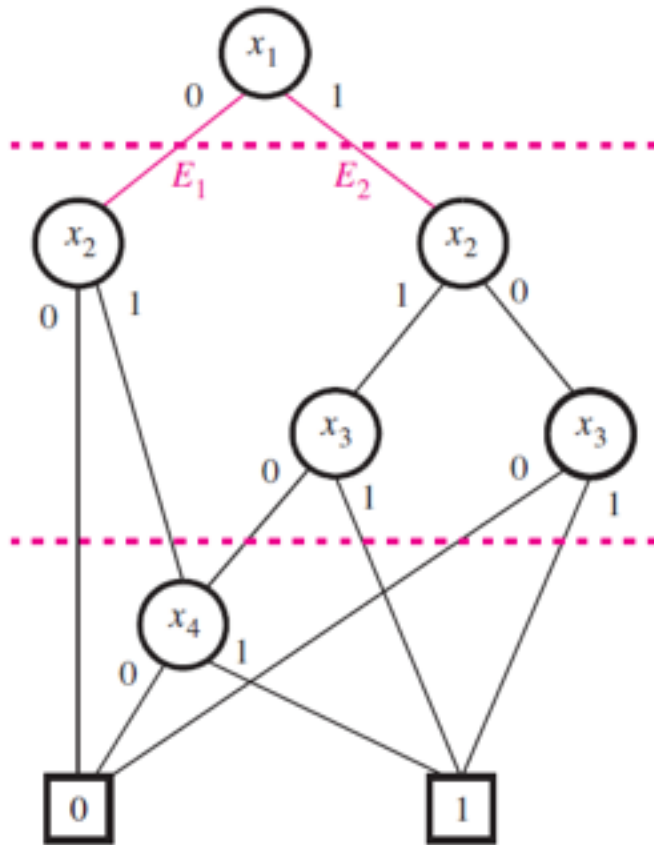


Figure 8.35. The BDD for Example 8.21.

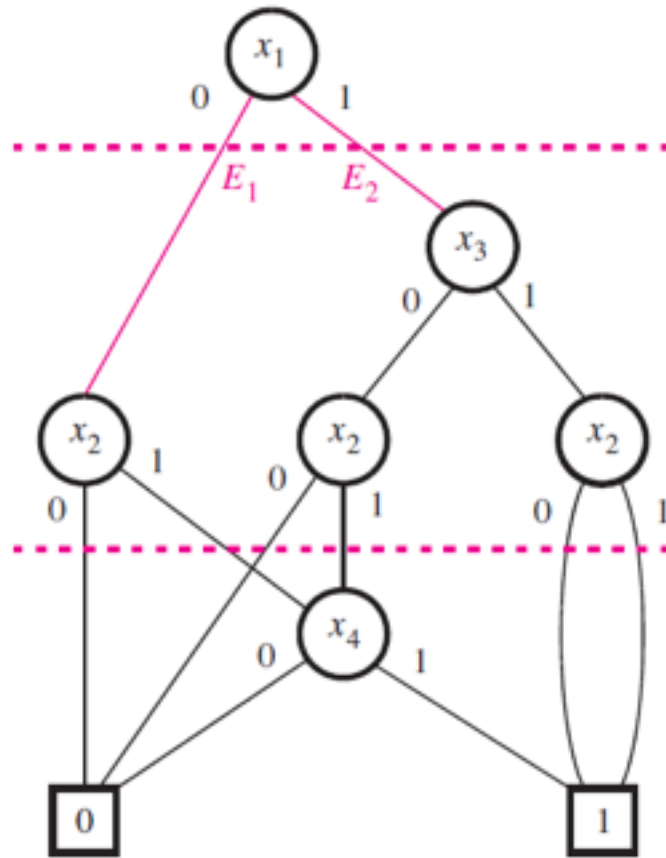


(a) Isolating x_2 and x_3

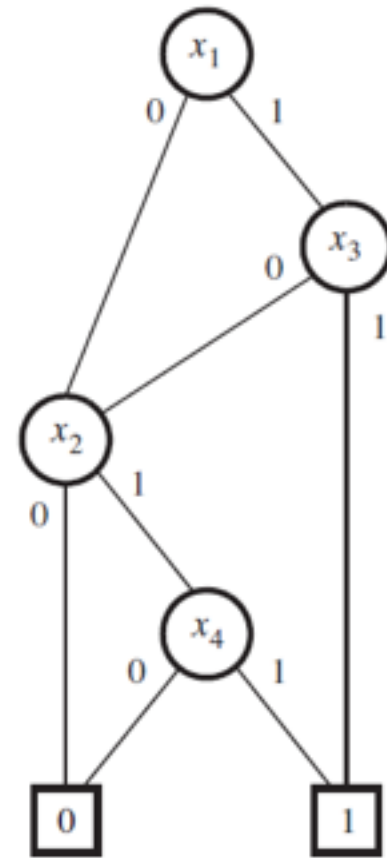
x_2	x_3	E_2
0	0	0
0	1	1
1	0	x_4
1	1	1

(b) Truth table

Figure 8.36. Reordering the BDD in Figure 8.35.



(a) Reordered tree



(b) Order x_1, x_3, x_2, x_4

Figure 8.37. The BDD for Example 8.22.

List 1			List 2			List 3	
0	0 0 0 0	✓	0,1	0 0 0 x		1,3,9,11	x 0 x 1
1	0 0 0 1	✓	0,4	0 x 0 0			
4	0 1 0 0	✓	1,3	0 0 x 1	✓	3,7,11,15	x x 1 1
3	0 0 1 1	✓	1,9	x 0 0 1	✓	9,11,13,15	1 x x 1
9	1 0 0 1	✓	4,12	x 1 0 0		12,13,14,15	1 1 x x
12	1 1 0 0	✓					
7	0 1 1 1	✓	3,7	0 x 1 1	✓		
11	1 0 1 1	✓	3,11	x 0 1 1	✓		
13	1 1 0 1	✓	9,11	1 0 x 1	✓		
14	1 1 1 0	✓	9,13	1 x 0 1	✓		
15	1 1 1 1	✓	12,13	1 1 0 x	✓		
			12,14	1 1 x 0	✓		
			7,15	x 1 1 1	✓		
			11,15	1 x 1 1	✓		
			13,15	1 1 x 1	✓		
			14,15	1 1 1 x	✓		

Figure 8.38. Generation of prime implicants for the function in Example 8.23.

Prime implicant	Minterm							
	0	1	3	4	7	11	13	15
$p_1 = 0\ 0\ 0\ x$	✓	✓						
$p_2 = 0\ x\ 0\ 0$	✓			✓				
$p_3 = x\ 1\ 0\ 0$				✓				
$p_4 = x\ 0\ x\ 1$		✓	✓			✓		
$p_5 = x\ x\ 1\ 1$			✓		✓	✓		✓
$p_6 = 1\ x\ x\ 1$						✓	✓	✓
$p_7 = 1\ 1\ x\ x$							✓	✓

(a) Initial prime implicant cover table

Prime implicant	Minterm			
	0	1	4	13
$p_1 = 0\ 0\ 0\ x$	✓	✓		
$p_2 = 0\ x\ 0\ 0$	✓		✓	
$p_4 = x\ 0\ x\ 1$		✓		
$p_6 = 1\ x\ x\ 1$				✓

(b) After the removal of rows p_3 , p_5 and p_7 , and columns 3, 7, 11 and 15

Figure 8.39. Selection of a cover for the function in Example 8.23.

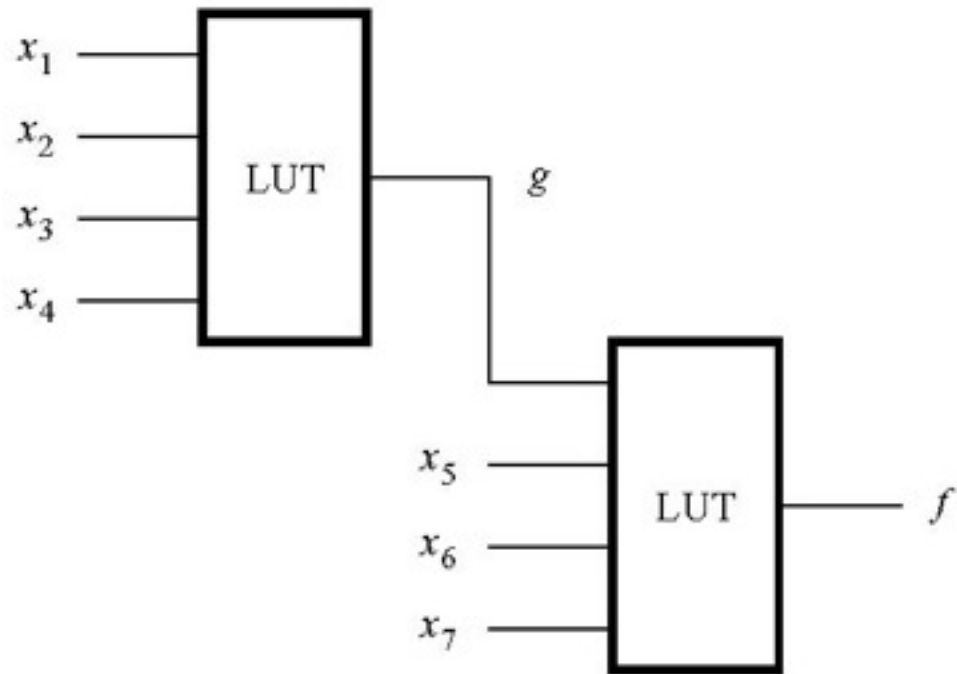


Figure 8.40. Circuit for Example 8.26.

		$x_1 x_2 x_3 x_4$				
		0000	0001	...	1110	1111
$x_5 x_6 x_7$						
000	m_0	m_8		m_{112}	m_{120}	
001	m_1	m_9		m_{113}	m_{121}	
010	m_2	m_{10}		m_{114}	m_{122}	
011	m_3	m_{11}	...	m_{115}	m_{123}	
100	m_4	m_{12}		m_{116}	m_{124}	
101	m_5	m_{13}		m_{117}	m_{125}	
110	m_6	m_{14}		m_{118}	m_{126}	
111	m_7	m_{15}		m_{119}	m_{127}	

Figure 8.41. A possible format for truth tables of seven-variable functions.

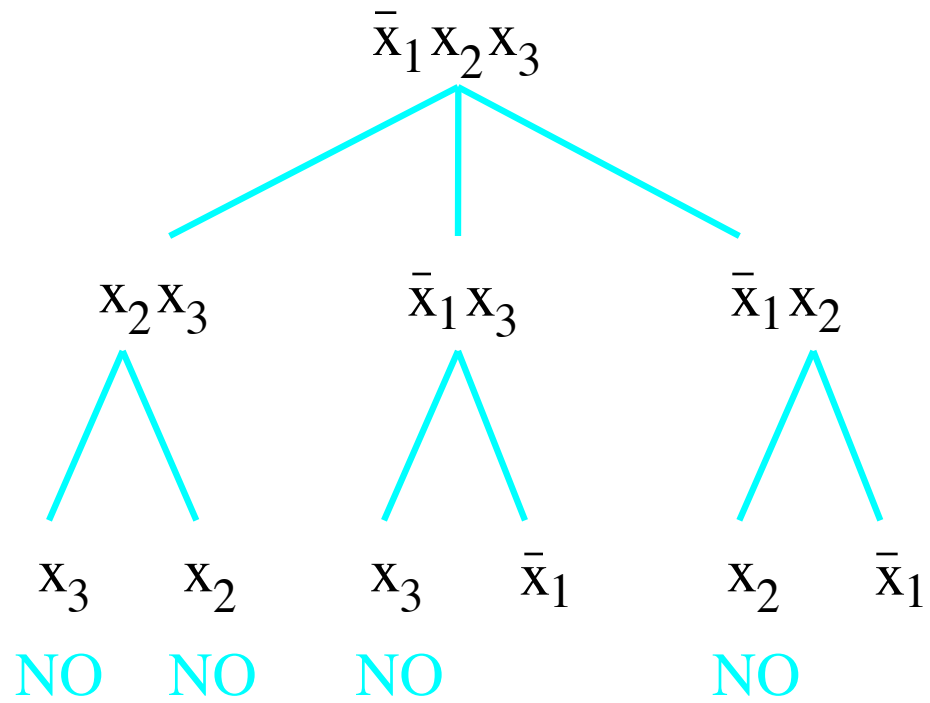


Figure P8.1. Expansion of implicant $x_1 x_2 x_3$.

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Figure P4.2. Circuit for problem 4.33.

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Figure P4.3. Circuit for problem 4.34.