

## Optimization of Finite State Machines

- State Equivalence and Distinguishability
- Minimization of FSMs: both Mealy & Moore type FSMs
- Machine equivalence
- Completely Specified and Incompletely Specified m/c
- Revisit Encoding Problems
- FSM Synthesis Demo + Verilog Design of FSMs.

## Completely + Incompletely Specified FSMs

- Complete Spec: For every input + state combination, every transition (next state) is specified. Ditto w/ every output.
- Incomplete Spec:
  - In some state, a specific input may never arrive. What's the next state? Unspecified! What about the output? Unspecified!
  - Sometimes, for an input + present state combination, next state is specified, but out value is not critical - and left unspecified.
- Completely Specified FSMs - easy to analyze. Not so with Incomp. specified m/c.

## State and M/C Equivalence

- What do we mean by equivalent states?
- How do you identify equivalent states?
- Subsequently, how do you prove/disprove equivalence of two FSMs. FSM Equivalence  $\iff$  Sequential circuit equivalence!
- States  $S_i$  and  $S_j$  of a machine  $M$  are equivalent if and only if, for every **possible input sequence, the same output sequence** will be produced regardless of whether  $S_i$  or  $S_j$  is the initial state.
- Identify ALL equivalent states, merge them = minimal FSM.
- A **unique minimal machine** exists for any (completely specified) FSM!

## State Table - Mealy Machine

Table 1: State Transition Table

P.S.	Next State, Z	
	$x = 0$	$x = 1$
A	E, 0	D, 1
B	F, 0	D, 0
C	E, 0	B, 1
D	F, 0	B, 0
E	C, 0	F, 1
F	B, 0	C, 0

- Minimize this machine!

## Minimized State Table

Table 2: State Transition Table

P.S.	Next State, Z	
	$x = 0$	$x = 1$
AC	E, 0	BD, 1
BD	F, 0	BD, 0
E	AC, 0	F, 1
F	BD, 0	AC, 0

- Encode this machine: AC: 00, BD: 01, E: 10, F: 11

Table 3: Encoded State Transition Table

P.S.	Next State, Z	
	$x = 0$	$x = 1$
$y_2y_1$	$Y_2Y_1, z$	$Y_2Y_1, z$
00	10, 0	01, 1
01	11, 0	01, 0
10	00, 0	11, 1
11	01, 0	00, 0

## State Table - Moore Machine (Fig. 8.51)

Table 4: State Transition Table

P.S.	Next State		Z
	$x = 0$	$x = 1$	
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0

## Incomp. Spec. FSM

Table 5: State Transition Table

P.S.	Next State, Z	
	$x = 0$	$x = 1$
A	C, 1	E, -
B	C, -	E, 1
C	B, 0	A, 1
D	D, 0	E, 1
E	D, 1	A, 0



## Incomp. Spec. FSM

Table 6: State Transition Table

P.S.	Next State, Z	
	$x = 0$	$x = 1$
A	B, 1	-, -
B	-, 0	C, 0
C	A, 1	B, 0