EX: If X has a gaussian (i.e., normal) distribution with mean $\mu_X = 1$ and standard deviation $\sigma_X = 2$, i.e., $X \sim n(1, 2)$, find the probability density function for $Y = -X + 9$.

SOL'N: For $Y = aX + b$, $(a \neq 0)$, the pdf for Y in terms of the pdf for X is given by the following formula:

$$f_Y(y) = \frac{1}{|a|} f_X\left(x = \frac{y-b}{a}\right)$$

For X, we have a gaussian or normal pdf:

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-(x-\mu_X)^2/2\sigma_X^2}$$

Using the formula for $f_Y(y)$, we have one form for $f_Y(y)$:

$$f_Y(y) = \frac{1}{|a|} \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-\left(\frac{y-b}{a} - \mu_X\right)^2/2\sigma_X^2}$$

Using $a = -1$ and $b = 9$ in the formula for $f_Y(y)$, we have one form of the final answer:

$$f_Y(y) = \frac{1}{|a|} \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-\left(\frac{y-b}{a} - \mu_X\right)^2/2\sigma_X^2}$$

This is again a gaussian or normal distribution and, after some algebraic manipulation, we find that the gaussian or normal distribution has the following mean and variance:

$$\mu_Y = a\mu_X + b = -1 \cdot 1 + 9 = 8 \qquad \sigma_Y^2 = a^2\sigma_X^2 = (-1)^2 2^2 = 4$$

Using $a = -1$ and $b = 9$, we have another form for the desired answer:

$$f_Y(y) = \frac{1}{\sqrt{2\pi(-2)^2}} e^{-[y-(-1+9)]^2/2(-2)^2} = \frac{1}{\sqrt{2\pi \cdot 4}} e^{-[y-8]^2/2\cdot 4}$$