
EX: If X is uniformly distributed on the interval [0, 1], i.e., $X \sim u[0, 1]$, find the probability density function (pdf) for $Y = 3X + 2$.

SOL'N: For $Y = aX + b$, $(a \neq 0)$, the pdf for Y in terms of the pdf for X is given by the following formula:

$$f_Y(y) = \frac{1}{|a|} f_X\left(x = \frac{y-b}{a}\right)$$

For X, we have a uniform pdf on [0, 1]:

$$f_X(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Using $a = 3$ and $b = 2$ in the formula for $f_Y(y)$, we make a literal substitution of $x = \frac{y-2}{3}$ in the expression for $f_X(x)$ and multiply by $\frac{1}{|3|}$:

$$f_Y(y) = f_X\left(x = \frac{y-2}{3}\right) = \frac{1}{3}\begin{cases} 1 & 0 < \frac{y-2}{3} < 1 \\ 0 & \text{otherwise} \end{cases}$$

Rewriting the inequality, we have the following form:

$$f_Y(y) = \begin{cases} \frac{1}{3} & 2 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$