TOOLS: The table below summarizes the values for means and variances of combinations of random variables.

MEAN (OR EXPECTED VALUE), 1 RANDOM VARIABLE

$$E(X) \equiv \mu_X = \int_{-\infty}^{\infty} x f_X(x)dx$$

$$E(aX+b) = aE(X)+b \qquad \mu_{aX+b} = a\mu_X + b$$

$$E(g(X)) = \int_{-\infty}^{\infty} g(x) f_X(x)dx = \int_{-\infty}^{\infty} g(x)\left[\int_{-\infty}^{\infty} f(x,y)dy\right]dx$$

$$\mu_{g(X)} = \int_{-\infty}^{\infty} g(x) f_X(x)dx = \int_{-\infty}^{\infty} g(x)\left[\int_{-\infty}^{\infty} f(x,y)dy\right]dx$$

NOTE: Typically, $E(g(X)) \neq g(E(X))$ and $\mu_{g(X)} \neq g(\mu_X)$ unless $g(\ )$ is a linear function.

MEAN (OR EXPECTED VALUE), ANY 2 RANDOM VARIABLES

$$E(aX+bY) = aE(X)+bE(Y) \qquad \mu_{aX+bY} = a\mu_X + b\mu_Y$$

$$E(g(X)+h(Y)) = E(g(X)) + E(h(Y)) = \int_{-\infty}^{\infty} g(x) f_X(x)dx + \int_{-\infty}^{\infty} h(y) f_Y(y)dy$$

$$\mu_{g(X)+h(Y)} = \mu_{g(X)} + \mu_{h(Y)}$$

$$E(XY) \equiv \mu_{XY} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xyf(x,y)dydx$$

$$E(aXbY) \equiv ab\,\mu_{XY}$$

MEAN (OR EXPECTED VALUE), 2 INDEPENDENT RANDOM VARIABLES

$$E(XY) = E(X)E(Y)$$

$$\mu_{XY} = \mu_X \mu_Y$$

VARIANCE (OR SQUARE OF STANDARD DEVIATION), 1 RANDOM VAR

$$\sigma_X^2 = E\left((X-\mu_X)^2\right) = \int_{-\infty}^{\infty} (x-\mu_X)^2 f_X(x)dx = E(X^2) - (\mu_X)^2$$

$$\sigma_{aX+b}^2 = a^2\sigma_X^2 \qquad\qquad \sigma_{aX+b} = a\sigma_X$$

VARIANCE (OR SQUARE OF STANDARD DEVIATION), ANY 2 RAND VARS

$$\sigma_{aX+bY}^2 = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\sigma_{XY} \text{ (see covariance below)}$$

VARIANCE (OR SQUARE OF STAND DEV), 2 INDEPENDENT RAND VARS

$$\sigma_{aX+bY}^2 = a^2\sigma_X^2 + b^2\sigma_Y^2$$

COVARIANCE, ANY 2 RANDOM VARIABLES

$$\sigma_{XY} = E(XY) - \mu_X\mu_Y = E((X-\mu_X)(Y-\mu_Y)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xyf(x,y)dydx - \mu_X\mu_Y$$

$$\sigma_{aXbY} = ab\sigma_{XY}$$

COVARIANCE, 2 INDEPENDENT RANDOM VARIABLES

$$\sigma_{XY} = 0$$

CORRELATION, ANY 2 RANDOM VARIABLES

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X\sigma_Y} = \frac{\sigma_{XY}}{\sqrt{\sigma_X^2\sigma_Y^2}}$$

$$\rho_{aXbY} = \rho_{XY}$$

CORRELATION, 2 INDEPENDENT RANDOM VARIABLES

$$\rho_{XY} = 0$$