Variance and Standard Deviation

def: Variance $\equiv \sigma^2 \equiv E\{(X-\mu)^2\} \equiv \int_{-\infty}^{\infty} (x-\mu)^2 p(x)\, dx$

$\equiv$ measures how spread out the distribution is

$\equiv$ similar to squared error as it measures squared distance from mean and weights it by the density at that distance.

tool: There are several equivalent formulas for variance:

$$\sigma^2 = E\{(X-\mu)^2\} = E\{X^2\} - (E\{X\})^2 \equiv E\{X^2\} - \mu^2$$

• These follow directly from the integral definition of σ^2.

ex: $X \sim u(0,1)$ uniform distribution on $[0,1]$

$$\sigma^2 = E\{(X-\mu)^2\} = E\{(X-\tfrac{1}{2})^2\} \quad \text{since } \mu = \tfrac{1}{2}$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 p(x)\, dx = \int_{-\infty}^{\infty} \left(x-\tfrac{1}{2}\right)^2 p(x)\, dx$$

$$\sigma^2 = \int_0^1 \left(x-\tfrac{1}{2}\right)^2 \cdot 1\, dx \quad \text{since } p(x) = \begin{cases} 1 & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\sigma^2 = \int_{-\frac{1}{2}}^{\frac{1}{2}} y^2\, dy \quad \text{change variable to } y = x - \tfrac{1}{2} \quad dy = dx$$

$$\sigma^2 = \left.\frac{y^3}{3}\right|_{-1/2}^{1/2} = \frac{2\left(\frac{1}{2}\right)^3}{3} = \frac{1}{12}$$

• Note: if we sum 12 independent uniform random vars on $[0,1]$ and subtract 12μ, we get a new random variable with mean $=0$, var $=1$. This Rand Var has almost a gaussian distribution.

def: Standard Deviation, $\sigma \equiv \sqrt{\sigma^2}$ • $\sigma > 0$ positive $\sqrt{\ }$