Random Variables

def: Random Variable $X \equiv$ a variable that represents the (numerical) outcome of an experiment before the experiment is performed.

Note: X is shorthand for "outcome of expt"

ex: $P\{X = 3\}$ means "Probability that outcome of expt $= 3$"

ex: $P\{X = x\}$ " " " " " " " $= x$"

def: Probability mass function (for discrete random variables) $\equiv$ the probabilities, $p_1, \ldots, p_n$, assigned to outcomes

Note: Σ probabilities $\equiv \sum_{i=1}^{n} p_i = 1$

ex: Dice $P\{X = i\} = \frac{1}{6}$ for $i = 1, \ldots, 6$

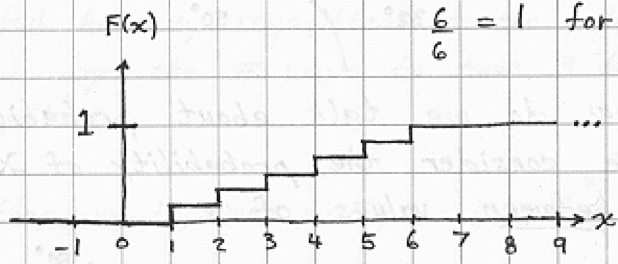
def: Cumulative distribution Function, $F(x) \equiv P\{X \leq x\}$

ex: Dice $P\{X \leq x\} = \frac{1}{6}$ for $x = 1$

$\frac{2}{6} = \frac{1}{3}$ for $x = 2$

$\vdots$

$\frac{6}{6} = 1$ for $x = 6$



tool: $F(x)$ is always monotonic increasing

tool: $F(-\infty) = P\{X \leq -\infty\} = 0$ and $F(\infty) = P\{X \leq \infty\} = 1$

def: continuous random variable, $X \equiv X$ can take on <u>any</u> value over an interval

ex: $X \equiv$ temperature at 12:00 tonight in °F

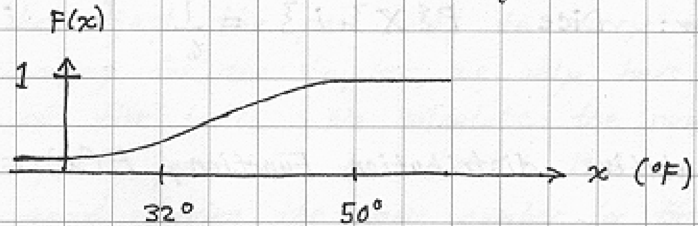
- Could be any value between -459.67 and ∞.

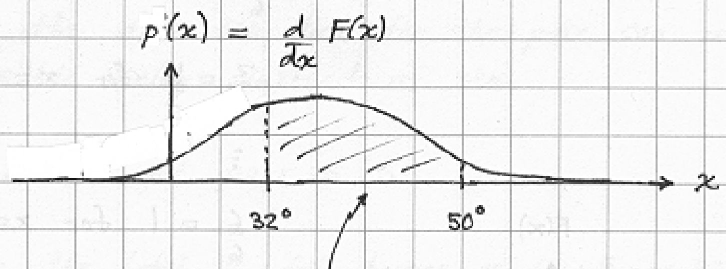
Note: $P\{X = x\} = 0$ for every x because the probability of X being <u>exactly</u> x is zero.

def: probability density function, $p(x) \equiv \frac{d}{dx} F(x)$

i.e., derivative of cumulative distribution function

ex: $X \equiv$ temperature at 12:00 tonight





~~tool~~: How do we talk about probabilities now? We consider the probability of X lying <u>between</u> values of x.

ex: $P\{32° \leq X \leq 50°\} = \int_{32°}^{50°} p(x)\, dx =$ area under $p(x)$ between 32° and 50°

$= F(50°) - F(32°)$

tool: $\int_{-\infty}^{\infty} p(x)\, dx = F(x \to \infty) = 1$