UU

ECE 3530 Mean = Balancing Point

ex: Show $E\{X\}$ = balancing point of teeter-totter with weight distributed as $p(x)$ upon it.

That is, show $$\int_{-\infty}^{E\{X\}} \underbrace{(E\{X\} - x)}_{\text{lever arm}} \cdot \underbrace{p(x)}_{\text{wt}}\, dx = \int_{E\{X\}}^{\infty} (x - E\{X\})\, p(x)\, dx.$$

pf: $$E\{X\} = \int_{-\infty}^{\infty} x\, p(x)\, dx \quad \text{and} \quad \int_{-\infty}^{\infty} p(x)\, dx = 1$$

$$\therefore\ E\{X\} \int_{-\infty}^{\infty} p(x)\, dx = E\{X\}$$

$$\therefore\ \int_{-\infty}^{\infty} E\{X\}\, p(x)\, dx = E\{X\}$$

So $$\int_{-\infty}^{\infty} x\, p(x)\, dx = \int_{-\infty}^{\infty} E\{X\}\, p(x)\, dx$$

or $$\int_{-\infty}^{\infty} (x - E\{X\})\, p(x)\, dx = 0$$

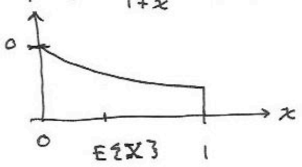
or $$\int_{-\infty}^{E\{X\}} (x - E\{X\})\, p(x)\, dx + \int_{E\{X\}}^{\infty} (x - E\{X\})\, p(x)\, dx = 0$$

or $$\int_{-\infty}^{E\{X\}} (x - E\{X\})\, p(x)\, dx = -\int_{E\{X\}}^{\infty} (x - E\{X\})\, p(x)\, dx$$

or (mult both sides by -1):

$$\int_{-\infty}^{E\{X\}} (E\{X\} - x)\, p(x)\, dx = \int_{E\{X\}}^{\infty} (x - E\{X\})\, p(x)\, dx.$$

ex: $p(x) = \dfrac{k}{1+x}$ for $0 \le x \le 1$



we will find $E\{X\}$, the expected value (or mean) of X given p.d.f. $p(x)$:

Find const k (scaling factor):

$$\int_{-\infty}^{\infty} p(x)\, dx = 1$$

$$= \int_0^1 \frac{k}{1+x}\, dx = k \int_0^1 \frac{1}{1+x}\, dx$$

$$= k \ln(1+x)\Big|_0^1 = k(\ln 2 - \xcancel{\ln 1}^{\,0})$$

$$= k \ln 2$$

So $k \ln 2 = 1 \Rightarrow k = \dfrac{1}{\ln 2} \doteq 1.44$

$E\{X\}$ = balancing point of teeter-totter with weight distributed as $p(x)$ upon it.

This idea allows us to make a rough visual estimate of where $E\{X\}$ is located. It should be a little to the left of $x = \frac{1}{2}$.

$E\{X\} \equiv \displaystyle\int_{-\infty}^{\infty} x\, p(x)\, dx \equiv$ weighted ave of x values

$$'' = \int_0^1 x \, \frac{1/\ln 2}{1+x}\, dx = \frac{1}{\ln 2} \int_0^1 \frac{x}{1+x}\, dx$$

$$'' = \frac{1}{\ln 2} \int_0^1 \left(\frac{1+x}{1+x} - \frac{1}{1+x} \right) dx = \frac{1}{\ln 2} \int_0^1 \left(1 - \frac{1}{1+x} \right) dx$$

$$'' = \frac{1}{\ln 2} \left(x\Big|_0^1 - \ln 2 \right) = \frac{1}{\ln 2} (1 - \ln 2)$$

$$'' = \frac{1}{\ln 2} - 1 \approx 0.44$$

$E\{X\} \doteq 0.44$ (slightly less than $\frac{1}{2}$, as expected)