Expected Value (or Mean Value)

def: Expected value $\equiv$ Mean value, $\mu \equiv E\{X\} \equiv$ ave value of expt outcome over time

$$\equiv \int_{-\infty}^{\infty} x\, p(x)\, dx$$

$\equiv$ center of mass of $p(x)$

ex: Die $\quad p(x) = \sum_{i=1}^{6} \frac{1}{6}\delta(x-i)$

Clearly, $E\{X\} =$ ave outcome $= \frac{1}{6}\cdot 1 + \frac{1}{6}\cdot 2 + \ldots + \frac{1}{6}\cdot 6 = 3.5$

Math: $E\{X\} = \int_{-\infty}^{\infty} x\, p(x)\, dx = \int_{-\infty}^{\infty} x \sum_{i=1}^{6} \frac{1}{6}\delta(x-i)\, dx$

$$= \int_{-\infty}^{\infty} \sum_{i=1}^{6} x \cdot \frac{1}{6}\, \delta(x-i)\, dx$$

$$= \sum_{i=1}^{6} \int_{-\infty}^{\infty} x \cdot \frac{1}{6}\, \delta(x-i)\, dx \qquad \int\Sigma = \Sigma\int$$

$$= \sum_{i=1}^{6} \frac{1}{6} \int_{-\infty}^{\infty} x\, \delta(x-i)\, dx$$

$$= \frac{1}{6}\sum_{i=1}^{6} \int_{-\infty}^{\infty} x\, \delta(x-i)\, dx$$

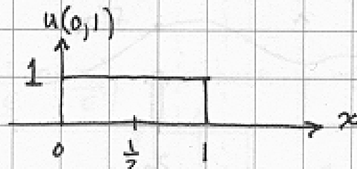
$$= \frac{1}{6} \sum_{i=1}^{6} i \qquad \int_{-\infty}^{\infty} x\, \delta(x-i)\, dx = x\Big|_{x=i} = i$$

$$= \frac{1}{6} \cdot \frac{6\cdot(6+1)}{2}$$

$$= \frac{7}{2}$$

$E\{X\} = 3.5$ ✓

ex: Uniform distribution, $u(0,1) \equiv p(x) = \begin{cases} 0 & x<0 \\ 1 & 0 \le x \le 1 \\ 0 & x>1 \end{cases}$



By inspection, we guess ave value of outcome is $\frac{1}{2}$.

Math: $E\{x\} = \int_{-\infty}^{\infty} x\, p(x)\, dx = \int_{0}^{1} x\cdot 1\, dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$ ✓

u u

ex: Semiconductor manufacturer finds number of bad bits in memory chips is distributed as $p(x) = \frac{1}{(x+1)^2}$ where x is the number of bad bits.

Find: ave number of bad bits for 1G bit memory chips.

soln: First check that $\int_{-\infty}^{\infty} p(x)\,dx = 1$

$$\int_{-\infty}^{\infty} p(x)\,dx = \int_{0}^{\infty} \frac{1}{(x+1)^2}\,dx = \left.\frac{-1}{x+1}\right|_0^{\infty} = \cancel{\frac{-1}{\infty}}^{\,0} - \frac{-1}{0+1}$$

$$= 1$$

\# bad bits ≥ 0

Note: In theory, we only have integer numbers of bad bits. We use the continuous model because it gives answers very close to the integer model and is much easier to deal with mathematically.

$$\text{Ave \# bad bits} = E\{X\} \equiv \int_{-\infty}^{\infty} x\,p(x)\,dx$$

$$\therefore E\{X\} = \int_0^{1G = 10^9} \frac{x}{(x+1)^2}\,dx = \int_0^{10^9} \frac{x+1}{(x+1)^2} - \frac{1}{(x+1)^2}\,dx$$

$$= \int_0^{10^9} \frac{1}{x+1}\,dx - \int_0^{10^9} \frac{1}{(x+1)^2}\,dx$$

$$= \left.\ln(x+1)\right|_0^{10^9} + \left.\frac{1}{x+1}\right|_0^{10^9}$$

$$= \ln(10^9+1) - \cancel{\ln 1}^{\,0} + \frac{1}{10^9+1} - 1$$

$$\approx \ln(10^9+1) - 1$$

$E\{X\} \approx 19.7$ bad bits on average

Note: As size of memory approaches ∞ we have $E\{x\} \to \infty$.