Linear Combinations of Random Vars

1

$$E\{g(X)\} = \int_{-\infty}^{\infty} g(x)\, p(x)\, dx$$

$$E\{aX+b\} = a\, E\{X\} + b$$

$$\int_{-\infty}^{\infty} (ax+b)\, p(x)\, dx = a\int_{-\infty}^{\infty} x\, p(x)\, dx + b \underbrace{\int_{-\infty}^{\infty} p(x)\, dx}_{1}$$

$$= a\, E\{X\} + b$$

$$\mathrm{Var}\{aX+b\} = E\{(aX+b-\mu_Y)^2\}$$

$$= E\{(aX - a\mu_X)^2\}$$

$$= \int_{-\infty}^{\infty} (ax - a\mu_X)^2\, p(x)\, dx$$

$$= a^2 \int_{-\infty}^{\infty} (x-\mu_X)^2\, p(x)\, dx$$

$$= a^2\ \mathrm{Var}\ X$$

2

$$E\{X_1 + X_2\} = E\{X_1\} + E\{X_2\}$$

Even if dependent vars

$$E\{X_1 + X_2\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x_1 + x_2)\, p(x_1, x_2)\, dx_2\, dx_1$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1\, p(x_1, x_2)\, dx_2\, dx_1 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_2\, p(x_1, x_2)\, dx_1\, dx_2$$

$$= \int_{-\infty}^{\infty} x_1 \underbrace{\left(\int_{-\infty}^{\infty} p(x_1, x_2)\, dx_2\right)}_{= p_{X_1}(x_1)} dx_1 + \int_{-\infty}^{\infty} x_2 \underbrace{\left(\int_{-\infty}^{\infty} p(x_1, x_2)\, dx_1\right)}_{= p_{X_2}(x_2)} dx_2$$

$$= E\{X_1\} + E\{X_2\}$$

3

ex: sum two gaussians

$$p(x_1) = \frac{1}{\sigma_{X_1}\sqrt{2\pi}}\, e^{-(x_1-\mu_{x1})^2/2\sigma_{x1}^2}$$

$$p(x_2) = \frac{1}{\sigma_{X_2}\sqrt{2\pi}}\, e^{-(x_2-\mu_{x2})^2/2\sigma_{x2}^2}$$

$$E\{X_1 + X_2\} = \mu_{x_1} + \mu_{x2}$$

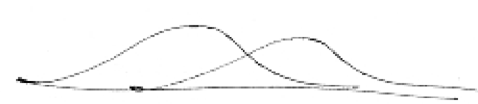
is it gaussian?

4

$$\mathrm{Var}\{a_1 X_1 + a_2 X_2\} = a_1^2 \sigma_{X_1}^2 + a_2^2 \sigma_{X2}^2$$

ex: sum two gaussians

$$\sigma_{x_1+x_2}^2 = \sigma_{x1}^2 + \sigma_{x2}^2$$



sum of μ's = μ of sum

sum of σ^2's = σ^2 of sum

see

$$\sigma^2_{X_1+X_2} = \sigma^2_{X_1} + 2\sigma^2_{X_1X_2} + \sigma^2_{X_2}$$

Does <u>not</u> require <u>ind</u> X_1 & X_2

$$\sigma^2_{a_1X_1+a_2X_2} = a_1^2\sigma^2_{X_1} + a_2^2\sigma^2_{X_2}$$

<u>Does</u> require ind X_1 & X_2

Central Limit Theorem: (p 264)

$Y = \frac{1}{N} \sum_{i=1}^{N} X_i$ $\quad$ X_i i.i.d R.V.'s mean μ, var σ^2

Then $Y \approx n(\mu, \frac{\sigma^2}{N})$

$\boxed{\text{Sum of outcomes from independent identically distributed random variables approaches gaussian dist.}}$

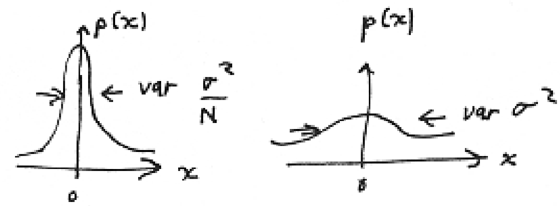
or $Z = \sum_{i=1}^{N} X_i$ no ave

Then $Z \approx n(n\mu, N\sigma^2)$

Note: for ave, var (ave) < var each X_i by factor of $\sqrt{N}$

Say $X_i \sim n(0,1)$ $\frac{1}{\sqrt{N}} X_i \sim n(0, \frac{1}{N})$ is dist of ave

ave acts like much narrower gaussian



narrower (ave of X_i) $\qquad$ wider (one X_i