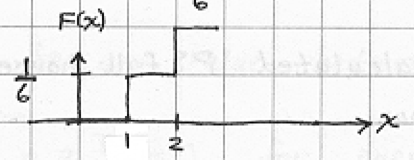
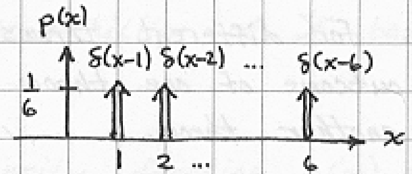
ECE 3530 Discrete Random Variables from Delta Functions

Discrete Random Variables in Continuous Framework

tool: Use delta function, $\delta(x)$, to represent discrete probabilities.

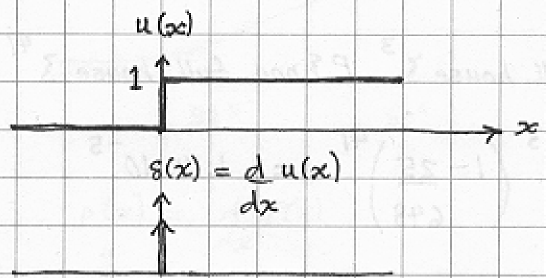
ex: Die $P\{1\} = \frac{1}{6}$ corresponds to $p(x) = \frac{1}{6}\,\delta(x-1)$ for rolling 1 on die





Comment: $p(x) = \frac{d}{dx}F(x)$ = derivative of stepped func

tool: Delta function, $\delta(x) = \frac{d}{dx}u(x)$ where $u(x) \equiv$ unit step

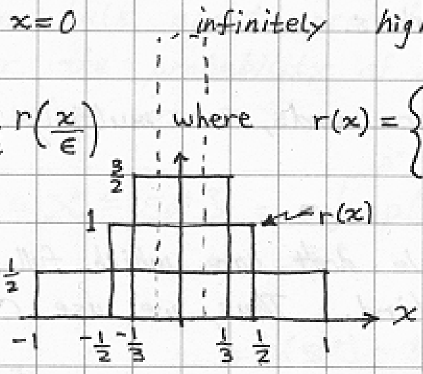


tool: $\int_{-\infty}^{x} \delta(x)\,dx = u(x)$ $\qquad \int_{-\epsilon}^{\epsilon} \delta(x)\,dx = 1$ area $=1$

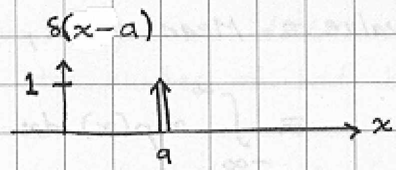
tool: $\delta(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ \infty & \text{for } x = 0 \end{cases}$ infinitely narrow / infinitely high

tool: $\delta(x) = \lim_{\epsilon \to 0} \frac{1}{\epsilon}\, r\!\left(\frac{x}{\epsilon}\right)$ where $r(x) = \begin{cases} 0 & |x| > \frac{1}{2} \\ 1 & |x| \leq \frac{1}{2} \end{cases}$

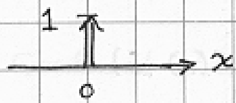
Gets narrower and taller, but area stays equal one, as $\epsilon \to 0$.



tool: $\delta(x-a)$ = delta function shifted to 'a':

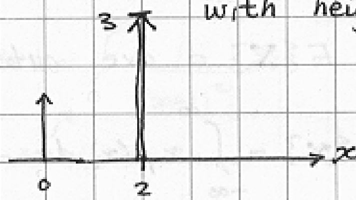


notn: $\delta(x) \equiv$

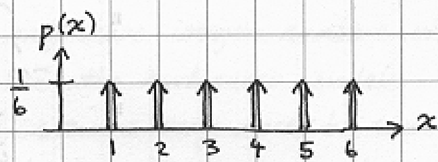


We draw $\delta(x)$ as double arrow with height = area under $\delta(x)$

ex: $3\delta(x-2) \equiv$



ex: Die $\quad p(x) = \sum_{i=1}^{6} \frac{1}{6}\delta(x-i) \equiv$



tool: Delta function is identity function for convolution:

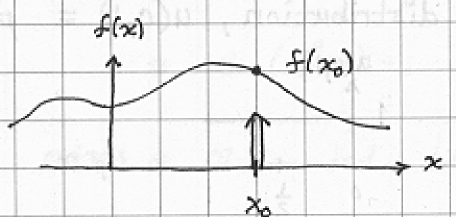
$$\int_{-\infty}^{\infty} f(x_0 - x)\,\delta(x)\,dx = f(x_0)$$

$$\parallel$$

$$f(x) \otimes \delta(x)\Big|_{x=x_0} = f(x_0)$$

tool: Delta function picks out function values when integrated:

$$\int_{-\infty}^{\infty} f(x)\,\delta(x-x_0)\,dx = f(x_0)$$



Comment: $\delta(x-x_0)$ has value only at $x = x_0$.

Thus, $\int_{-\infty}^{\infty} f(x)\,\delta(x-x_0)\,dx = \int_{-\infty}^{\infty} f(x_0)\,\delta(x-x_0)\,dx$

$= f(x_0)\int_{-\infty}^{\infty} \delta(x-x_0)\,dx = f(x_0)\cdot 1 = f(x_0).$