Probability

def: Outcomes ≡ All Possible ~~outcomes~~ results of hypothetical experiment.
User decides how detailed to make these "atoms" that exhaust the possibilities of what the ~~outcome~~ results of an experiment will be.

ex: Roll dice is experiment.

Outcomes: 1, 2, 3, 4, 5, or 6 · obvious choice
or Outcomes: even #, odd # · ok, our choice

· Must be exhaustive · For playing cards suit, or # would be possible outcome choices

def: $\mathcal{S} \equiv S \equiv$ Sample Space ≡ set of all possible outcomes

ex: For dice $S = \{1, 2, 3, 4, 5, 6\}$
or $S = \{$even #, odd #$\}$

For two dice thrown at once $\mathcal{S} = \{(1,1), (1,2), \ldots, (2,1), (2,2), \ldots, (6,1), (6,2), \ldots (6,6)\}$

def: Probabilities ≡
- $P\{O_i\} = 0$ means outcome O_i never happens
- $P\{O_i\} = 1$ " " " always "
- $P\{O_i\}$ between 0 and 1 indicates how often outcome i happens
- Sum of $P\{O_i\} = 1$ <u>something</u> happens

ex: Dice $\underbrace{P\{1\}}_{\frac{1}{6}} + \underbrace{P\{2\}}_{\frac{1}{6}} + \ldots + \underbrace{P\{6\}}_{\frac{1}{6}} = 1$

def: Event ≡ Set of outcomes (not necessarily all of them)
· Molecules of outcome atoms.

ex: Dice
event: # is 5 or 6
event: # is 5
event: # is 1, 2, 5, or 6
event: # is even or greater than 3, i.e. 2, 4, 5, or 6

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def: Complement of event $A \equiv A' \equiv$ set of outcomes not in A

ex: Dice $A = \{1,2,5,6\}$ $A' = \{3,4\}$

tool: $P\{A'\} = 1 - P\{A\}$

pf: $P\{A\} + P\{A'\} = P\{\mathcal{S}\} = 1$

• Don't you wish it could always be this simple?...

not'n: $A \cap B \equiv A$ intersect $B \equiv$ ~~events~~ outcomes in both A and B

ex: $A = \{1,2,5,6\}$ $B = \{2,4,6\}$ $A \cap B = \{2,6\}$

not'n: $A \cup B \equiv A$ union $B \equiv$ outcomes in A or B

ex: $A = \{1,2,5,6\}$ $B = \{2,4,6\}$ $A \cup B = \{1,2,4,5,6\}$

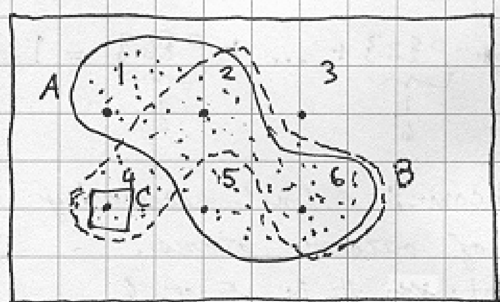
def: A, B mutually exclusive $\equiv A \cap B = \emptyset$ empty set

ex: $A = \{1,2,5,6\}$ $C = \{4\}$ $A \cap C = \emptyset$

• $P\{A \cup C\} = P\{A\} + P\{C\}$ if mutually exclusive

• $P\{\text{event}\} =$ sum of $P\{\text{outcome in event}\}$'s

Venn Diagrams • Helps us see what combined event is



$P\{A \cup B\} \neq \underset{4/6}{P\{A\}} + \underset{3/6}{P\{B\}}$

counted outcomes 2&6 twice

$P\{A \cup B\} = P\{A\} + P\{B\} - \underset{2/6}{P\{A \cap B\}} = \frac{5}{6}$ ✓

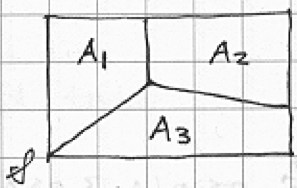
$P\{A \cap B\} = P\{(A' \cup B')'\} = 1 - P\{A' \cup B'\}$

$= 1 - (P\{A'\} + P\{B'\} - P\{A' \cap B'\})$ $P\{A\}+P\{B\}$

$= 1 - (1 - P\{A\} + 1 - P\{B\} - P\{A' \cap B'\})$ " $-P\{A \cup B\}$

def: Partition $A_1, \ldots, A_n$ of Sample Space, $\mathcal{S} \equiv A_1, \ldots, A_n$ are events that include all possible outcomes and are mutually exclusive (i.e., their intersections are all empty, i.e. they don't overlap). $\therefore \mathcal{S} = A_1 \cup \ldots \cup A_n$

ex:



ex: Playing cards

A_1 = odd numbered cards
A_2 = even " "
A_3 = face cards

tool: Law of Total Probability $\equiv$ $P\{B\} = P\{B \cap A_1\} + \ldots + P\{B \cap A_n\}$ for any partition $A_1, \ldots, A_n$

pf: $P\{B\} = P\{B \cap \mathcal{S}\} = P\{B \cap (A_1 \cup \ldots \cup A_n)\}$

$= P\{(B \cap A_1) \cup (B \cap A_2) \cup \ldots \cup (B \cap A_n)\}$

$= P\{B \cap A_1\} + \ldots + P\{B \cap A_n\}$

ex: Playing cards

$P\{$suit is hearts$\} = P\{$hearts and odd number$\}$
$+ P\{$ " " even " $\}$
$+ P\{$ " " face card$\}$

- Seems trivial but it leads to Bayes' theorem.