

tool: Bayes' Theorem $\equiv P\{A_i|B\} = \frac{P\{B|A_i\}P\{A_i\}}{P\{B\}}$

" $= \frac{P\{B|A_i\}P\{A_i\}}{\sum_{i=1}^n P\{B|A_i\}P\{A_i\}}$

where $A_1, \dots, A_n$ is Partition of $\mathcal{S}$

pf: $P\{B\} = \sum_{i=1}^n P\{B|A_i\}P\{A_i\}$ by Law of Total Prob

So now we only need to prove 1st eq'n.

$$P\{A_i \cap B\} = P\{A_i|B\}P\{B\}$$

and $P\{A_i \cap B\} = P\{B|A_i\}P\{A_i\}$

$$\therefore P\{A_i|B\}P\{B\} = P\{B|A_i\}P\{A_i\}$$

or $P\{A_i|B\} = \frac{P\{B|A_i\}P\{A_i\}}{P\{B\}}$

def: priors $\equiv P\{A_i\}$ for partition $A_1, \dots, A_n \equiv$ Probabilities before outcomes known

def: Posterior probability $\equiv P\{A_i|B\} \equiv$ Probability of A_i after outcome B has occurred

comment: We use Bayes' theorem to revise (or improve) our estimate of $P\{A_i\}$ after observing that outcome B occurred.

note: If we don't know $P\{B\}$ we still know it is the same for each $P\{A_i|B\}$ and we can compare values of $P\{A_1|B\}$ and $P\{A_2|B\}$ to see which of A_1 and A_2 was more likely.

ex: Lie detector test

• Problem of false positives

- $\equiv$ Detector says you Lie
 + $\equiv$ " " " tell truth

Lie $\equiv$ you did lie
 Truth $\equiv$ " told truth

Given: $P\{- | \text{Lie}\} = 0.89$ • caught you
 $P\{+ | \text{Lie}\} = 0.11$ • con artist

$P\{- | \text{Truth}\} = 0.1$ • falsely accused
 $P\{+ | \text{Truth}\} = 0.9$ • exonerated

What is probability you lied if lie detector result says you lied? $= P\{\text{Lie} | -\}$

• Intuition says $\approx 90\%$, perhaps.

soln: Use Bayes Rule

$$P\{\text{Lie} | -\} = \frac{P\{- | \text{Lie}\} P\{\text{Lie}\}}{P\{-\}}$$

$$\text{but } P\{-\} = P\{- | \text{Lie}\} P\{\text{Lie}\} + P\{- | \text{Truth}\} P\{\text{Truth}\}$$

Need more info. Need $P\{\text{Lie}\}$ and $P\{\text{Truth}\}$.

Suppose $P\{\text{Lie}\} = 0.05$, $P\{\text{Truth}\} = 0.95$

• Most people tell the truth.

$$P\{\text{Lie} | -\} = \frac{0.89 \cdot 0.05}{0.89 \cdot 0.05 + 0.1 \cdot 0.95} \doteq 0.32$$

• Only 32% chance you were lying if detector says so. % gets higher if more people tell the truth!

ex: Speech Recognition

Parent says "yes" or "no".

Baby hears "yoyo." Does that mean "yes"? or "no"?

$$P\{yes\} = \frac{5}{6} \cdot (\text{const} < 1) \text{ permissive parent}$$

$$P\{no\} = \frac{1}{6} \cdot (\text{const} < 1) \cdot \text{Only "yes" or "no" ever spoken to baby? Strict parents? No. Other words also used. Ratio } P\{yes\} \text{ is known.}$$

$$\text{Given: } P\{yoyo \text{ heard} | \text{"yes" said}\} = \frac{1}{2} P\{yoyo \text{ heard} | \text{"no" said}\}$$

• Only ratios of probabilities known.

Which was more likely, "yes" or "no" if we heard "yoyo"?

soln: Use Bayes' Thm

$$P\{yes | yoyo\} = \frac{P\{yoyo | yes\} P\{yes\}}{P\{yoyo\}}$$

$$P\{no | yoyo\} = \frac{P\{yoyo | no\} P\{no\}}{P\{yoyo\}}$$

Note: $P\{yoyo\} \equiv$ Probability of someone saying "yoyo" at any arbitrary time. For example, what is probability of turning on radio and 1st word announcer says is "yoyo"?
Not conditioned on anything.

$$P\{yoyo\} \approx 0 \quad \text{Exact value unknown.}$$

soln: Just compute ratio of $\frac{P\{yes | yoyo\}}{P\{no | yoyo\}}$.

Both have same denominator, $P\{yoyo\}$, that cancels out.

3. 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840.