Spring 2007 ECE 6962 Advanced topics in communication Homework 1

Assign date: 1/16/07 Due date: 1/30/07 (Tuesday) at the beginning of class.

1. Small Scale Fading. Consider the delay profile with the 13 paths, and the following parameters. The sequence of squared path gains $(\alpha_0^2, \alpha_1^2, \cdots, \alpha_{12}^2)$ is given by

$$(0.5, 0.025, 0.075, 0.1, 0.075, 0.025, 0.025, 0.075, 0.025, 0.025, 0.02, 0.02, 0.01)$$

and the corresponding sequence of path delays $(\tau_0, \tau_1, \cdots, \tau_{12})$ is given by:

$$(0, 1.2, 1.4, 1.6, 1.8, 2.0, 3.4, 3.6, 3.8, 4.0, 5.2, 5.4, 5.6)\mu s$$

(a) Carefully sketch the delay profile $|h(t;\xi)|$ with the heights of the δ functions drawn approximately to scale to match the path gains.

(b) Is there an LOS path? If so, what is the Rice factor?

(c) Argue that a signal with bandwidth $W = 20$ KHz sees the channel as a flat fading channel, and write down an expression for the pdf $p_\alpha(\alpha)$ of the effective channel gain α.

(d) Argue that a signal with bandwidth $W = 1$ MHz sees the channel as a frequency selective channel, and draw the tapped delay line model for $\frac{1}{W}$-spaced taps.

(e) Suppose the carrier frequency $f_c = 1$ GHz, the mobile velocity is 60 km/hr, and the symbol rate is 10^5 symbols/s. Find the coherence time. Is the fading slow?

2. Simulating Ricean flat fading. Using the direct approach we discussed in class to simulate a Ricean flat fading process with Rice factor $\kappa = 1$. Assume that the LOS component arrives at angle $\theta_0 = \pi/4$, and that the diffuse components are uniformly distributed in angle and power (isotropic). Let the total number of diffuse components be 12, and assume $f_m = 60$ Hz.

Plot the channel gain $|V(t)|$ in dB as a function of t, for t ranging from 0 to 250 ms.

3. Ricean Random Variables. Assume $X \sim N(a, \sigma^2)$ and $Y \sim N(b, \sigma^2)$ are independent Gaussian random variables, where a and b are deterministic constants. Define R and Θ by:

$$R = \sqrt{X^2 + Y^2}, \text{ and } \Theta = \tan^{-1}(\frac{Y}{X})$$

(Assume $\Theta \in [-\pi, \pi]$.) Find the joint pdf of R and Θ and from this the marginal pdf of R. Express the latter in term of the modified Bessel function of the first kind:

$$I_0(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp(x \cos\phi) d\phi$$

The pdf of R in this case is called Ricean.

4. Auto-correlation function (ACF) of flat fading process $\{V(t)\}$. Consider the flat fading process $\{V(t)\}$ for purely diffuse (Rayleigh) fading. We showed in class that for fixed t, $V(t)$ is a zero mean, unit variance proper complex Gaussian (PCG) random variable with a Rayleigh envelope. Since $\{V(t)\}$ is Gaussian, all that is needed to characterize it completely (statistically) is its ACF.
(a) Use the approximation $\phi_n(t+\tau) - \phi_n(t) \approx 2\pi f_m \tau \cos\theta_n$ to show that

$$R_V(t+\tau, t) \approx \sum_n \beta_n^2 e^{j2\pi f_m \tau \cos\theta_n}$$

Is $\{V(t)\}$ is a stationary process?

(b) Consider the situation where the propagation environment is such that power β_n is received roughly uniformly from all directions. We may approximate such isotropic scattering by continuum of paths and let

$$R_V(\tau) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j2\pi f_m \tau \cos\theta} d\theta$$

Show that $R_V(\tau)$ equals $J_0(2\pi f_m \tau)$, where $J_0(\cdot)$ is the zero-th order Bessel function of the first kind.

$$J_0(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos(x\cos\theta) d\theta$$