Full Name: ______________________________

ECE 6534 (Spring 2017) – Exam **Date: March 20, 2017**

Question	# of Points Possible	# of Points Obtained	Grader
# 1	20		
# 2	19		
# 3	20		
# 4	20		
# 5	21		
Total	100		

For justifications: Justifications need to show you understand the underlying reasons *why* your answer is correct.

Before starting the exam, read and sign the following agreement.
By signing this agreement, I agree to solve the problems of this exam while adhering to the policies and guidelines of the University of Utah and ECE 6534 and without additional external help. The guidelines include, but are not limited to,

- One 8.5 inch by 11 inch cheat sheet is allow
- No textbooks may be used
- No calculators or computers may be used
- No collaboration is allowed
- No cheating is allowed

______________________________ ______________________________

Student Date

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Question #1: Let $\mathcal{S}_2$ and $\mathcal{S}_1$ be defined by the following sets

$$\mathcal{S}_1 = \{ \quad x = |\alpha| + 1 \quad | \quad \alpha \in \mathbb{R} \quad \} \qquad , \qquad \mathcal{S}_2 = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \right\}$$

(a) *(5 pts)* Is $\mathcal{S}_1$ a subspace of $\mathbb{R}$? **Briefly justify why.**

(b) *(5 pts)* Are the elements of $\mathcal{S}_1$ linearly independent? **Justify why.**

(c) *(5 pts)* Determine $\dim(\mathcal{S}_2)$. **Justify why.**

(d) *(5 pts)* Are the elements of $\mathcal{S}_2$ orthogonal? **Justify why.**

Question #2: Let I be the $N \times N$ identity matrix, where all of the diagonal values (top-left to bottom-right) are 1. Let J be the $N \times N$ exchange matrix, where all of the anti-diagonal values (top-right to bottom-left) are 1. Assume $N = 4$ and let $A = (I + J)/\sqrt{2}$.

(a) *(6 pts)* Determine the range space of A (i.e., $\mathcal{R}(A)$).

(b) *(6 pts)* Determine the null space of A (i.e., $\mathcal{N}(A)$).

(c) *(7 pts)* (True / False) A is a projection operator. **Justify why.**

Question #3: Consider a finite-length rank-one matrix defined by

$$R = vv^H$$

where v is an $N \times 1$ vector.

(a) *(7 pts)* Can R be time-invariant? If so, with what condition on the values of v is this true?

(b) *(7 pts)* Can R be memoryless? If so, with what condition on the values of v is this true?

(c) *(6 pts)* Can R be causal? If so, with what condition on the values of v is this true?

Question #4:

(a) *(7 pts)* (True / False) If the eigenvaluess of a matrix H are only 1 and -1, then H is an orthogonal matrix. **Justify why.**

(b) *(6 pts)* (True / False) The product of all singular values of a matrix is a norm. **Justify why.**

(c) *(7 pts)* (True / False) A tight frame must be full rank (i.e., it must have the maximum possible number of linearly independent columns). **Justify why.**

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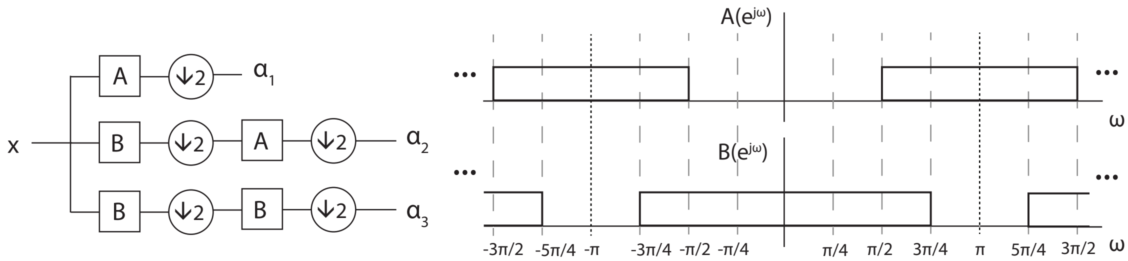
Question #5:

(a) *(10 pts)* Consider the filter bank and filters below. Simplify each branch. Draw the equivalent, simplified filter bank and sketch the equivalent filter frequency responses.



(b) *(5 pts)* Sketch the block diagram for

$$y = AU_3D_3A^*x + U_3D_3x$$

(c) *(6 pts)* Sketch the time-frequency tiling for the following wavelet packet. Assume the top filters are high pass and the bottom filters are low pass and the downsamplings are by 2.