

Full Name: _____
ECE 6534 (Spring 2015) – Exam Date: March 12, 2015

Question	# of Points Possible	# of Points Obtained	Grader
# 1	16		
# 2	18		
# 3	17		
# 4	17		
# 5	16		
# 6	16		
Total	100		

For justifications: Justifications need to show you understand the underlying reasons *why* your answer is correct.

Before starting the exam, read and sign the following agreement.

By signing this agreement, I agree to solve the problems of this exam while adhering to the policies and guidelines of the University of Utah and ECE 6534 and without additional external help. The guidelines include, but are not limited to,

- Notes are allowed
- No textbooks may be used
- No calculators or computers may be used
- No collaboration is allowed
- No cheating is allowed

Student

Date

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Question #1: Let $\mathcal{S}$ be defined by the set of all real, Toeplitz $N \times N$ matrices, a subspace of $\mathbb{R}^{N \times N}$ over the field of real numbers $\mathbb{R}$.

(a) (5 pts) Is $\mathcal{S}_1 = \{\alpha I | \alpha \in \mathbb{R}\}$, where I is identity, a subspace of $\mathcal{S}$? **Briefly justify why.**

(b) (5 pts) Is the set of all Toeplitz $N \times N$ matrices with strictly positive elements a subspace of $\mathcal{S}$? **Briefly justify why.**

(c) (6 pts) Determine $\dim(\mathcal{S})$. **Justify why.**

Question #2: A permutation matrix P is a $N \times N$ matrix in which exactly one element in each row and each column is equal to 1 and all other elements are equal to 0.

(a) (5 pts) (True / False) P has a null space of strictly $\mathcal{N}(P) = \mathbf{0}$. **Justify why.**

(b) (4 pts) (True / False) P is an orthogonal operator. **Justify why.**

(c) (4 pts) (True / False) P is a projection operator. **Justify why.**

(d) (5 pts) (True / False) $\sqrt{x^H P x}$ is a norm. **Justify why.**

Question #3: Let A be a linear, memoryless, and time-invariant system.
Let $B = D_3 U_2 U_3 D_2 A$

(a) (4 pts) Is B time-invariant? **Justify why.**

(b) (4 pts) Is B memoryless? **Justify why.**

(c) (4 pts) Is B causal? **Justify why.**

(d) (5 pts) Is $D_3 U_2 U_3 D_2$ a projection operator? **Justify why.**

Question #4:

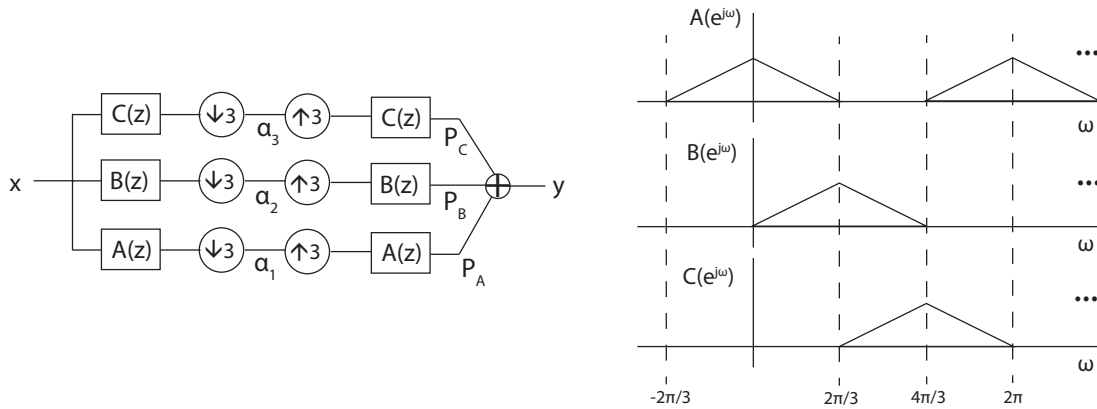
(a) (6 pts) Show that $I + A^*A$ is invertible for any $A \in \mathbb{R}^{N \times N}$.

(b) (5 pts) Show that if B is a λ -tight frame, then $I - \lambda^{-1}B^*B$ is orthogonal to B^*B .

(c) (6 pts) Determine the singular values of D_N . **Justify why.**

Question #5:

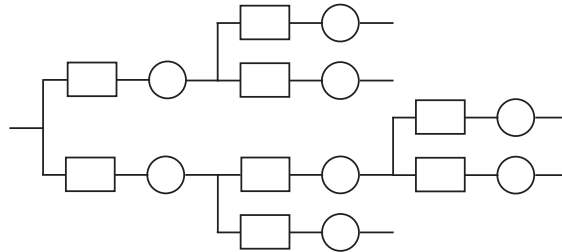
- (a) (8 pts) Consider the filter bank and filters below. For an impulse input, sketch the frequency response of α_1 , α_2 , and α_3 as well as P_A , P_B , and P_C . [the maximum amplitude of each frequency response is $\sqrt{3}$]



- (b) (8 pts) Does the filter bank above satisfy the conditions for perfect reconstruction in general (for any input x)? **Justify why.**

Question #6:

- (a) (8 pts) Sketch the time-frequency tiling for the following wavelet packet. Assume the top filters are high pass and the bottom filters are low pass and the downsamplings are by 2.



(b) (8 pts) Design a wavelet packet tree for the following time-frequency tiling.

