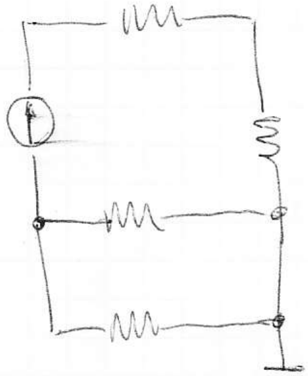
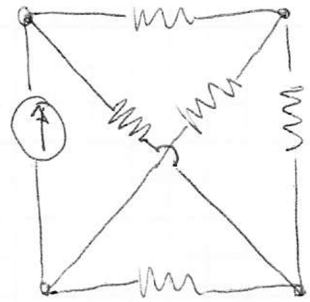
PLANAR



← PLANAR →

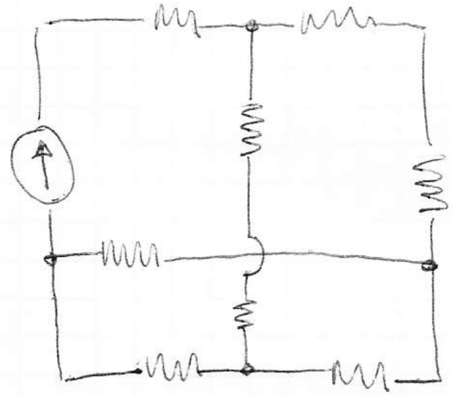
NON-PLANAR OR PLANAR?



KVL OR KCL FOR PLANAR

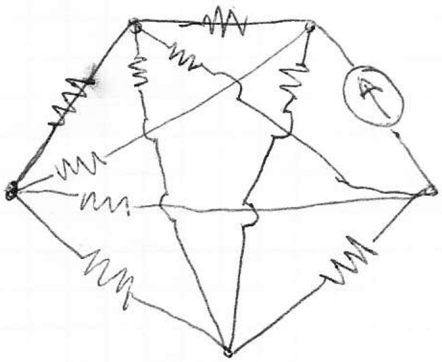
ONLY KCL FOR NON-PLANAR

HOW ABOUT THIS?



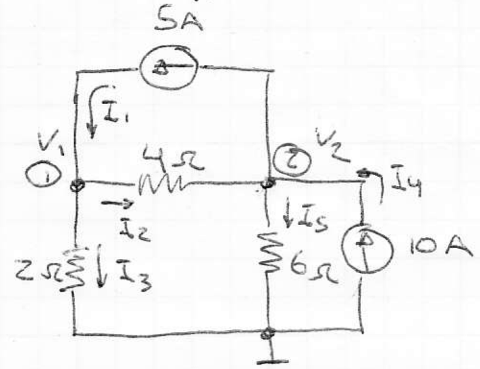
← PLANAR

AND THIS ONE?



← NON-PLANAR

CONSIDER THE FOLLOWING CIRCUIT:



APPLYING KCL @ 1,

$$I_1 = I_2 + I_3 \longrightarrow 5 = \frac{V_1 - V_2}{4} + \frac{V_1}{2}$$

$\times 4$, (LEAST COMMON MULTIPLE OF 4 & 2)

$$20 = V_1 - V_2 + 2V_1$$

$$\boxed{3V_1 - V_2 = 20}$$

APPLYING KCL @ 2

$$I_2 + I_4 = I_5 + I_1 \longrightarrow \frac{V_1 - V_2}{4} + 10 = \frac{V_2}{6} + 5$$

$\times 12$ (LCM)

$$3V_1 - 3V_2 + 120 = 2V_2 + 60$$

$$\boxed{-3V_1 + 5V_2 = 60}$$

IN MATRIX FORM,

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix} \longrightarrow \boxed{\begin{array}{l} V_1 = 13.33V \\ V_2 = 20\ V \end{array}}$$

$$G \quad V = I$$

I → KNOWN (SOURCES)

V → UNKNOWN

G → KNOWN (SYSTEM) HAS DIMENSIONS OF ADMITTANCE $1/\Omega$ = SIEMENS OR MHO $℧$.

2

Static Solution for Linear Networks

adapted from Carlos J. Cela PhD dissertation, April 2012

The static solution constitutes the simplest case for a linear network, in which all current sources are DC, and all equivalent capacitances in the circuit are completely charged. Under these conditions, capacitors are replaced by open circuits, and the network can be considered purely resistive.

Nodal analysis is used to create a suitable linear system, considering m nodes, and excluding ground. The system is solved for the voltage value at each node.

$$G\ V = I \qquad (3.9)$$

This is equivalent to use Kirchoff Current Law. KCL is chosen as a method because in 3D structures the topologies may not be planar, leading to cases that cannot be solved using KVL. The resulting linear system is of the form (3.9), where G is the sparse symmetric admittance matrix representing the model's admittance, V is the unknown voltage vector, and I the vector describing the currents being injected at each node.

Forming the Admittance Matrix G

If the network has m + 1 nodes, G is the $m \times m$ conductance matrix; each node of the network is present in both the rows and the columns of the matrix. The ground node is implicit. G is formed from the network resistors by using the following steps derived from nodal analysis:

1. Nodes are sequentially labeled from 1 to m. Ground node is not counted.
2. All values of G are initialized to zero.
3. For each pair of nodes (a,b):
   (a) If a = b, and there are K resistances connected to the node:

$$G_{a,b} = \sum_{k=1}^{K} \frac{1}{R_k}$$

   (b) If $a \neq b$, and the nodes have K resistances connected between them:

$$G_{a,b} = G_{b,a} = -\sum_{k=1}^{K} \frac{1}{R_k}$$

The resulting system will yield a sparse symmetric G matrix.

Forming the Current Vector I

The I current vector describes the independent current sources connected to the network. These are the vehicle to inject energy into the model. There is one entry for each node in the system, labeled from 1 to m.

1. Nodes are sequentially labeled from 1 to m, using the same order used in G. Ground node is not counted.

2. All values of I are initialized to zero.

3. For each current source i having its positive terminal connected to node a and its negative terminal connected to node b:

   (a) If a is the ground node:

   $$Ib = Ib - i$$

   (b) If b is the ground node:

   $$Ia = Ia + i$$

   (c) If neither a or b are the ground node:

   $$Ia = Ia + i$$
   $$Ib = Ib - i$$

Solving the Linear System

In order to minimize computer memory usage, it is desirable to keep the admittance matrix sparse while solving the linear system; this makes direct methods such a Gaussian elimination a poor choice for this problem. Some authors report using successive overrelaxation (SOR) and Gauss-Seidel. In our experience, Krylov sub-space iterative gradient- descent methods have proved effective.

USING THE GENERIC METHOD, WE FIRST SETUP THE ADMITTANCE MATRIX G,

$$\begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c}\begin{array}{cc} 1 & 2 \end{array} \\ \left[\begin{array}{c|c} \; & \; \\ \hline \; & \; \end{array}\right]\end{array}$$

HAVING ONE ROW AND ONE COLUMN PER NODE, WITH OUT COUNTING THE GROUND NODE. APPLYING THE STEPS DISCUSSED, G IS

$$G = \begin{bmatrix} \frac{1}{4} + \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} + \frac{1}{6} \end{bmatrix} = \begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.416\hat{6} \end{bmatrix}$$

AND I IS

$$\begin{array}{c} 1 \\ 2 \end{array}\left[\begin{array}{c} \; \\ \hline \; \end{array}\right] \Rightarrow \quad I = \begin{bmatrix} 5 \\ -5+10 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

RESULTING ON A LINEAR SYSTEM EQUIVALENT TO THE PREVIOUS SOLUTION,

$$\begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.4166 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix} \Rightarrow \boxed{\begin{array}{l} V_1 = 13.33V \\ V_2 = 20V \end{array}}$$