FDTD IN DISPERSIVE, LOSSY MEDIA

THE TERM 'DISPERSIVE' REFERS TO THE PROPERTY OF CERTAIN MATERIALS TO HAVE DIFFERENT SPEED OF PROPAGATION FOR DIFFERENT FREQUENCIES. IN THE TIME DOMAIN, THIS CAUSES WAVEFORM DISTORTION.

THERE ARE MULTIPLE APPROACHES TO MODEL DISPERSION, ONE OF THE MOST USED IS THE 'DEBYE MODEL'

DEBYE MODEL

STARTING FROM MAXWELL'S EQUATIONS, AND CONSIDERING DIELECTRIC POLARIZATION, IN OUR 1-D CASE,

$$\begin{cases} \varepsilon \dfrac{\partial E_x}{\partial t} = -\dfrac{\partial H_y}{\partial z} - \sigma E_x - \dfrac{\partial P}{\partial t} \\[2ex] \dfrac{\partial H_y}{\partial t} = -\dfrac{\partial E_x}{\partial z} \end{cases}$$

WHERE

$$D_x = \varepsilon E_x + P$$ AND P IS THE DIELECTRIC POLARIZATION

WE CAN DEFINE P IN TERMS OF A CONVOLUTION,

$$P(t,z) = g * E_x(t,z) = \int_0^t g(t-s, x, \nu)\, E_x(s,z)\, ds$$

WHERE g IS A GENERAL DIELECTRIC RESPONSE FUNCTION AND ν IS SOME PARAMETER SET.

ON THE DEBYE MODEL,

$$g(t,z) = \varepsilon_0 (\varepsilon_s - \varepsilon_\infty)/\tau \; e^{-t/\tau}$$

OR

$$\tau \frac{\partial P}{\partial t} + P = \varepsilon_0 (\varepsilon_s - \varepsilon_\infty) E$$

CONVERTING TO FREQUENCY DOMAIN VIA FOURIER TRANSFORMS

$$D(\omega) = \varepsilon(\omega)\, E$$

AND THE DEBYE MODEL IS

$$\boxed{\varepsilon(\omega) = \varepsilon_r + \frac{\sigma}{j\omega\varepsilon_0} + \frac{\chi_1}{1 + j\omega t_0}}$$

WHERE

$$\chi_1 = (\varepsilon_S - \varepsilon_\infty)$$

$$t_0 = \tau$$

IN TERMS OF CODE,

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Dx(K) = Dx(K) + 0.5 * (Hy(K-1) - Hy(K))
Ex(K) = gax(K) * (Dx(K) - Ix(K) - DELEXP * Sx(K))
Ix(K) = Ix(K) + gbx(K) * Ex(K)
Sx(K) = DELEXP * Sx(K) + gbc(K) * Ex(K)
Hy(K) = Hy(K) + 0.5 * (Ex(K) - Ex(K-1))
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WHERE

$$gax(K) = 1/(\varepsilon r(K) + (\sigma(K) * dt/\varepsilon_0) + (chi1(K) * dt/t_0))$$

$$gbx(K) = \sigma(K) * dt/\varepsilon_0$$

$$gbc(K) = chi1(K) * dt/t_0$$

AND

$$DEL\ EXP = EXP(-dt/t_0)$$