FDTD D-H FORMULATION

THE E-H FORMULATION WE HAVE BEEN USING IS IDEAL FOR LEARNING THE BASICS, BUT HAS THE DRAWBACK THAT MIXES THE ITERATIVE EQUATIONS WITH THE MATERIAL MODEL. THE D-H FORMULATION ALLOWS FOR SEPARATE TREATMENT OF THESE TWO ASPECTS.

STARTING FROM

$$\begin{cases} \dfrac{\partial \overline{D}}{\partial t} = \nabla \times \overline{H} \\[2ex] \overline{D}(\omega) = \varepsilon_0 \varepsilon_r^*(\omega) \cdot \overline{E}(\omega) \\[2ex] \dfrac{\partial \overline{H}}{\partial t} = -\dfrac{1}{\mu_0} \nabla \times \overline{E} \end{cases}$$

WHERE $\overline{D}$ IS THE ELECTRIC FLUX DENSITY VECTOR, AND

$\varepsilon_r^*(\omega) = \varepsilon_r + \dfrac{\sigma}{j\omega \varepsilon_0}$ (THIS IS JUST ONE OF THE POSSIBLE FORMULATIONS FOR ε_r^*).

AS BEFORE, WE NORMALIZE THE $\overline{E}$ (AND $\overline{D}$) FIELD BY

$$\underline{E} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot \overline{E}$$

$$\underline{D} = \sqrt{\frac{1}{\varepsilon_0 \mu_0}} \cdot \overline{D}$$

WHICH RESULTS IN

$$\begin{cases} ① \quad \dfrac{\partial \underline{D}}{\partial t} = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}} \nabla \times \overline{H} \\[2ex] ② \quad \underline{D}(\omega) = \varepsilon_r^*(\omega) \cdot \underline{E}(\omega) \\[2ex] ③ \quad \dfrac{\partial \overline{H}}{\partial t} = -\dfrac{1}{\sqrt{\varepsilon_0 \mu_0}} \nabla \times \underline{E} \end{cases}$$

(1) AND (3) RESULT IN VERY SIMPLE FINITE DIFFERENCE EXPRESSIONS. BUT (2) IS EXPRESSED IN THE FREQUENCY DOMAIN, AND WE NEED IT IN THE TIME DOMAIN.

WE CAN WRITE (2) AS:

$$\underline{D}(\omega) = \varepsilon_r \cdot \underline{E}(\omega) + \frac{\sigma}{j\omega\varepsilon_0} \cdot \underline{E}(\omega)$$

IN TIME DOMAIN, $\frac{1}{j\omega} \rightarrow$ INTEGRATION, SO

$$\underline{D}(t) = \varepsilon_r \cdot \underline{E}(t) + \frac{\sigma}{\varepsilon_0} \int_0^t \underline{E}(t)dt$$

IN SAMPLED-TIME DOMAIN, WE APPROXIMATE THE INTEGRAL AS SUMMATON OVER TIME,

$$\underline{D}^n = \varepsilon_r \cdot \underline{E}^n + \frac{\sigma \Delta t}{\varepsilon_0} \sum_{i=0}^{n} \underline{E}^i$$, WHERE D & E ARE CALCULATED AT $t = \Delta t \cdot n$

SINCE ~~FOR~~ WE NEED D_n TO CALCULATE E_n, AND VICEVERSA, WE REWRITE THE LAS EXPRESSION AS:

$$\underline{D}^n = \varepsilon_r \underline{E}^n + \frac{\sigma \Delta t}{\varepsilon_0} E^n + \frac{\sigma \Delta t}{\varepsilon_0} \sum_{i=0}^{n-1} \underline{E}^i$$

AND FINALLY,

(4) $$\underline{E}^n = \frac{\underline{D}^n - \frac{\sigma \Delta t}{\varepsilon_0} \sum_{i=0}^{n-1} E_i}{\varepsilon_r + \frac{\sigma \cdot \Delta t}{\varepsilon_0}}$$

THEN, WE CAN CALCULATE $\underline{E}^n$ FROM THE CURRENT VALUE OF $\underline{D}^n$ AND PREVIOUS VALUES OF $\underline{E}$. WE DEFINE AN AUXILIARY VALUE

$$I^n = \frac{\sigma \cdot \Delta t}{\varepsilon_0} \sum_{i=0}^{n} \underline{E}^i$$

NOTE: I^n IS NOT IN UNITS OF CURRENT!

4 CAN NOW BE FORMULATED AS:

$$\begin{cases} \underline{E}^n = \dfrac{\underline{D}^n - \underline{I}^{n-1}}{\varepsilon_r + \dfrac{\sigma \cdot \Delta t}{\varepsilon_0}} \\ \\ I^n = I^{n-1} + \dfrac{\sigma \Delta t}{\varepsilon_0} \underline{E}^n \quad (5) \end{cases}$$

WHERE (5) SIMPLY ACCUMMULATES THE VALUE AT EACH TIME-STEP.

WRITING THIS IN TERMS OF COMPUTER CODE, ASSUMING OUR 1D E_x, D_x, H_y FORMULATION,

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Dx(K) = Dx(K) + 0.5 * ( Hy(K-1) - Hy(K) )
Ex(K) = gax(K) * ( Dx(K) - Ix(K) )
Ix(K) = Ix(K) + gbx(K) * Ex(K)
Hy(K) = Hy(K) + 0.5 ( Ex(K) - Ex(K+1) )
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WHERE

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gax(K) = 1/(EPSILONR +((SIGMA*dt)/EPSILONZ))
gbx(K) = SIGMA * dt / EPSILONZ
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