## PROPAGATION IN A LOSSY DIELECTRIC MEDIUM

WE START FROM THE MAXWELL CURL EQUATIONS, THIS TIME CONSIDERING THE CURRENT DENSITY TERM

$$\begin{cases} \dfrac{\partial \bar{E}}{\partial t} = \dfrac{1}{\varepsilon}\left(\nabla \times \bar{H} - \bar{J}\right) \\ \\ \dfrac{\partial \bar{H}}{\partial t} = -\dfrac{1}{\mu_0} \nabla \times \bar{E} \end{cases}$$

THE TERM $\bar{J}$ CAN BE WRITTEN AS

$$\bar{J} = \sigma . \bar{E}$$

WHERE $\sigma$ IS THE CONDUCTIVITY. THEN, AMPERE'S LAW CAN BE WRITTEN AS:

$$\frac{\partial \bar{E}}{\partial t} = \frac{1}{\varepsilon_0 \varepsilon_r} \nabla \times \bar{H} - \frac{\sigma}{\varepsilon_0 \varepsilon_r} \bar{E}$$

IN OUR ONE-DIMENSIONAL FORMULATION, USING NORMALIZED EXPRESSIONS FOR THE ELECTRIC FIELD,

$$\begin{cases} \dfrac{\partial \underline{\mathcal{E}}_x}{\partial t} = -\dfrac{1}{\varepsilon_r \sqrt{\varepsilon_0 \mu_0}} \dfrac{\partial H_y}{\partial z} - \dfrac{\sigma}{\varepsilon_r \varepsilon_0} \underline{\mathcal{E}}_x \quad (A) \\ \\ \dfrac{\partial H_y}{\partial t} = -\dfrac{1}{\sqrt{\varepsilon_0 \mu_0}} \dfrac{\partial \underline{\mathcal{E}}_x}{\partial z} \end{cases}$$

TAKING THE FINITE DIFFERENCE APPROXIMATION, AND APPROXIMATING THE $\bar{J}$ TERM AS A TIME-AVERAGE,

$$(A) \quad \frac{\underline{\mathcal{E}}_x^{n+1/2}(k) - \underline{\mathcal{E}}_x^{n-1/2}(k)}{\Delta t} = -\frac{1}{\varepsilon_r \sqrt{\varepsilon_0 \mu_0}} \frac{H_y^{n}(k+1/2) - H_y^{n}(k-1/2)}{\Delta z}$$

$$- \frac{\sigma}{\varepsilon_r \varepsilon_0} \frac{\underline{\mathcal{E}}_x^{n+1/2}(k) + \underline{\mathcal{E}}_x^{n-1/2}(k)}{2}$$

AS IN THE PREVIOUS FORMULATIONS,

$$\frac{1}{\sqrt{\varepsilon_0 \mu_0}} \frac{\Delta t}{\Delta z} = \frac{1}{2}$$

THEN, EQ (A) BECOMES

$$\text{(A)} \quad \underline{E}_x^{n+1/2}(k)\left[1+\frac{\Delta t\,\sigma}{2\varepsilon_r\varepsilon_0}\right] = \underline{E}_x^{n-1/2}(k)\left[1-\frac{\Delta t\,\sigma}{2\varepsilon_0\varepsilon_r}\right] - \frac{1/2}{\varepsilon_r}\left[H_y^n(k+\tfrac{1}{2}) - H_y^n(k-\tfrac{1}{2})\right]$$

AND

$$\underline{E}_x^{n+1/2}(k) = \frac{\left(1-\frac{\Delta t\,\sigma}{2\varepsilon_r\varepsilon_0}\right)}{\left(1+\frac{\Delta t\,\sigma}{2\varepsilon_r\varepsilon_0}\right)} \underline{E}_x^{n-1/2}(k) - \frac{1/2}{\varepsilon_r\left[1+\frac{\Delta t\,\sigma}{2\varepsilon_0\varepsilon_r}\right]}\left[H_y^n(k+\tfrac{1}{2}) - H_y^n(k-\tfrac{1}{2})\right]$$

IN TERMS OF COMPUTING CODE:

```
Ex(k) = ca(k) * Ex(k) + cb(k) * (Hy(k-1) - Hy(k))
Hy(k) = Hy(k) + 0.5 * (Ex(k) - Ex(k+1))
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WHERE

```
EAF = dt * SIGMA/(2 * EPSZ * EPSR)
ca(k) = (1 - EAF) /(1 + EAF)
cb(k) = 0.5/ (EPSR * (1 + EAF))
```

SEE EXAMPLE CODE

FDTD1DLDM.M