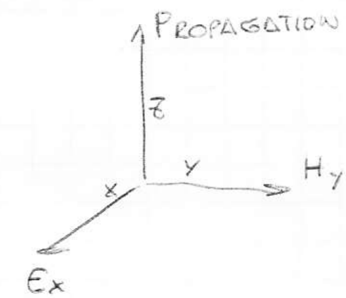
PROPAGATION IN DIELECTRIC MEDIUM

STARTING FROM AMPERE'S AND FARADAY'S LAWS, CONSIDERING A RELATIVE PERMITTIVITY,

$$\begin{cases} \dfrac{\partial \bar{E}}{\partial t} = \dfrac{1}{\varepsilon_r \varepsilon_0} \nabla \times \bar{H} \\[2ex] \dfrac{\partial \bar{H}}{\partial t} = -\dfrac{1}{\mu_0} \nabla \times \bar{E} \end{cases}$$

ASSUMING A ONE-DIMENSIONAL PROBLEM SPACE, WITH FIELDS AND PROPAGATION ORIENTED AS



THE EQUATIONS ABOVE SIMPLIFY TO (ASSUMING $\underline{E} = \sqrt{\dfrac{\varepsilon_0}{\mu_0}}\, \bar{E}$)

$$\begin{cases} \dfrac{\partial \underline{E}_x}{\partial t} = \dfrac{1}{\varepsilon_r \sqrt{\varepsilon_0 \mu_0}} \dfrac{\partial H_y}{\partial z} \\[2ex] \dfrac{\partial H_y}{\partial t} = -\dfrac{1}{\sqrt{\varepsilon_0 \mu_0}} \dfrac{\partial \underline{E}_x}{\partial z} \end{cases}$$

AND REMEMBERING THAT

$$\frac{1}{\sqrt{\varepsilon_0 \mu_0}} \frac{\Delta t}{\Delta z} = \frac{1}{2}$$

AND TAKING A FINITE DIFFERENCE APPROXIMATION FOR THE DERIVATIVES, WE OBTAIN:

$$\begin{cases} \underline{E}_x^{n+1/2}(k) = \underline{E}_x^{n+1/2}(k) + \dfrac{1}{2\,\varepsilon_r}\left[H_y^n(k+1/2) - H_y^n(k+1/2)\right] \\ H_y^{n+1}(k+1/2) = H_y^n(k+1/2) - \dfrac{1}{2}\left[\underline{E}_x^{n+1/2}(k+1) - \underline{E}_x^{n+1/2}(k)\right] \end{cases}$$

FROM WHICH WE CAN DERIVE EXPRESSIONS TO INCLUDE IN OUR NUMERICAL CODE:

$$\begin{cases} Ex(k) = Ex(k) + 0.5/er * (hy(k-1) - hy(k)) \\ Hy(k) = Hy(k) + 0.5 * (Ex(k) - Ex(k+1)) \end{cases}$$

$\boxed{\text{SEE EXAMPLE FDTD1dDM.m}}$ (H)