ECE5340/6340: Homework 11
Method of Moments

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University of Utah, Salt Lake City, Utah
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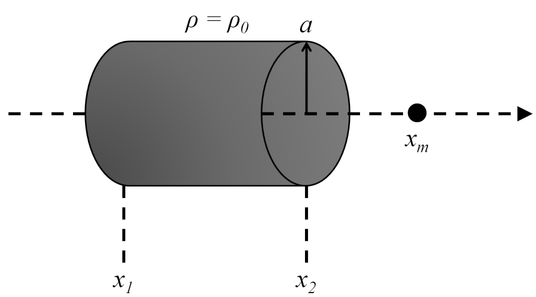
Write your section (ECE5340 or ECE6340) by your name. Turn in a printed copy containing the problem solutions, plots, and the code used to generate them. Remember to comment and format the code so it is legible to the graders. Label the plots appropriately, including units for each axis and for the values plotted. Assume all units to be SI units unless stated differently. Due Wednesday 4/4 BEFORE class begins.

ASSIGNMENT

1. (10 points) Calculate the voltage potential V at the point x_m along the axis of a hollow tube. Assume the tube has a uniform charge density ρ_0 spread out along the surface with a radius a. See the figure below:



   HINT: Set the problem up as the following integral:

$$V(x_m) = \frac{a}{4\pi\epsilon_0} \int_{x_1}^{x_2} \int_{0}^{2\pi} \frac{\rho_0}{\sqrt{(x_m - x')^2 + a^2}} \, d\theta dx'$$

   What is the voltage potential at the midpoint of the tube where $x_m = (x_1 + x_2)/2$?

2. (10 points) Consider a thin metal rod fixed at a constant voltage V_0 with some charge density $\rho(x)$ along its surface. If we break up the rod into N uniform segments, we may approximate the true distribution of charge using an expansion function with the form

$$\hat{\rho}(x') = \sum_{n=1}^{N} \alpha_n u_n(x') \ .$$

   Using the Dirac delta function as the basis u_n, calculate the residual function $R(x)$ that results from this approximation.

3. (10 points) How many equations and how many unknowns does the residual R from the previous problem represent? Is this an overdetermined or an underdetermined system of equations? Describe how we might manipulate this problem in order to generate a *unique* solution for the unknown expansion coefficients in α_n.

4. (10 points) Based on your answer to the previous problem, generate a system of linear equations that will uniquely determine the α_n coefficients. Explain your choice of *test* locations for x_m. Write out the general form for matrix coefficients $\mathbf{A}_{mn}$, including the self-terms.

5. (10 points) Explain why a charged metal rod must necessarily rest at a fixed voltage potential in a static system. What is the electric field inside of a charged metal rod? What would the charges do if the voltage were not constant along the rod?

6. (50 points) Using the method of moments, write a Matlab function that calculates the estimation function $\hat{\rho}$ for the charge distribution along a thin metal wire held at constant potential. Use the following parameters as input arguments:

$$\begin{aligned} L &= \text{Length of the wire (m)} \\ a &= \text{Wire radius (m)} \\ V_0 &= \text{Wire potential (V)} \\ h &= \text{Length of the wire subdivisions (m)} \end{aligned}$$

   Assume the use of delta functions for your basis in $\hat{\rho}$. Demonstrate your code by plotting $\hat{\rho}$ for the case of $L = 10$ m, $a = L/100$, $V_0 = 1.0$ V, and $h = L/60$. Calculate the total charge on the rod and comment on any peculiar behavior you notice.

7. ECE 6340 Only: (10 points) Plot $V(x)$ along $x \in [-L, 2L]$, where the rod sits along $[0, L]$. Assume your point charges in $\hat{\rho}$ are distributed uniformly along hollow tube segments with length h as shown in Problem 1. Comment on any observations you make.

8. EXTRA CREDIT: (20 points) Repeat the MoM problem using rectangle functions as the basis for $\hat{\rho}$. In other words, assume that the entire rod is a hollow cylindrical tube rather than point charges, and that the charges are uniformly distributed along little segments with length h. Plot $V(x)$ along $x \in [-L, 2L]$, where the rod sits along $[0, L]$. Comment on any observations you make. How could we better approximate a true metal rod?

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