ITERATIVE METHOD: GAUSS-SEIDEL

USED TO SOLVE PROBLEMS HAVING THE FORM

$$A x = b \qquad \text{(A IS A SQUARE } n \times n \text{ MATRIX)}$$

CONVERGENCE: GUARANTEED IF THE MATRIX IS DIAGONALLY DOMINANT (E.G. THE DIAGONAL ENTRY OF EACH ROW IS IN MAGNITUDE LARGER THAN THE SUM OF THE MAGNITUDES OF ALL THE OTHER ELEMENTS OF THAT ROW) OR SYMMETRIC AND POSITIVE-DEFINITE (E.G. $z^T M z > 0$ WHERE z IS A REAL VECTOR). MAY CONVERGE IN SOME OTHER CASES.

ITERATION FORMULA

$$x_i^{k+1} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k+1} - \sum_{j=i+1}^{n} a_{ij} x_j^{k} \right]$$

WHERE

$i = 1, 2, \ldots n$

$k = 1, 2 \ldots$ UNTIL CONVERGENCE (ITERATION #)

CONVERGENCE CRITERIA

$$\frac{\sum_{i=1}^{n} \left| x_i^{k+1} - x_i^{k} \right|}{\sum_{i=1}^{n} \left| x_i^{k+1} \right|} < \text{TOLERANCE}$$

WHERE TOLERANCE IS A SUITABLY SMALL NUMBER, (I.E. 1E-5).

(VIEW MATLAB EXAMPLE CODE)