DIRECT METHOD: GAUSSIAN ELIMINATION

PROCEDURE HAS TWO PARTS:

PART ① : REDUCE SYSTEM TO TRIANGULAR FORM USING ELEMENTARY ROW OPERATIONS. THIS IS KNOWN AS FORWARD ELIMINATION. WE USE THE AUGMENTED MATRIX FOR THIS STEP.

PART ② : USE BACK SUBSTITUTION TO FIND THE SOLUTION.

ELEMENTARY ROW OPERATIONS ARE:

(A) ROW SWITCHING $R_i \leftrightarrow R_j$

(B) ROW MULTIPLICATION $KR_i \rightarrow R_i \quad K \neq 0$

(C) ROW ADDITION $R_i + KR_j \rightarrow R_i$

EXAMPLE: CONSIDER THE LINEAR SYSTEM

$$x_1 + x_2 + x_3 = 0$$
$$x_1 - 2x_2 + 2x_3 = 4$$
$$x_1 + 2x_2 - x_3 = 2$$

PART ①
WE FORM THE CORRESPONDING AUGMENTED MATRIX

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -2 & 2 & 4 \\ 1 & 2 & -1 & 2 \end{array}\right]$$

SUBTRACTING NOW $R_3 - R_1 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -2 & 2 & 4 \\ 0 & 1 & -2 & 2 \end{array}\right]$$

$R_2 - R_1 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & 1 & 4 \\ 0 & 1 & -2 & 2 \end{array}\right]$$

$R_2 + 3R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & 1 & 4 \\ 0 & 0 & -5 & 10 \end{array}\right]$$

SINCE THE MATRIX IS TRIANGULAR, WE WRITE THE SYSTEM BACK AS A SET OF EQUATIONS,

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ -3x_2 + x_3 = 4 \\ -5x_3 = 10 \end{cases}$$

PART 2

WE USE BACK SUBSTITUTION TO SOLVE FOR x_1, x_2, AND x_3. STARTING FROM THE LAST EQUATION,

$$\boxed{x_3 = -2}$$

THEN, FROM THE SECOND EQUATION,

$$\boxed{x_2 = -2}$$

AND FROM THE FIRST EQUATION, $\boxed{x_1 = 4}$