SOLVING LINEAR SYSTEMS

A SYSTEM OF m LINEAR EQUATIONS WITH n UNKNOWNS CAN BE WRITTEN AS:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

WHERE

$x_1, x_2, \cdots, x_n \equiv$ UNKNOWNS

$a_{11}, a_{12}, \cdots, a_{mn} \equiv$ COEFFICIENTS

$b_1, b_2, \cdots, b_m \equiv$ CONSTANT TERMS

MATRIX EQUIVALENT

$$Ax = b$$

WHERE

$A \equiv m \times n$ COEFFICIENT MATRIX

$b \equiv m$-ELEMENT CONSTANT TERM COLUMN VECTOR

$X \equiv n$-ELEMENT UNKNOWNS COLUMN VECTOR

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}}_{b}$$

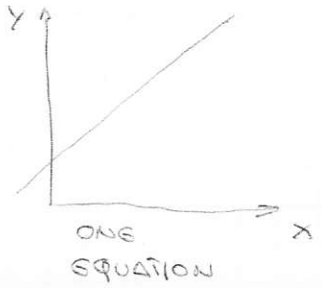
SOLUTION SET

A LINEAR SYSTEM MAY

1. HAVE INFINITE NUMBER OF SOLUTIONS
2. HAVE A UNIQUE SOLUTION
3. HAVE NO SOLUTION

USUALLY (BUT NOT ALWAYS),

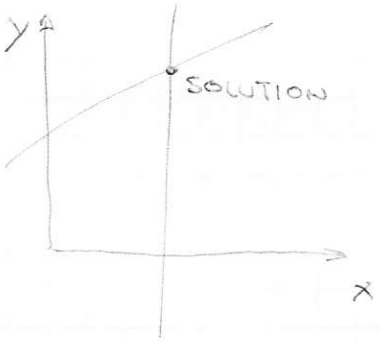
1. A SYSTEM WITH FEWER INDEPENDENT EQUATIONS THAN UNKNOWNS HAS INFINITE SOLUTIONS (UNDERDETERMINED)



ONE EQUATION

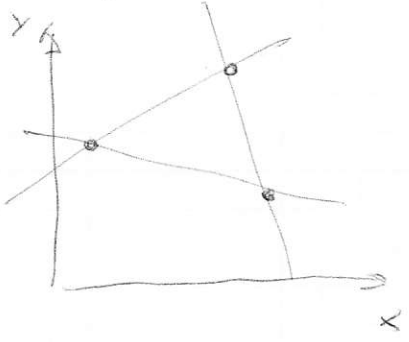
EXAMPLE: ONE EQUATION TWO UNKNOWNS (LINE).

2. A SYSTEM WITH THE SAME NUMBER OF EQUATIONS THAN UNKNOWNS HAS ONE SOLUTION.



2 EQUATIONS
2 UNKNOWNS

3. A SYSTEM WITH MORE EQUATIONS (LINEARLY INDEPENDENT) THAN UNKNOWNS IS OVERDETERMINED AND HAS NO SOLUTION (I.E. ALL EQUATIONS CANNOT BE SATISFIED SIMULTANEOUSLY).



3 EQUATIONS,
2 UNKNOWNS