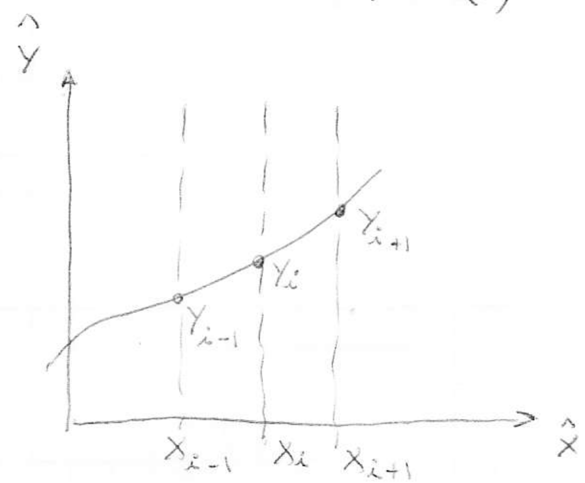
NUMERICAL DIFFERENTIATION

CONSIDER A FUNCTION

$$Y = F(X)$$



OF WHICH WHICH WE KNOW THE VALUE OF Y ONLY AT A FINITE SET OF POINTS. THE DERIVATIVE OF F(X) CAN BE NUMERICALLY APPROXIMATED AT THESE POINTS USING DIFFERENT FORMULATIONS.

FORWARD DIFFERENCE

$$F'(X_i) \cong \frac{Y_{i+1} - Y_i}{X_{i+1} - X_i}$$

ERROR $= O(h)$

BACKWARD DIFFERENCE

$$F'(X_i) \cong \frac{Y_i - Y_{i-1}}{X_i - X_{i-1}}$$

ERROR $= O(h)$

CENTRAL DIFFERENCE

$$F'(X_i) \cong \frac{Y_{i+1} - Y_{i-1}}{X_{i+1} - X_{i-1}}$$

ERROR $= O(h^2)$

HIGHER ORDER DERIVATIVES (CENTRAL DIFFERENCE FORMULAS)

$$F''(x_i) \simeq \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

$$\begin{cases} h = x_{i+1} - x_i \\ h = x_i - x_{i-1} \end{cases}$$

$$F'''(x_i) \simeq \frac{1}{2h^3}\left[y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}\right]$$

$$F^4(x_i) \simeq \frac{1}{h^4}\left[y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}\right]$$

PARTIAL DERIVATIVES

SUPPOSE $U = U(x,y,z)$ IS A FUNCTION OF 3 VARIABLES THAT WE ONLY KNOW AT GRID POINTS (x_i, y_j, z_k). WE USE THE NOTATION

$$U_{(i,j,k)} = U(x_i, y_j, z_k)$$

WE ASSUME THE GRID POINTS ARE EVENLY SPACED IN EACH OF THE 3 DIRECTIONS, WITH AN INCREMENT OF a IN THE $\hat{x}$ DIRECTION, b IN $\hat{y}$, AND c IN $\hat{z}$.

$$U_x(x_i, y_i, z_i) \simeq \frac{1}{2a}\left(U_{(i+1,j,k)} - U_{(i-1,j,k)}\right)$$

$$U_y(x_i, y_i, z_i) \simeq \frac{1}{2b}\left(U_{(i,j+1,k)} - U_{(i,j-1,k)}\right)$$

$$U_z(x_i, y_j, z_i) \simeq \frac{1}{2c}\left(U_{(i,j,k+1)} - U_{(i,j,k-1)}\right)$$

ALL THE ABOVE FORMULAS ARE BASED ON THE CENTRAL DIFFERENCE.