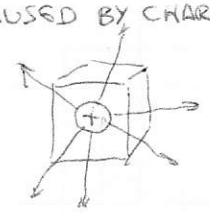
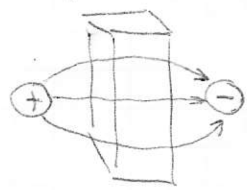
GAUSS'S LAW FOR $\bar{E}$ FIELDS

$\bar{E}$ FIELD : ELECTRICAL FORCE PER UNIT CHARGE EXERTED ON A CHARGED OBJECT.

CAUSED BY VARYING $\bar{B}$ FIELDS:

CAUSED BY CHARGE:





$$\bar{E} \equiv \frac{F_e}{q_0} \quad \text{WHEN } q_0 \to 0 \qquad \text{(FROM LORENTZ EQ)}$$

① $\bar{E}$ IS PROPORTIONAL TO $\bar{F}_E$

② $\bar{E}$ HAS UNITS OF $\frac{N}{C} = \frac{V}{m}$ BECAUSE $V = \frac{N\,m}{C}$

③ FOR A POINT CHARGE q,

$$\bar{E} = \frac{1}{4\pi\varepsilon_0 r^2}\, q\, \hat{r} \qquad \text{COULOMB'S LAW}$$

④ $\bar{E}$ FIELD LINES ORIGINATE ON POSITIVE CHARGES AND TERMINATE IN NEGATIVE CHARGES.

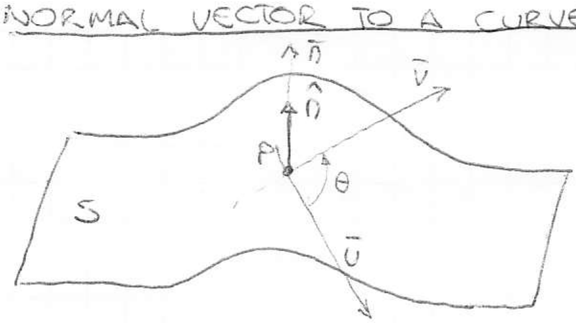
⑤ THE NET $\bar{E}$ FIELD AT A POINT IS THE VECTOR SUM OF ALL FIELDS AT THAT POINT.

⑥ $\bar{E}$ FIELD LINES NEVER CROSS (BECAUSE OF ⑤)

⑦ $\bar{E}$ FIELD LINES ARE PERPENDICULAR TO THE SURFACE OF A CONDUCTOR IN EQUILIBRIUM.

⑧ $\bar{E}$ FIELDS THAT CIRCULATE ON THEMSELVES ARE CAUSED BY VARYING MAGNETIC FIELDS.

NORMAL VECTOR TO A CURVED SURFACE



$$\bar{n} = \bar{U} \times \bar{V}$$

$$\hat{n} = \frac{\bar{U} \times \bar{V}}{|U \times V|}$$

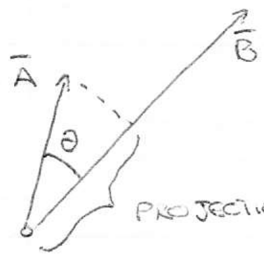
WHERE $\bar{U}$ AND $\bar{V}$ ARE TANGENT TO SURFACE S AT POINT P, AND

$$\bar{U} \times \bar{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ U_x & U_y & U_z \\ V_x & V_y & V_z \end{vmatrix} \qquad \text{CROSS PRODUCT}$$

$\hat{n}$ DIRECTION GIVEN BY RIGHT HAND RULE.

ALSO, $|\bar{U} \times \bar{V}| = |\bar{U}|\,|\bar{V}| \sin\theta$

PROJECTION OF $\bar{A}$ ONTO $\bar{B}$



WE USE DOT PRODUCT

PROJECTION $= |\bar{A}||\bar{B}| \cos\theta$

FLUX OF A VECTOR FIELD $\bar{A}$

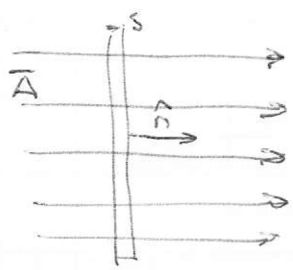
$$\boxed{\Phi = \int_S \bar{A} \cdot \hat{n}\, ds}$$

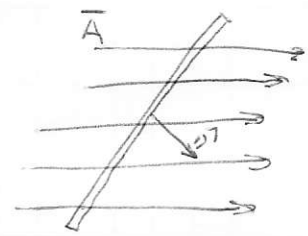
FLUX OF $\bar{A}$ THROUGH S

Φ_E DENOTES $\bar{E}$ FIELD FLUX AND IT IS MEASURED IN $V\,m$

$$\Phi = \left(\vec{A} \cdot \hat{n}\right) S$$

↑ AREA OF SURFACE





GAUSS'S LAW FOR $\bar{E}$ FIELDS (INTEGRAL FORM)

$$\boxed{\oint_S \bar{E} \cdot \hat{n}\, ds = \frac{Q_V}{\varepsilon_0}}$$

THE ELECTRIC FLUX ACROSS A CLOSED SURFACE IS EQUAL TO THE CHARGE ENCLOSED DIVIDED BY THE PERMITTIVITY.

Q_V CAN ALSO BE WRITTEN AS $Q_V = \rho \Delta V$, WHERE ρ IS THE AVERAGE CHARGE DENSITY AND ΔV IS THE VOLUME ENCLOSED BY THE SURFACE.

ε_0 IS THE PERMITTIVITY OF FREE SPACE, AND

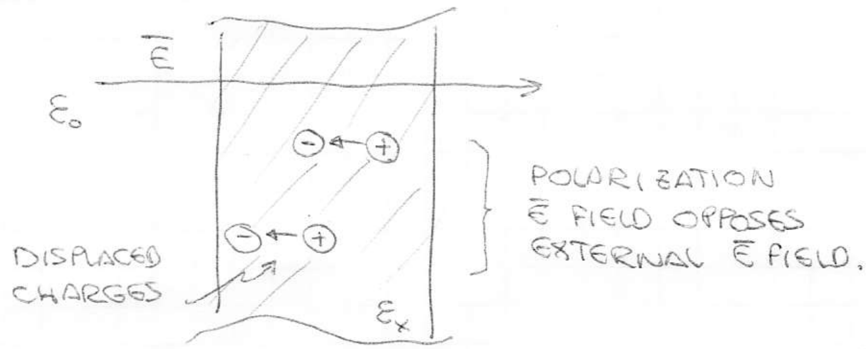
$$\varepsilon_0 \simeq 8.85 \times 10^{-12} \; \frac{C}{V\,m}$$

PERMITTIVITY DETERMINES THE RESPONSE OF DIELECTRIC

MATERIALS TO AN APPLIED $\bar{E}$ FIELD. IN DIELECTRICS, CHARGES ARE NOT FREE TO MOVE, BUT ~~CAN~~ MAY BE SLIGHTLY DISPLACED FROM THEIR EQUILIBRIUM POSITION BY THE EFFECT OF AN $\bar{E}$ FIELD.

GAUSS LAW IS GENERAL, AND CAN BE APPLIED TO DIFFERENT MATERIALS IF THEIR PERMITTIVITY IS USED, AND IF ALL THE ENCLOSED CHARGE IS ACCOUNTED FOR.

THE EFFECT OF BOUND CHARGES CAUSES POLARIZATION, WHICH GENERALLY LESSENS THE AMPLITUDE OF AN $\bar{E}$ FIELD INSIDE DIELECTRICS.



PERMITTIVITY OF A DIELECTRIC IS OFTEN EXPRESSED AS

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

USING ε_r

VERIFYING COULOMB'S LAW FROM GAUSS LAW

THIS IS ONLY VALID IF ALL CHARGES ARE STATIONARY.

FROM

$$\oint_S \bar{E} \cdot \hat{n} \, ds = \frac{Q_v}{\varepsilon_0}$$

ASSUMING A SPHERICAL CLOSED SURFACE, BECAUSE OF SPHERICAL SYMMETRY, THE INTEGRAND IS CONSTANT, SO:

$$4 \pi r^2 \hat{n} \cdot \bar{E} = \frac{Q_v}{\varepsilon_0}$$

AND

$$\bar{E} = \frac{Q_V}{4\pi\varepsilon_0} \frac{\hat{n}}{r^2}$$

WHERE $\bar{E}$ IS A FUNCTION OF POSITION.

GAUSS'S LAW EXAMPLES

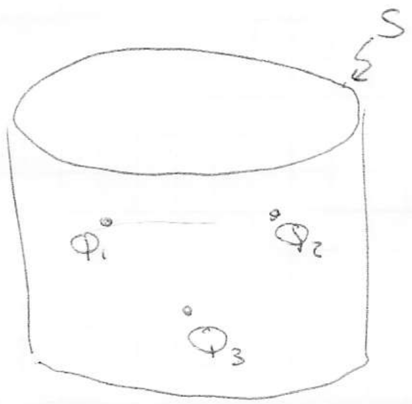
1) FIND $\bar{E}$ FLUX THROUGH CLOSED SURFACE, KNOWING ENCLOSED CHARGE.

PROBLEM: 3 POINT CHARGES ARE ENCLOSED IN A CYLINDRICAL SURFACE.

$Q_1 = 3nC$

$Q_2 = -2nC$

$Q_3 = 2nC$



WHAT IS THE TOTAL FLUX THROUGH S?

SOLUTION:

$$\Phi_E = \oint_S \bar{E} \cdot \hat{n}\, ds = \frac{Q_{TOTAL}}{\varepsilon_0}$$

$$\Phi_E = \frac{Q_{TOTAL}}{\varepsilon_0} = \frac{3\times10^{-9}\ C}{8.85\times10^{-12}\ C/Vm} = 338.98\ Vm$$