# ECE5340/6340: Homework 7
ANSWER KEY

Department of Electrical and Computer Engineering
University of Utah, Salt Lake City, Utah
February 29, 2012

Write your section (ECE5340 or ECE6340) by your name. Turn in a printed copy containing the problem solutions, plots, and the code used to generate them. Remember to comment and format the code so it is legible to the graders. Label the plots appropriately, including units for each axis and for the values plotted. Assume all units to be SI units unless stated differently. Due Wednesday 2/29 BEFORE class begins.

ASSIGNMENT

1. Starting from Ampere's law in the phasor domain, derive a complex-valued Poisson equation for voltage potential by applying the quasi-static approximation.

   See notes. More copy and paste for theoretical background.

2. Use the definition of magnetic vector potential to derive a vector-Poisson equation for magnetostatic systems.

   Yet again, copy and paste for theoretical background.

3. Write a Matlab function that numerically computes the curl of a sampled vector field in two dimensions. Assume the vector field is strictly polarized along the z-direction so that the computation only produces field components along the x- and y-directions (hint: this will be very similar to your 2D gradient calculator).

   See codes.

4. Simulate the electric field distribution due to a circular rod of uniform charge density 1 meter in diameter. Use a charge density of $\rho = 1.0 \times 10^{-12}$ C/m^3. Use a simulation domain size of 10×10 m^2 with a grid step size of $h = 0.05$ m. Use a quiver plot on top of the field intensity plot to show the direction of the field vectors (NOTE: see if you can thin out the quivers so that you only plot every 5th quiver rather than all 200×200 of them. It looks MUCH prettier this way).

   Things actually look a little better if you plot every 10th quiver, but that's okay. The goal is to just try and get an image of the E-field color map with some quivers on top. Notice how the charge density just acts as a source term for voltage potential and electric fields. The only difference is that before we would explicitly define V by using some "hard" (Dirichlet) boundary condition.

Electric Field (V/m)
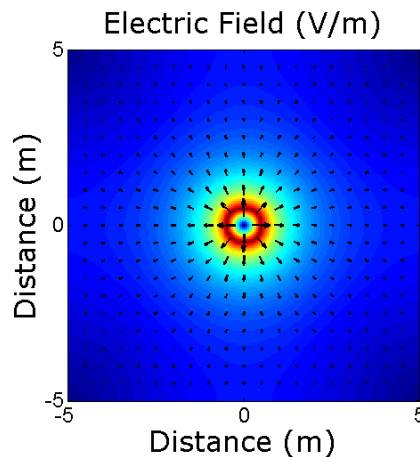
5. Repeat the previous simulation, but this time treat the charge density as if it were a current density oriented along the z-direction. So instead of a voltage potential function V, your FDM simulation will represent the z-component to the magnetic vector potential (A_z). Use your 2D curl calculator to find the B-field of the current source (NOTE: do not worry about the exact scale of things; just get a basic image plot with the quivers again).

   This step should not even require you to run another simulation. Just take the V-matrix from the previous problem and pretend it is now A_z. Once you take the curl, you'll find that it generates the exact same color map, but now all the vectors are swirling around the source instead of radiating away. Also remember that the fields due to the current source must follow the *right hand rule*. That means the B-fields must swirl counter-clockwise or else there is an error in your code.

Magnetic Field (a.u.)
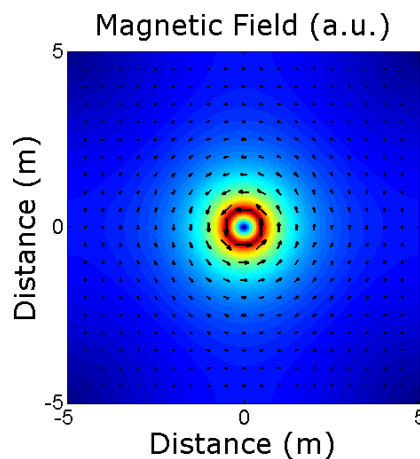
6. Download my 2D Poisson solver from the website. This code fills the system matrix **A** by using Matlab's *sparse* function and then directly inverts it to find a solution. Try repeating one of the previous simulations using this method. How much faster did it solve the system? What are the advantages/disadvantages to using a direct matrix solver rather than an iterative solution?

   The only point of this problem is to get students playing with my solver and see how direct matrix inversion is way faster than iterative methods. It also helps out the 6000-level student who need it for the next problem. The trade-off for direct inversion is that it requires a ton of book-keeping and clever Matlab trickery. The direct method also requires a ton of memory in order to invert the matrix while SOR can probably handle much larger simulations. Another downside with SOR is that it cannot handle simulations with large dielectric constants because it takes forever to converge.

7. <u>ECE 6340 ONLY</u>: Use the direct Poisson solver to simulate the current density on a conductive slab of metal. Use the system geometry depicted below:
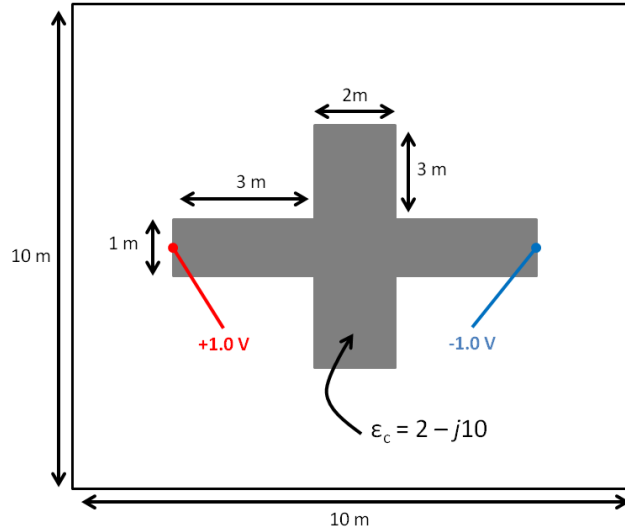
Use a grid spacing of $h = 0.05$ m. Excite the system by placing a single Dirichlet boundary of 1.0 V at the left tip of the conductor. Place a -1.0 V boundary at the right tip. Place Dirichlet boundaries of 0.0 V around the simulate edges. Specify a complex dielectric constant of $\epsilon_c = 2 - j10$ for the conductive object and $\epsilon_c = 1$ everywhere else. Generate a plot of the current density $\mathbf{J} = \sigma\mathbf{E}$. Comment on anything physically significant that you observe in the output.

This actually works better using something like $\epsilon_c = 20 - j10000$ since it produces a better distribution on the current density.
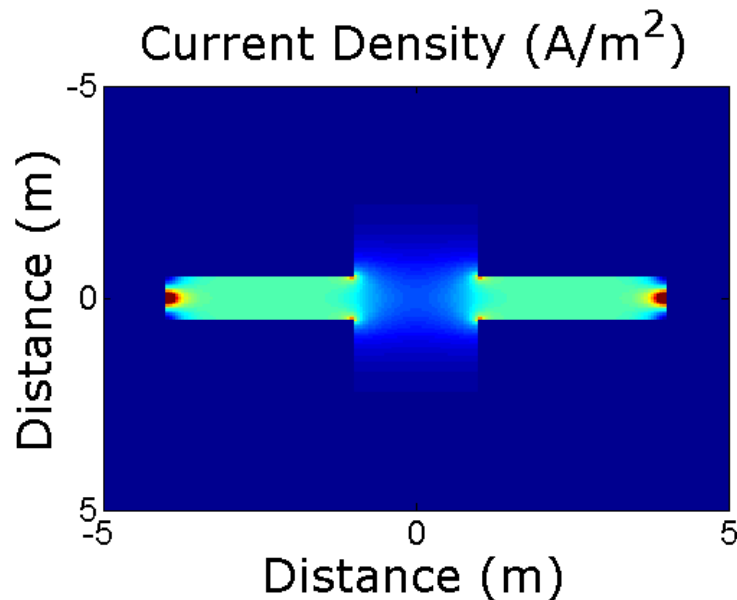
Notice the strong peaks around the insertion points. The current then flows mostly uniformly down the slab until reaching the thicker section. It then spreads out and crams back in to the right slab.

Here is a nice little close-up of the current vectors for kicks.