ECE5340/6340: Homework 7
Conductivity and Magnetism with FDM

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University of Utah, Salt Lake City, Utah
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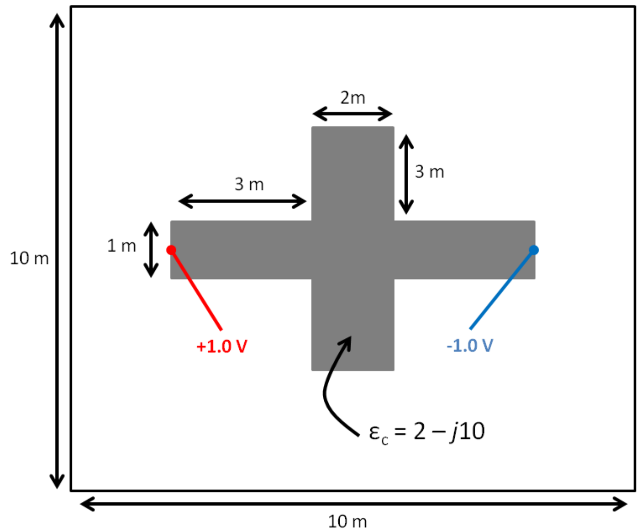
Write your section (ECE5340 or ECE6340) by your name. Turn in a printed copy containing the problem solutions, plots, and the code used to generate them. Remember to comment and format the code so it is legible to the graders. Label the plots appropriately, including units for each axis and for the values plotted. Assume all units to be SI units unless stated differently. Due Wednesday 2/29 BEFORE class begins.

ASSIGNMENT

1. Starting from Ampere's law in the phasor domain, derive a complex-valued Poisson equation for voltage potential by applying the quasi-static approximation.
2. Use the definition of magnetic vector potential to derive a vector-Poisson equation for magnetostatic systems.
3. Write a Matlab function that numerically computes the curl of a sampled vector field in two dimensions. Assume the vector field is strictly polarized along the z-direction so that the computation only produces field components along the x- and y-directions (hint: this will be very similar to your 2D gradient calculator).
4. Simulate the electric field distribution due to a circular rod of uniform charge density 1 meter in diameter. Use a charge density of $\rho = 1.0 \times 10^{-12}$ C/m^3. Use a simulation domain size of 10×10 m^2 with a grid step size of $h = 0.05$ m. Use a quiver plot on top of the field intensity plot to show the direction of the field vectors (NOTE: see if you can thin out the quivers so that you only plot every 5th quiver rather than all 200×200 of them. It looks MUCH prettier this way).
5. Repeat the previous simulation, but this time treat the charge density as if it were a current density oriented along the z-direction. So instead of a voltage potential function V, your FDM simulation will represent the z-component to the magnetic vector potential (A_z). Use your 2D curl calculator to find the B-field of the current source (NOTE: do not worry about the exact scale of things; just get a basic image plot with the quivers again).
6. Download my 2D Poisson solver from the website. This code fills the system matrix **A** by using Matlab's *sparse* function and then directly inverts it to find a solution. Try repeating one of the previous simulations using this method. How much faster did it solve the system? What are the advantages/disadvantages to using a direct matrix solver rather than an iterative solution?

7. ECE 6340 ONLY: Use the direct Poisson solver to simulate the current density on a conductive slab of metal. Use the system geometry depicted below:



Use a grid spacing of $h = 0.05$ m. Excite the system by placing a single Dirichlet boundary of 1.0 V at the left tip of the conductor. Place a -1.0 V boundary at the right tip. Place Dirichlet boundaries of 0.0 V around the simulate edges. Specify a complex dielectric constant of $\epsilon_c = 2 - j10$ for the conductive object and $\epsilon_c = 1$ everywhere else. Generate a plot of the current density $\mathbf{J} = \sigma\mathbf{E}$. Comment on anything physically significant that you observe in the output.