Lecture 2
Material Properties
Dielectrics, Polarization and Permittivity

→ Dielectrics are materials whose dominant charges in atoms and molecules are bound negative and positive charges that are held in place by atomic and molecular forces, and they are not free to travel.

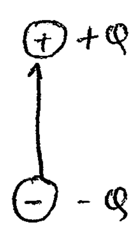
→ Dielectrics donot contain any free charges.

→ When external forces (fields) are applied, these bound negative and positive charges do not move to the surface of the material as in conductors, but their respective centroids can shift slightly in position relative to each other, thus creating numerical electrical dipoles.

→ The formation of electric dipoles is usually referred to as orientational polarization.

→ For each dipole the dipole moment is given by

$$dp_i = \underbrace{Q}_{\text{magnitude of charge}} \overbrace{l_i}^{\text{distance}}$$



$dp_i = Q l_i$

The total dipole moment P_t

$$P_t = \sum_{i=1}^{N_e \Delta v} dp_i$$

N_e → electric dipoles per unit volume

Δv → volume

$N_e \Delta v$ → Electric dipoles.

Electric polarization vector $\bar{P}$ can be defined as the dipole moment per unit volume.

$$\bar{P} = \lim_{\Delta v \to 0}\left[\frac{1}{\Delta v} P_t\right] = \lim_{\Delta v \to 0}\left[\frac{1}{\Delta v}\sum_{i=1}^{N_e \Delta v} dp_i\right] \quad (C/m^2)$$

$\bar{P}$ represents surface charge density which is bound. Within a volume an integral number of positive & negative pairs with an overall zero net charge must exist.

Assuming an average dipole moment of

$$dp_i = dp_{av} = Q I_{av}$$

The electric polarization vector can be written as.

$$\bar{P} = N_e dp_{av} = N_e Q I_{av}$$

Electric polarization for dielectrics can be produced by any of the following mechanisms.

1) Dipole or Orientational Polarization

This polarization is evident in material that, in the absence of an applied field and owing to their structure posses permanent dipoles moments that are randomly oriented. However when an electric field is applied these dipoles tend to allign with the applied fields. (polar)

eg water is a good polar material

Ionic or Molecular Polarization:- Evident in materials, such as NaCl that possess +ve & -ve ions and that tend to displace themselves when electric field is applied

Electronic polarization:- It exists when an applied electric field displaces the electric ~~cloud~~ center of an atom relative to the center of the nucleus.

Dipole, polar, Non Polar

If the charges in a material, in the absence of an applied electric field $\bar{E}_a$, are averaged in such a way that +ve and -ve charges cancel each other throughout the material, then there are no individual dipoles formed and the total dipole moment and electric polarization vector $\bar{P}$ are zero. However when an electric field is applied, it exhibits a net nonzero polarization. Such a material is referred to as non polar material. Polar materials are those whose charges in the absence of an applied electric field are distributed so that there are individual dipoles formed each with a dipole moment $\bar{p}_i$ but with a net total dipole moment $\bar{p}_t = 0$ and electric polarization vector $\bar{P} = 0$. This is usually due to the random orientation of the dipole. Typical dipole moments are of the order of 10^{-30} C-m

Materials that in the absence of an applied electric field, E_a, possess nonzero net dipole moment and electric polarization vector $\bar{P}$ are referred as electrets

When an electric field is applied to a nonpolar or polar dielectric material, the charges in each medium are aligned in such as way that individual dipoles with nonzero dipole moments are formed within the material.

When we see the material in macroscopic scale we can make the following items

1. On the lower surface there exists a net positive surface charge density q_s^+
2. On the upper surface there exists a net negative surface charge density q_s^-
3. The volume charge density q_v inside the material is zero because the positive and negative charges of adjacent dipoles cancel each other

From the figure 2.6.a in the Textbook we can ~~make~~ give the following explanation

A dc voltage source is connected and remains across two parallel plates separated by distance s. Half of the space between the two plates is occupied by dielectric & half is filled with air (free space).

Assume five free charges on each part of the plates separated by free space. The same number appears on the part of the plates separated by dielectric material. Because of the realignment of bound charges in the dielectric material and formation of electric dipoles and cancellation of adjacent opposite charges, a polarization electric vector $\bar{P}$ is formed within the dielectric material.

The net effect is that between the lower and upper surfaces of the dielectric there is a net electric polarization vector $\bar{P}$ directed from the upper toward the lower surfaces, in the same direction as the applied electric field $\bar{E}_a$, whose amplitude is given by

$$\bar{P} = q_{sp}$$

The electric flux density for free space is given by

$$\bar{D} = \varepsilon_0 \bar{E}_a$$

In dielectric portion, the electric flux density $\bar{D}$ is related to that in free space as

$$\bar{D} = \varepsilon_0 \bar{E}_a + \bar{P}$$

$$\bar{D} = \varepsilon_s \bar{E}_a$$

$$\bar{P} = \varepsilon_0 \chi_e \bar{E}_a$$

$$\chi_e = \frac{1}{\varepsilon_0} \frac{P}{E_a}$$

$\chi_e \Rightarrow$ electric susceptibility

$$\bar{D} = \varepsilon_0 \bar{E}_a + \varepsilon_0 \chi_e \bar{E}_a = \varepsilon_0 (1+\chi_e) \bar{E}_a = \varepsilon_s \bar{E}_a$$

$$\varepsilon_s = \varepsilon_0 (1+\chi_e)$$

$\varepsilon_s \Rightarrow$ static permittivity of medium whose relative value ε_{sr} is given by

$$\varepsilon_{sr} = \frac{\varepsilon_s}{\varepsilon_0} = 1+\chi_e$$

referred to as relative permittivity (dielectric constant)

Index of refraction $n = \sqrt{\varepsilon_r}$

dielectric constant of a dielectric material is a parameter that indicates the relative (compared to free space) charge (energy) storage capabilities of a dielectric material, the larger its value, the greater the ability to store charge.

Magnetics, Magnetization and Permeability

Magnetic materials are those that exhibit magnetic polarization when they are subjected to an applied magnetic field

The phenomenon is represented by the alignment of magnetic dipoles of the material with the applied magnetic field

The magnetic flux density across the slab is increased by the presence of $\bar{M}$ so that the net magnetic flux density at any interior point of the slab is given by

$$\bar{B} = \mu_0(\bar{H}_a + \bar{m})$$

$$\bar{m} = \chi_m \bar{H}_a$$

$\chi_m \rightarrow$ magnetic suseptibility

$$\bar{B} = \mu_0(\bar{H}_a + \chi_m \bar{H}_a) = \mu_0(1+\chi_m)\bar{H}_a = \mu_s \bar{H}_a$$

$$\mu_s = \mu_0(1+\chi_m)$$

static permeability

$$\mu_{sr} = \frac{\mu_s}{\mu_0} = 1+\chi_m$$

Within the material, a bound magnetic current density $\bar{J}_m$ is induced that is related to the magnetic polarization vector $\bar{m}$ by

$$\bar{J}_m = \nabla \times \bar{m} \quad (A/m^2)$$

$$\nabla \times \bar{H} = \bar{J}_i + \bar{J}_c + \bar{J}_m + \bar{J}_d$$

The bound magnetization current I_m flowing through a cross section S_0 of the material can be obtained by using

$$I_m = \iint_S \bar{J}_m \cdot \bar{ds} = \iint_{S_0} (\nabla \times \bar{m}) \cdot \bar{ds}$$

Currents, Conductors and Conductivity

Assume electric volume charge density ρ_v distributed uniformly in an infinitesimal circular cylinder of cross section area Δs. and volume ΔV. The total electric charge ΔQ within volume ΔV is moving in the 'z' direction with uniform velocity v_z

$$\frac{\Delta Q_e}{\Delta t} = \rho_v \frac{\Delta V}{\Delta t} = \rho_v \frac{\Delta s \Delta z}{\Delta t} = \rho_v \Delta s \frac{\Delta z}{\Delta t}$$

As $\Delta t \to 0$

$$\Delta I = \lim_{\Delta t \to 0} \left[\frac{\Delta Q_e}{\Delta t} \right] = \rho_v v_z \Delta s$$

$$\bar{J}_z = \frac{\Delta I}{\Delta s} = \cancel{\lim_{\Delta s \to 0} [\rho_v v_z \Delta s]} \; \rho_v v_z$$

Total current density

$$\bar{J} = \rho_v \bar{v} \; (A/m^2)$$

Convection current density

current between cathode & anode of a vaccum tube

<u>Conductors</u>

Materials whose atomic outer shell electrons are not held very tightly and can migrate from one atom to another. For metal conductors free electrons are large in numbers

When a free charge ρ_{v_0} is placed inside a conductor that is subject to a ~~charge~~ static field. The charge density at the point over time is given as

$$\rho_v(t) = \rho_{v_0} e^{-t/t_r} = \rho_{v_0} e^{-(\sigma/\varepsilon)t}$$

because charge migrates to the surface of the conductor. The time t_r that it takes for the free charge density placed inside a conductor to decay to $e^{-1} = 0.368$ or 36.8% of its initial value, is known as the relaxation time constant

$t_r = \frac{\varepsilon}{\sigma}$ $\varepsilon \rightarrow$ permittivity σ - conductivity

Free charges of a very good conductor ($\sigma \rightarrow \infty$) which is subject to electric field, migrate very rapidly and distribute themselves as surface charge density ρ_s to surface of conductor within an extremely short period of time.

When a conductor is subjected to an electric field, the electrons still move in random directions but drift slowly (with drift velocity v_e) in negative direction of the applied electric field, thus creating a conduction current in the conductor.

$v_e = -\mu_e \bar{E}$

$\mu_e \Rightarrow$ electron mobility

$\bar{J} = \rho_v v_e = \rho_v (-\mu_e \bar{E}) = -\rho_v \mu_e \bar{E}$ — (A)

ρ_v - electron charge density

We know that

$\bar{J}_c = \sigma \bar{E}$ — (B)

Comparing the 2 eqns (A) & (B)

$\sigma = -\rho_v \mu_e$ (S/m)

$\frac{1}{\sigma}$ = Resistivity.

The conductivity of a conductor is a parameter that characterizes free electron conductive properties of a conductor. As temperature increases, the increased thermal energy of the conductor lattice structure increases lattice vibrations. Thus the possibility of moving free electrons colliding increases, which results in decrease is conductivity of conductor. Materials with vary low value of. conductivity are classified as dielectrics.

Linear, Homogeneous, Isotropic, and nondispersive media

The electrical behavior of materials when they are subjected to electromagnetic fields is characterized by their constitutive parameters. Many material exhibit almost linear characteristics as long as the applied fields are within certain ranges. Beyond those points, the material may exhibit a high degree of nonlinearity.

→ Air is nearly linear for applied electric fields up to about 1×10^{6} V/m. Beyond that air breaks down and exhibits a high degree of nonlinearity.

Dispersive materials are those whose constitutive parameter varies as a function of frequency. Else they are nondispersive. The permittivities and conductivities, especially of dielectric material, and permeabilities of ferromagnetic material and ferrites exhibit rather pronounced dispersive characteristics.

Nonisotropic materials are those whose constitutive parameters are a function of the direction of the applied field.

Dielectric materials in which each component of their electric flux density $\bar{D}$ depends on more than one component of the electric field $\bar{E}$, are called anisotropics dielectrics. For such material the permittivities and susceptibilities cannot be represented by a single value. These values take the form of a 3×3 tensor, which is known as permittivity tensors.

$$\bar{D} = \bar{\varepsilon}\bar{E}$$

ε – permittivity tensor

$$\begin{bmatrix} \bar{D}_x \\ \bar{D}_y \\ \bar{D}_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} \bar{E}_x \\ \bar{E}_y \\ \bar{E}_z \end{bmatrix}$$

The diagonal terms are referred to as principal permittivities.
For physically realizable materials, the entries ε_{ij} of permittivity tensor satisfy the relation

$$\varepsilon_{ij} = \varepsilon_{ji}^{*}$$

Matrices whose entries satisfy the above equation are referred to as Hermitian.