5.28

a. $$\frac{\sigma}{\omega\epsilon} = \frac{1}{2\pi(10^6)\left(\frac{81\times10^{-9}}{36\pi}\right)} = \frac{18}{81}\times10^3 = \frac{2}{9}\times10^3 >> 1 \Rightarrow \text{Good conductor}$$

$$\eta = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j) = \sqrt{\frac{2\pi\times10^6(4\pi\times10^{-7})}{2(1)}}(1+j) = 1.987(1+j) = 2.810\underline{\angle 45^\circ}$$

$$\Gamma = \frac{\eta-\eta_0}{\eta+\eta_0} = \frac{1.987(1+j)-377}{1.987(1+j)+377} = -\frac{375.013+j1.987}{378.987+j1.987} = 0.9895\underline{\angle 179.4^\circ}$$

$$E^r = |\Gamma|E^i = 0.9895(1\times10^{-3}) = 0.9895\times10^{-3}\ \text{V/m}$$

b. $$SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.9895}{1-0.9895} = 189.476$$

c. $$S^i = \frac{1}{2\eta_0}|E^i|^2 = \frac{1}{2(377)}(10^{-3})^2 = 1.326\times10^{-9}\ \text{W/m}^2$$

$$S^r = \frac{1}{2\eta_0}|E^r|^2 = \frac{1}{2\eta}|\Gamma E^i|^2 = |\Gamma|^2 S^i = (0.9895)^2(1.326\times10^{-9}) = 1.2983\times10^{-9}\ \text{W/m}^2$$

d. $$T = \frac{2\eta}{\eta+\eta_0} = \frac{2(2.810\underline{\angle 45^\circ})}{378.987+j1.987} = \frac{2(2.810\underline{\angle 45^\circ})}{378.992\underline{\angle 0.3}} = 0.0148\underline{\angle 44.7^\circ}$$

$$E^t = |T|E^i = 0.0148\times10^{-3}\ \text{V/m}$$

e. $$S_L^t = S^i - S^r = S^i - |\Gamma|^2 S^i = (1-|\Gamma|^2)S^i = (1-|0.9895|^2)1.326\times10^{-9} = 0.0277\times10^{-9}\ \text{W/m}^2$$

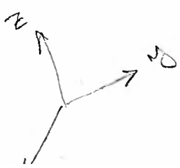
f. $$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}} = \sqrt{\frac{1}{\pi(10^6)(4\pi\times10^{-7})(1)}} = \frac{1}{2\pi}\sqrt{10} = 0.5033\ \text{m}$$

g. $$\lambda = 2\pi\sqrt{\frac{2}{\omega\mu\sigma}} = 2\pi\delta = 2\pi(0.5033) = 3.1623\ \text{m}$$

$$d = 20\lambda = 2(3.1623) = 63.2455\ \text{m}$$

h. $$v = \sqrt{\frac{2\omega}{\mu\sigma}} = \sqrt{\frac{2(2\pi\times10^6)}{4\pi\times10^{-7}(1)}} = \sqrt{10}(10^6) = 3.1623\times10^6\ \text{m/sec.}$$

$$t = \frac{d}{v} = \frac{100}{3.1623\times10^6} = 31.623\times10^{-6}\ \text{sec.}$$



i. $$\frac{v}{v_0} = \frac{3.1623\times10^6}{3\times10^8} = 1.054\times10^{-2}$$

5.30

a. For right hand circular $\psi = 90^\circ$

b. $\underline{H}^i = (j\hat{a}_y - \hat{a}_z)\frac{E_0}{\eta_0} e^{+j\beta_0 x}$

c. $\frac{\sigma}{\omega\varepsilon} = \frac{10^{-1}}{2\pi(10^9)\left(\frac{81\times10^{-9}}{36\pi}\right)} = \frac{18}{81}\times10^{-1} << 1 \Rightarrow$ Good dielectric

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0} \simeq \frac{\sqrt{\mu_0/\varepsilon} - \sqrt{\mu_0/\varepsilon_0}}{\sqrt{\mu_0/\varepsilon} + \sqrt{\mu_0/\varepsilon_0}} = \frac{\sqrt{\varepsilon_0} - \sqrt{\varepsilon}}{\sqrt{\varepsilon_0} + \sqrt{\varepsilon}} = \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} = \frac{1 - \sqrt{81}}{1 + \sqrt{81}} = \frac{1-9}{1+9} = -0.8$$

$$\underline{E}^r = -0.8(\hat{a}_y + j\hat{a}_z)E_0 e^{-j\beta_0 x}$$

$$\underline{H}^r = -\frac{0.8}{377}(\hat{a}_z - j\hat{a}_y)E_0 e^{-j\beta_0 x} = 2.122\times10^{-3}(j\hat{a}_y - \hat{a}_z)E_0 e^{-j\beta_0 x}$$

Cont'd.

d. Circular, ccw

e. $$T = \frac{2\eta}{\eta + \eta_0} \simeq \frac{2\sqrt{\frac{\mu_0}{\varepsilon}}}{\sqrt{\frac{\mu_0}{\varepsilon}} - \sqrt{\frac{\mu_0}{\varepsilon_0}}} = 2\frac{\frac{1}{\sqrt{\varepsilon}}}{\frac{1}{\sqrt{\varepsilon}} - \frac{1}{\sqrt{\varepsilon_0}}} = 2\frac{\sqrt{\varepsilon_0}}{\sqrt{\varepsilon_0} - \sqrt{\varepsilon}} = \frac{2}{1 + \sqrt{\varepsilon_r}} = \frac{2}{1+9} = 0.2$$

$$\underline{E}^t = 0.2(\hat{a}_y + j\hat{a}_z)e^{-\gamma x} = 0.2(\hat{a}_y + j\hat{a}_z)e^{-(\alpha + j\beta)x}$$

$$\underline{H}^t = \frac{0.2}{377}(j\hat{a}_y - \hat{a}_z)E_0 e^{-(\alpha + j\beta)x} = 5.3\times10^{-4}(j\hat{a}_y - \hat{a}_z)E_0 e^{-(\alpha + j\beta)x}$$

$$\alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu_0}{\varepsilon}} = \frac{10^{-1}}{2}\left(\frac{377}{9}\right) = 2.0944 \ \text{N/m}$$

$$\beta \simeq \omega\sqrt{\mu_0\varepsilon} = \beta_0\sqrt{81} = 9\beta_0 = 9\omega\sqrt{\mu_0\varepsilon_0} = \frac{9(2\pi\times10^9)}{3\times10^8} = 9(20.944) = 188.5 \ \text{rad}$$

f. Circular, CW

g. $$\frac{S^r}{S^i} = |\Gamma|^2 = |-0.8|^2 = 0.64 = 64\%$$

$$\frac{S^t}{S^i} = 1 - |\Gamma|^2 = (1 - |-0.8|^2) = 0.36 = 36\%$$

$\boxed{7.1}$ a. $H_\phi = \frac{E_\theta}{\eta} = j\frac{\beta I_0 l e^{-j\beta r}}{4\pi r}\sin\theta\left[2\cos(\beta h\cos\theta)\right]$

b. $\underline{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\underline{E}\times\underline{H}^*\right] = \frac{1}{2}\mathrm{Re}\left[\hat{a}_\theta E_\theta \times \hat{a}_\phi H_\phi^*\right] = \frac{\hat{a}_r}{2}\mathrm{Re}\left[E_\theta H_\phi^*\right]$

$= \hat{a}_r \frac{1}{2}\mathrm{Re}\left[E_\theta \frac{E_\theta^*}{\eta}\right] = \hat{a}_r \frac{1}{2}\frac{|E_\theta|^2}{\eta}$

$\underline{S}_{av} = \hat{a}_r \frac{\eta}{2}\left|\frac{\beta I_0 l}{4\pi r}\right|^2 \sin^2\theta \left|2\cos(\beta h\cos\theta)\right|^2 = \hat{a}_r 2\eta\left|\frac{\beta I_0 l}{4\pi r}\right|^2 \sin^2\theta\cos^2(\beta h\cos\theta)$

c. $P_{av} = \oiint_S \underline{S}_{av}\cdot d\underline{s} = \int_0^{2\pi}\int_0^{\pi/2} \hat{a}_r S_{av}\cdot\hat{a}_r r^2\sin\theta\, d\theta\, d\phi = \int_0^{2\pi}\int_0^{\pi/2} S_{av} r^2 \sin\theta\, d\theta\, d\phi$

$P_{av} = 4\pi\eta\left|\frac{\beta I_0 l}{4\pi}\right|^2 \int_0^{\pi/2}\sin^3\theta\cos^2(\beta h\cos\theta)\,d\theta = \pi\eta\left|\frac{I_0 l}{\lambda}\right|^2\int_0^{\pi/2}\sin^3\theta\cos^2(\beta h\cos\theta)\,d\theta$

$P_{rad} = P_{av} = \pi\eta\left|\frac{I_0 l}{\lambda}\right|^2 I$

where $I = I_1 + I_2 = \int_0^{\pi/2}\sin^3\theta\cos^2(\beta h\cos\theta)\,d\theta = \int_0^{\pi/2}\sin^3\theta\left[\frac{1+\cos(2\beta h\cos\theta)}{2}\right]d\theta$

$I = I_1 + I_2 = \frac{1}{2}\int_0^{\pi/2}\sin^3\theta\, d\theta + \frac{1}{2}\int_0^{\pi/2}\sin^3\theta\cos(2\beta h\cos\theta)\,d\theta$

$I_1 = \frac{1}{2}\int_0^{\pi/2}\sin^3\theta\,d\theta = -\frac{1}{6}\cos\theta\left(\sin^2\theta + 2\right)\Big|_0^{\pi/2} = \frac{1}{3}$

$I_2 = \frac{1}{2}\int_0^{\pi/2}\sin^3\theta\cos(2\beta h\cos\theta)\,d\theta = \frac{1}{2}\int_0^{\pi/2}\sin^2\theta\cos(2\beta h\cos\theta)\sin\theta\,d\theta$

Let $u = \sin^2\theta$ $\quad dv = -\frac{\cos(2\beta h\cos\theta)}{2\beta h}\,d(2\beta h\cos\theta)$, $v = -\frac{1}{2\beta h}\sin(2\beta h\cos\theta)$

$du = 2\sin\theta\cos\theta\,d\theta$

Thus

$I_2 = \underbrace{-\frac{\sin^2\theta}{4\beta h}\sin(2\beta h\cos\theta)\Big|_0^{\pi/2}}_{=0} + \frac{1}{2\beta h}\int_0^{\pi/2}\cos\theta\sin(2\beta h\cos\theta)\sin\theta\,d\theta$

Let $u = \cos\theta$ $\quad dv = -\frac{1}{2\beta h}\sin(2\beta h\cos\theta)\,d(2\beta h\cos\theta)$

$du = -\sin\theta\,d\theta$ $\quad v = \frac{1}{2\beta h}\cos(2\beta h\cos\theta)$

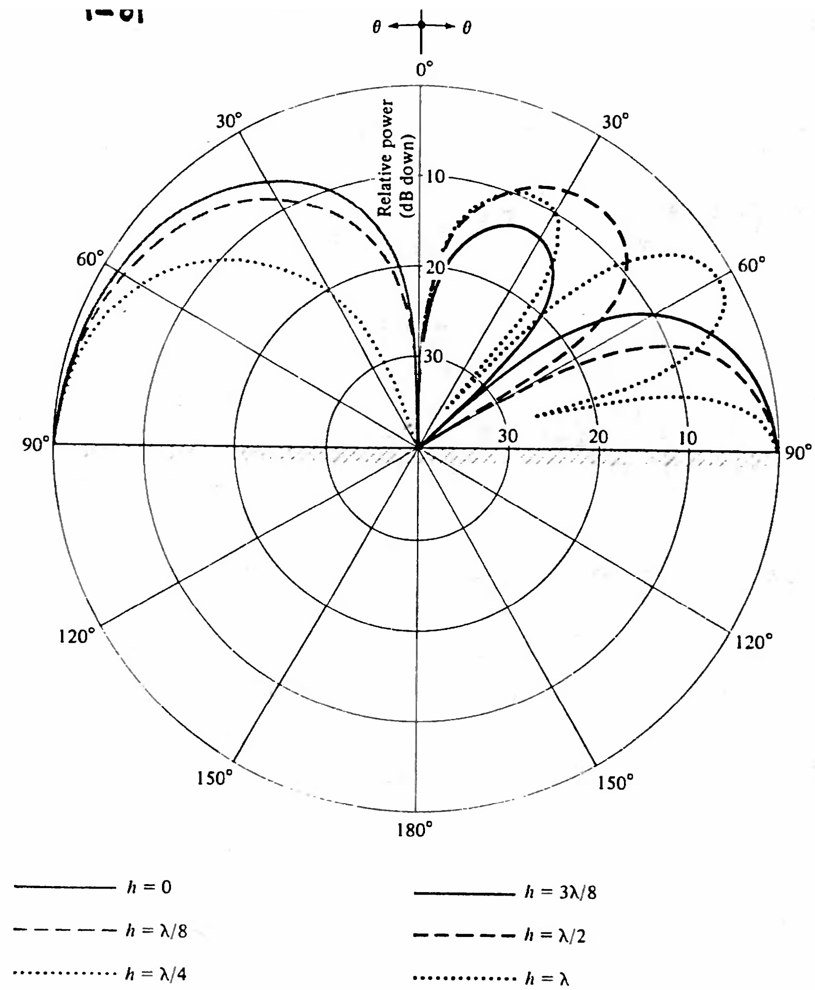
$I_2 = 0 + \frac{1}{2\beta h}\left\{\frac{\cos\theta}{2\beta h}\cos(2\beta h\cos\theta)\Big|_0^{\pi/2} + \frac{1}{2\beta h}\int_0^{\pi/2}\cos(2\beta h\cos\theta)\sin\theta\,d\theta\right\}$

$I_2 = \frac{1}{2\beta h}\left\{-\frac{1}{2\beta h}\cos(2\beta h) - \frac{1}{(2\beta h)^2}\sin(2\beta h\cos\theta)\Big|_0^{\pi/2}\right\} = \left\{-\frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right\}$

Therefore

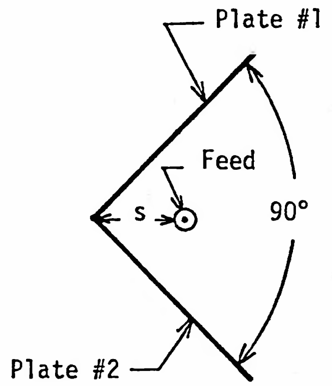
$P_{rad} = \pi\eta\left|\frac{I_0 l}{\lambda}\right|^2\left[\frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right]$

7.3

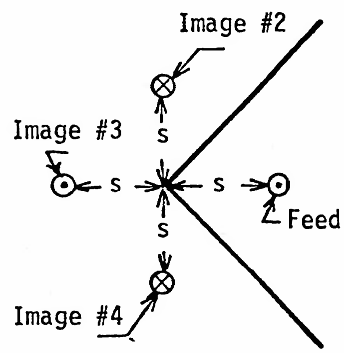


θ θ
0°
30°
30°
Relative power (dB down)
10
20
30
60°
60°
90°
30
20
10
90°
120°
120°
150°
150°
180°
$h = 0$
$h = \lambda/8$
$h = \lambda/4$
$h = 3\lambda/8$
$h = \lambda/2$
$h = \lambda$

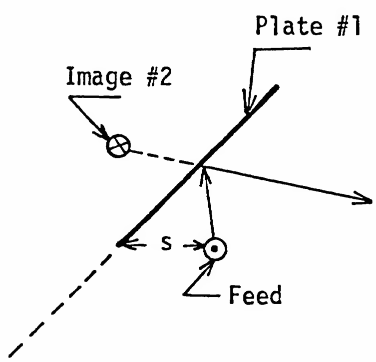
$\boxed{7.5}$ a. The number of images (3 of them), their polarizations, and their positions are shown below.



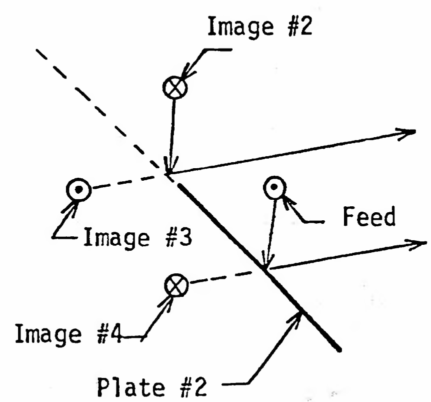
(a) 90° corner reflector



(b) images for 90° corner reflector



(c) placement of image #2 (due to feed)



(d) placement of images #4 and #3 (due to feed and image #2)

7.5 cont'd. b. The total field is derived by summing the contributions from the feed and its images. Thus

$$E^t(r,\theta,\phi) = E_1(r_1,\theta,\phi) + E_2(r_2,\theta,\phi) + E_3(r_3,\theta,\phi) + E_4(r_4,\theta,\phi) \qquad (1)$$

In the far zone, the normalized field can be written as

$$E^t(r,\theta,\phi) = f(\theta,\phi)\frac{e^{-j\beta r_1}}{r_1} - f(\theta,\phi)\frac{e^{-j\beta r_2}}{r_2} + f(\theta,\phi)\frac{e^{-j\beta r_3}}{r_3} - f(\theta,\phi)\frac{e^{-j\beta r_4}}{r_4}$$

$$= \left[e^{j\beta s\cos\psi_1} - e^{j\beta s\cos\psi_2} + e^{j\beta s\cos\psi_3} - e^{j\beta s\cos\psi_4}\right] f(\theta,\phi)\frac{e^{-j\beta r}}{r} \qquad (2)$$

where

$$\cos\psi_1 = \hat{a}_x \cdot \hat{a}_r = \sin\theta\cos\phi$$

$$\cos\psi_2 = \hat{a}_y \cdot \hat{a}_r = \sin\theta\sin\phi$$

$$\cos\psi_3 = -\hat{a}_x \cdot \hat{a}_r = -\sin\theta\cos\phi \qquad (3)$$

$$\cos\psi_3 = -\hat{a}_y \cdot \hat{a}_r = -\sin\theta\sin\phi$$

since

$$\hat{a}_r = \hat{a}_x\sin\theta\cos\phi + \hat{a}_y\sin\theta\sin\phi + \hat{a}_z\cos\theta \qquad (3a)$$

Substituting (3) into (2) we can write it as

$$E^t(r,\theta,\phi) = 2\left[\cos(\beta s\sin\theta\cos\phi) - \cos(\beta s\sin\theta\sin\phi)\right] f(\theta,\phi)\frac{e^{-j\beta r}}{r} \qquad (4)$$

By letting the field of the isolated element (in this case the dipole) to be

$$E_\theta^o = f(\theta,\phi)\frac{e^{-j\beta r}}{r} = j\eta\frac{\beta I_o \ell e^{-j\beta r}}{4\pi r}\sin\theta \qquad (5)$$

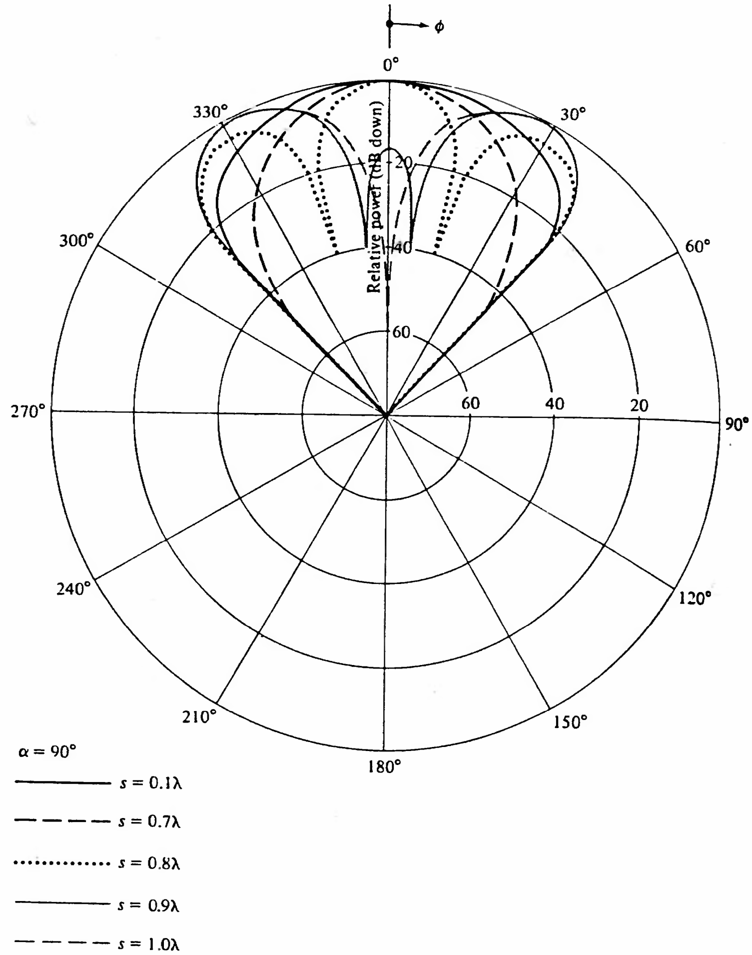
then (4) can be written as

$$E^t(r,\theta,\phi) = E_\theta^o\, F(\beta s)$$

where

$$F(\beta s) = 2\left[\cos(\beta s\sin\theta\cos\phi) - \cos(\beta s\sin\theta\sin\phi)\right]$$

7.7



7.9 The period of each is

$\alpha = 60^\circ$: $\Delta s = 2.0\lambda$

$\alpha = 45^\circ$: $\Delta s = 16.69\lambda$

$\alpha = 30^\circ$: $\Delta s = 30.0\lambda$

while the peak values are

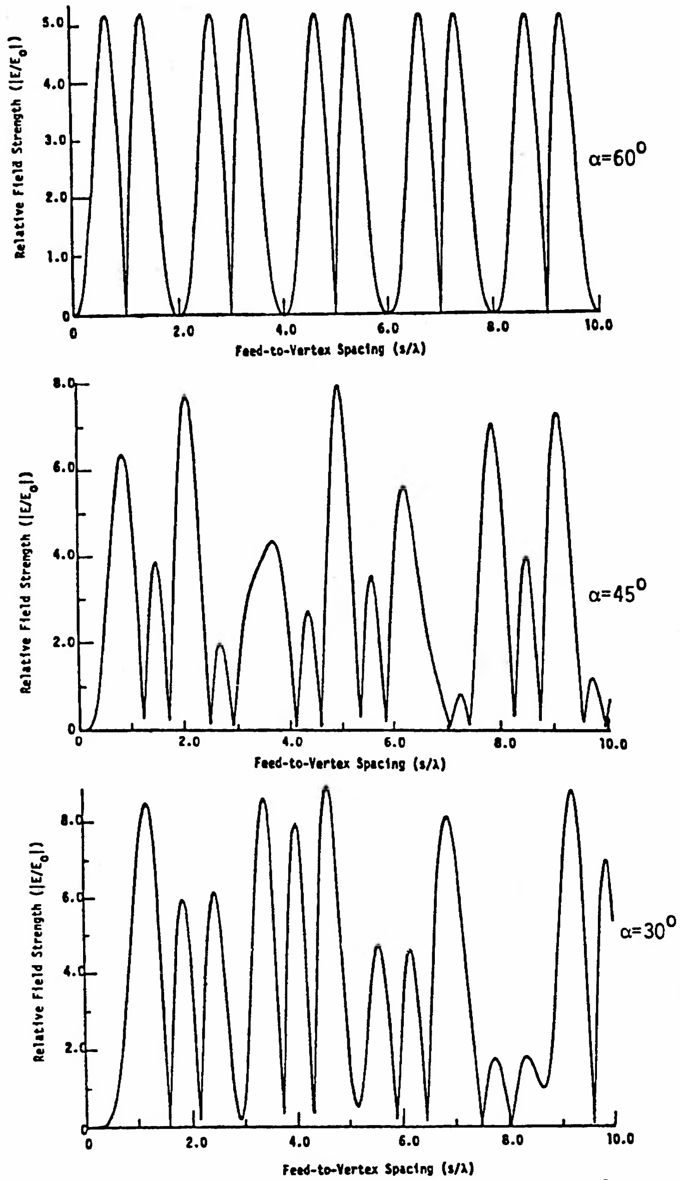
$\alpha = 60^\circ$: $|E/E_0|_{max} = 5.2$

$\alpha = 45^\circ$: $|E/E_0|_{max} = 8.0$

$\alpha = 30^\circ$: $|E/E_0|_{max} = 9.0$

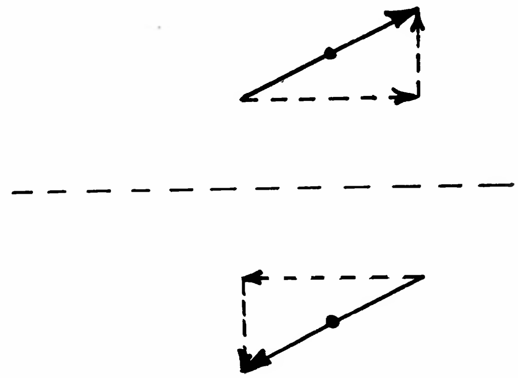
The plots of $|E/E_0|$ as a function of s in the range $0 \le s \le 10\lambda$

7.9 cont'd.

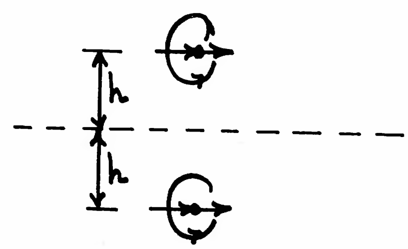


.Relative field strengths along the axis ($\theta=90^\circ$, $\phi=0^\circ$) for $\alpha=60^\circ$, 45°, 30° corner reflectors as a function of feed-to-vertex spacing.

$\boxed{7.10}$



$\boxed{7.11}$ Since a small loop can be modeled as a linear magnetic dipole, as shown in the figure, then using the sources and images of Figure 7-2(a) we choose the image to be another magnetic horizontal linear dipole as shown in the figure. The corresponding small loop to represent the image is also shown.



$\boxed{7.13}$ Since $\underline{E}_a = \hat{a}_\rho E_\rho + \hat{a}_\phi E_\phi$ exists only over the circular aperture where $\rho' \le a$, and it is zero elsewhere, then over the aperture and outside it along a flat plane coincident with the circular aperture we can form the electric and magnetic equivalent current, as shown below

$\underline{M}_a = \hat{n} \times \underline{E}_a$

$\underline{M}_s = 0,\ \underline{J}_s \neq 0$ $\quad$ $\underline{J}_s,\ \underline{M}_a$ $\hat{n}$ $\quad$ $\underline{M}_s = 0,\ \underline{J}_s \neq 0$ $\quad$ ϵ_0, μ_0

$a \quad a$

Now if we move a perfect electric conductor to coincide with the interface we have the following equivalent.

$\underline{M}_s = \underline{J}_s = 0$ $\quad$ $\underline{J}_s = 0, \underline{M}_a$ $\hat{n}$ $\quad$ $\underline{M}_s = \underline{J}_s = 0$ $\quad$ ϵ_0, μ_0

$\sigma = \infty$

which is equivalent to

$\underline{M}_s = 2\hat{n} \times \underline{E}_a$ $\quad$ $\underline{J}_s = \underline{M}_s = 0$ $\quad$ $\underline{J}_s = 0, \underline{M}_s = 2\underline{M}_a$ $\quad$ $\underline{J}_s = \underline{M}_s = 0$ $\quad$ ϵ_0, μ_0

$\underline{M}_s = 2\hat{a}_z \times (\hat{a}_\rho E_\rho + \hat{a}_\phi E_\phi) = 2\hat{a}_\phi E_\rho - 2\hat{a}_\rho E_\phi$ $\quad$ $a \quad a$ $\quad$ ϵ_0, μ_0

$\boxed{7.14}$ $\underline{E}^i = \hat{a}_x E_o e^{-j\beta_o(y\sin\theta_i - z\cos\theta_i)}$

$\underline{H}^i = \frac{E_o}{\eta}(-\hat{a}_y\cos\theta_i - \hat{a}_z\sin\theta_i)e^{-j\beta_o(y\sin\theta_i - z\cos\theta_i)}$

a. <u>Induction Equivalent</u>

$$\underline{M}_i = 2\hat{n}\times\underline{E}^i\Big|_{\substack{z=0\\y=y'}} = 2\hat{a}_z\times\hat{a}_x E_x\Big|_{\substack{z=0\\y=y'}} = \hat{a}_y 2E_o e^{-j\beta_o y\sin\theta_i}$$

$$M_x = M_z = 0,\quad M_y = 2E_o e^{-j\beta_o y'\sin\theta_i}$$

Using (6-125a)-(6-125d)

$$N_\theta = N_\phi = 0$$

$$L_\theta = \iint_{S_a} M_y\cos\theta_s\sin\phi_s e^{j\beta r'\cos\psi}ds' = 2E_o\cos\theta\sin\phi\iint_{S_a} e^{-j\beta_o y'\sin\theta_i}e^{j\beta r'\cos\psi}ds'$$

Using (6-130b), (6-132a) and (6-132b) we can write

$$r'\cos\psi = x'\sin\theta_s\cos\phi_s + y'\sin\theta_s\sin\phi_s$$

$$ds' = dx'dy' = \rho'd\rho'd\phi'$$

Thus

$$L_\theta = 2E_o\cos\theta_s\sin\phi_s\iint_{S_a} e^{j\beta_o[x'\sin\theta_s\cos\phi_s + y'(\sin\theta_s\sin\phi_s - \sin\theta_i)]}\rho'd\rho'd\phi'$$

Since $x' = \rho'\cos\phi'$
$y' = \rho'\sin\phi'$

then the exponent of the exponential can be written as

$$x'\sin\theta_s\cos\phi_s + y'(\sin\theta_s\sin\phi_s - \sin\theta_i) = \rho'[\underbrace{\sin\theta_s\cos\phi_s}_{A}\cos\phi' + \sin\phi'(\underbrace{\sin\theta_s\sin\phi_s - \sin\theta_i}_{B})]$$

$$= \rho'(A\cos\phi' + B\sin\phi')$$

$$= \rho'\sqrt{A^2+B^2}\left(\frac{A}{\sqrt{A^2+B^2}}\cos\phi' + \frac{B}{\sqrt{A^2+B^2}}\sin\phi'\right)$$

$$= \rho'\sqrt{A^2+B^2}(\cos\phi_o\cos\phi' + \sin\phi_o\sin\phi')$$

$$x'\sin\theta_s\cos\phi_s + y'(\sin\theta_s\sin\phi_s - \sin\theta_i) = \sqrt{A^2+B^2}\,\rho'\cos(\phi' - \phi_o)$$

where $A = \sin\theta_s\cos\phi_s$
$B = \sin\theta_s\sin\phi_s - \sin\theta_i$
$\phi_o = \tan^{-1}\left(\frac{B}{A}\right)$

7.14 Cont'd. Thus we can write L_θ as

$$L_\theta = 2E_0 \cos\theta_s \sin\phi_s \int_0^a \left\{ \int_0^{2\pi} e^{j\beta_0 \sqrt{A^2+B^2}\, \rho' \cos(\phi'-\phi_0)} d\phi' \right\} \rho' d\rho'$$

$$= 2E_0 \cos\theta_s \sin\phi_s \int_0^a \left[2\pi J_0\left(\beta_0 \sqrt{A^2+B^2}\, \rho'\right) \right] \rho' d\rho' = 4\pi E_0 \cos\theta_s \sin\phi_s \int_0^a J_0\left(\beta_0 \sqrt{A^2+B^2}\right) \rho' d\rho'$$

$$L_\theta = 4\pi E_0 \cos\theta_s \sin\phi_s \left[a^2 \, \frac{J_1\left(\beta_0 a \sqrt{A^2+B^2}\right)}{\beta_0 a \sqrt{A^2+B^2}} \right] = 4\pi a^2 E_0 \cos\theta_s \sin\phi_s \frac{J_1\left(\beta_0 a \sqrt{A^2+B^2}\right)}{\beta_0 a \sqrt{A^2+B^2}}$$

Using (6-125d)

$$L_\phi = \iint_{S_a} M_y \cos\phi_s e^{j\beta_0 r' \cos\psi} ds' = 4\pi a^2 E_0 \cos\phi_s \frac{J_1\left(\beta_0 a \sqrt{A^2+B^2}\right)}{\beta_0 a \sqrt{A^2+B^2}}$$

The electric and magnetic scattered fields can be written using (6-122a) - (6-122f) as

$$E_r^s \simeq H_r^s = 0$$

$$E_\theta^s \simeq -j \frac{\beta_0 e^{-j\beta_0 r}}{4\pi r} \left(L_\phi + \cancelto{0}{\eta N_\theta} \right) = -j \frac{\beta_0 a^2 E_0 e^{-j\beta_0 r}}{r} \cos\phi_s \frac{J_1\left(\beta_0 a \sqrt{A^2+B^2}\right)}{\beta_0 a \sqrt{A^2+B^2}}$$

$$E_\phi^s \simeq j \frac{\beta_0 e^{-j\beta_0 r}}{4\pi r} \left(L_\theta - \cancelto{0}{\eta N_\phi} \right) = j \frac{\beta_0 a^2 E_0 e^{-j\beta_0 r}}{r} \cos\theta_s \sin\phi_s \frac{J_1\left(\beta_0 a \sqrt{A^2+B^2}\right)}{\beta_0 a \sqrt{A^2+B^2}}$$

$$H_\theta^s \simeq -\frac{E_\phi^s}{\eta}, \quad H_\phi^s = \frac{E_\theta^s}{\eta}$$

<u>For backscatter observations</u>

$$\theta_s = \theta_i, \quad \phi_s = 3\pi/2$$

$$\left.\begin{aligned} A &= \sin\theta_s \cos\phi_s = 0 \\ B &= \sin\theta_s \sin\phi_s - \sin\theta_i = -\sin\theta_i - \sin\theta_i = -2\sin\theta_i \end{aligned}\right\} \sqrt{A^2+B^2} = 2\sin\theta_i$$

Thus

$$E_\theta^s \simeq 0$$

$$E_\phi^s \simeq -j \frac{\beta_0 a^2 E_0 e^{-j\beta_0 r}}{r} \cos\theta_i \, \frac{J_1(2\beta_0 a \sin\theta_i)}{2\beta_0 a \sin\theta_i}$$

$$H_\theta^s \simeq -\frac{E_\phi^s}{\eta}$$

$$H_\phi^s \simeq \frac{E_\theta^s}{\eta}$$

cont'd.

7.14 Cont'd.

b. Physical Equivalent

$$\underline{J}_p = 2\hat{n}\times\underline{H}^i\Big|_{\substack{z=0\\y=y'}} = 2\hat{a}_z\times(\hat{a}_y H_y^i + \hat{a}_z H_z^i)\Big|_{\substack{z=0\\y=y'}} = -\hat{a}_x 2H_y^i\Big|_{\substack{z=0\\y=y'}} = \hat{a}_x 2\frac{E_0}{\eta}\cos\theta_i e^{-j\beta_0 y'\sin\theta_i}$$

$$J_y = J_z = 0, \quad J_x = \frac{2E_0}{\eta}\cos\theta_i e^{-j\beta_0 y\sin\theta_i}$$

Using (6-125a)-(6-125d) and the results of the induction equivalent, we can write

$$N_\theta = \iint_{S_a} J_x\cos\theta_s\cos\phi_s e^{j\beta_0 r'\cos\psi}\,ds' = \frac{4\pi a^2 E_0}{\eta}\cos\theta_i\cos\theta_s\cos\phi_s \frac{J_1(\beta_0 a\sqrt{A^2+B^2})}{\beta_0 a\sqrt{A^2+B^2}}$$

$$N_\phi = \iint_{S_a} -J_x\sin\phi_s e^{j\beta_0 r'\cos\psi}\,ds' = -\frac{4\pi a^2 E_0}{\eta}\cos\theta_i\sin\phi_s \frac{J_1(\beta_0 a\sqrt{A^2+B^2})}{\beta_0 a\sqrt{A^2+B^2}}$$

$$A = \sin\theta_s\cos\phi_s, \quad B = \sin\theta_s\sin\phi_s - \sin\theta_i$$

Now using (6-122a)-(6-122f) we can write the scattered electric and magnetic fields as

$$E_r^s \simeq H_r^s \simeq 0$$

$$E_\theta^s \simeq -j\frac{\beta_0 e^{-j\beta_0 r}}{4\pi r}(\cancelto{0}{L_\phi} + \eta N_\theta) = -j\frac{\beta_0 a^2 E_0 e^{-j\beta_0 r}}{r}\cos\theta_i\cos\theta_s\cos\phi_s \frac{J_1(\beta_0 a\sqrt{A^2+B^2})}{\beta_0 a\sqrt{A^2+B^2}}$$

$$E_\phi^s \simeq j\frac{\beta_0 e^{-j\beta_0 r}}{4\pi r}(\cancelto{0}{L_\theta} - \eta N_\phi) = j\frac{\beta_0 a^2 E_0 e^{-j\beta_0 r}}{r}\cos\theta_i\sin\phi_s \frac{J_1(\beta_0 a\sqrt{A^2+B^2})}{\beta_0 a\sqrt{A^2+B^2}}$$

$$H_\theta^s \simeq -\frac{E_\phi^s}{\eta}, \quad H_\phi^s \simeq \frac{E_\theta^s}{\eta}$$

For backscatter observations

$$\theta_s = \theta_i, \quad \phi_s = 3\pi/2$$

$$\left.\begin{aligned} A &= 0 \\ B &= -2\sin\theta_i \end{aligned}\right\}\sqrt{A^2+B^2} = 2\sin\theta_i$$

$$E_\theta^s \simeq 0$$

$$E_\phi^s \simeq -j\frac{\beta_0 a^2 E_0 e^{-j\beta_0 r}}{r}\cos\theta_i \frac{J_1(2\beta_0 a\sin\theta_i)}{2\beta_0 a\sin\theta_i}$$

$$H_\theta^s \simeq -\frac{E_\phi^s}{\eta}$$

$$H_\phi^s \simeq \frac{E_\theta^s}{\eta}$$

$\boxed{7.15}$ $\underline{H}^i = \hat{a}_x H_o e^{-j\beta_o(y\sin\theta_i - z\cos\theta_i)}$

$\underline{E}^i = \eta H_o(\hat{a}_y \cos\theta_i + \hat{a}_z \sin\theta_i) e^{-j\beta_o(y\sin\theta_i - z\cos\theta_i)}$

a. Induction Equivalent

$$\underline{M}_i = 2\hat{n}\times\underline{E}^i\Big|_{\substack{z=0\\y=y'}} = 2\hat{a}_z \times (\hat{a}_y E_y + \hat{a}_z E_z)\Big|_{\substack{z=0\\y=y'}} = -\hat{a}_x 2E_y\Big|_{\substack{z=0\\y=y'}} = -\hat{a}_x 2\eta H_o \cos\theta_i e^{-j\beta_o y'\sin\theta_i}$$

$M_y = M_z = 0,\quad M_x = -2\eta H_o \cos\theta_i e^{-j\beta_o y'\sin\theta_i}$

Using (6-125a)-(6-125d) and the solution of Problem 7.14, we can write

$$L_\theta = \iint_{S_a} M_x \cos\theta_s \cos\phi_s e^{j\beta_o r'\cos\psi} ds' = -4\pi a^2 \eta H_o \cos\theta_i \cos\theta_s \cos\phi_s \frac{J_1(\beta_o a\sqrt{A^2+B^2})}{\beta_o a\sqrt{A^2+B^2}}$$

$$L_\phi = \iint_{S_a} -M_x \sin\phi_s e^{j\beta_o r'\cos\psi} ds' = +4\pi a^2 \eta H_o \cos\theta_i \sin\phi_s \frac{J_1(\beta_o a\sqrt{A^2+B^2})}{\beta_o a\sqrt{A^2+B^2}}$$

$A = \sin\theta_s\cos\phi_s,\quad B = \sin\theta_s\sin\phi_s - \sin\theta_i$

Now using (6-122a)-(6-122f) we can write the scattered electric and magnetic fields as

$E_r^s \simeq H_r^s \simeq 0$

$$E_\theta^s \simeq -\frac{j\beta_o e^{-j\beta_o r}}{4\pi r}(L_\phi + \cancel{\eta N_\theta}^{\,0}) = -j\frac{\beta_o a^2 \eta H_o e^{-j\beta_o r}}{r}\cos\theta_i \sin\phi_s \frac{J_1(\beta_o a\sqrt{A^2+B^2})}{\beta_o a\sqrt{A^2+B^2}}$$

$$E_\phi^s \simeq j\frac{\beta_o e^{-j\beta_o r}}{4\pi r}(L_\theta - \cancel{\eta N_\phi}^{\,0}) = -j\frac{\beta_o a^2 \eta H_o e^{-j\beta_o r}}{r}\cos\theta_i \cos\theta_s \cos\phi_s \frac{J_1(\beta_o a\sqrt{A^2+B^2})}{\beta_o a\sqrt{A^2+B^2}}$$

$H_\theta^s \simeq -\dfrac{E_\phi^s}{\eta},\quad H_\phi^s \simeq \dfrac{E_\theta^s}{\eta}$

For backscatter observations

$\theta_s = \theta_i,\ \phi_s = 3\pi/2$

$\left.\begin{aligned} A &= 0 \\ B &= -2\sin\theta_i \end{aligned}\right\} \sqrt{A^2+B^2} = 2\sin\theta_i$

$$E_\theta^s \simeq +j\frac{\beta_o a^2 \eta H_o e^{-j\beta_o r}}{r}\cos\theta_i \frac{J_1(2\beta_o a \sin\theta_i)}{2\beta_o a\sin\theta_i}$$

$E_\phi^s \simeq 0$

$H_\theta^s \simeq -\dfrac{E_\phi^s}{\eta}$

$H_\phi^s \simeq E_\theta^s/\eta$

cont'd.

b. Physical Equivalent

$$\underline{J}_P = 2\hat{n}\times\underline{H}^i\Big|_{\substack{z=0\\y=y'}} = 2\hat{a}_z\times\hat{a}_x H_x = \hat{a}_y 2H_0 e^{-j\beta_0 y'\sin\theta_i}$$

$$J_x = J_z = 0, \quad J_y = 2H_0 e^{-j\beta_0 y'\sin\theta_i}$$

Using (6-125a)-(6-125d) and the solution of Problem 7.14, we can write

$$\left.\begin{aligned} N_\theta &= \iint_{S_a} J_y \cos\theta_s \sin\phi_s e^{j\beta_0 r'\cos\psi} ds' = 4\pi a^2 H_0 \cos\theta_s \sin\phi_s \frac{J_1(\beta_0 a\sqrt{A^2+B^2})}{\beta_0 a\sqrt{A^2+B^2}} \\ N_\phi &= \iint_{S_a} J_y \cos\phi_s e^{j\beta_0 r'\cos\psi} ds' = 4\pi a^2 H_0 \cos\phi_s \frac{J_1(\beta_0 a\sqrt{A^2+B^2})}{\beta_0 a\sqrt{A^2+B^2}} \\ A &= \sin\theta_s\cos\phi_s, \; B = \sin\theta_s\sin\phi_s - \sin\theta_i \end{aligned}\right\}$$

Now using (6-122a)-(6-122f) we can write the scattered electric and magnetic fields as

$$E_\theta^s \simeq -j\frac{\beta_0 e^{-j\beta_0 r}}{4\pi r}(\cancelto{0}{L_\phi}+\eta N_\theta) = -j\frac{\beta_0 a^2\eta H_0 e^{-j\beta_0 r}}{r}\cos\theta_s\sin\phi_s \frac{J_1(\beta_0 a\sqrt{A^2+B^2})}{\beta_0 a\sqrt{A^2+B^2}}$$

$$E_\phi^s \simeq j\frac{\beta_0 e^{-j\beta_0 r}}{4\pi r}(\cancelto{0}{L_\theta}-\eta N_\phi) = -j\frac{\beta_0 a^2\eta H_0 e^{-j\beta_0 r}}{r}\cos\phi_s \frac{J_1(\beta_0 a\sqrt{A^2+B^2})}{\beta_0 a\sqrt{A^2+B^2}}$$

$$H_\theta^s \simeq -\frac{E_\phi^s}{\eta}$$

$$H_\phi^s \simeq \frac{E_\theta^s}{\eta}$$

For backscatter directions

$$\theta_s = \theta_i, \; \phi_s = 3\pi/2$$

$$\left.\begin{aligned} A &= 0 \\ B &= -2\sin\theta_i \end{aligned}\right\}\sqrt{A^2+B^2} = 2\sin\theta_i$$

$$E_\theta^s \simeq j\frac{\beta_0 a^2\eta H_0 e^{-j\beta_0 r}}{r}\cos\theta_i \frac{J_1(2\beta_0 a\sin\theta_i)}{2\beta_0 a\sin\theta_i}$$

$$E_\phi^s \simeq 0$$

$$H_\theta^s \simeq -\frac{E_\phi^s}{\eta}$$

$$H_\phi^s \simeq \frac{E_\theta^s}{\eta}$$