University of Utah
Advanced Electromagnetics
Image Theory

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Volume Equivalence

- Used to determine scattered fields when a material obstacle is introduced in the free space environment where the fields $\mathbf{E_0}$ and $\mathbf{H_0}$
- These fields must satisfy Maxwell's equation:

$$\nabla \times \mathbf{E}_0 = -\mathbf{M}_i - j\omega\mu_0\mathbf{H}_0$$

$$\nabla \times \mathbf{H}_0 = \mathbf{J}_i + j\omega\varepsilon_0\mathbf{E}_0$$

Volume Equivalence

When the same sources $(\mathbf{J}_i, \mathbf{M}_i)$ radiate in a medium represented by (ε, μ), they generate fields $(\mathbf{E}, \mathbf{H})$ that satisfy Maxwell's equations

$$\nabla \times \mathbf{E} = -\mathbf{M}_i - j\omega\mu\mathbf{H} \quad (7\text{-}36a)$$

$$\nabla \times \mathbf{H} = \mathbf{J}_i + j\omega\varepsilon\mathbf{E} \quad (7\text{-}36b)$$

Subtracting the two equations we get:

$$\nabla \times (\mathbf{E} - \mathbf{E}_0) = -j\omega(\mu\mathbf{H} - \mu_0\mathbf{H}_0) \quad (7\text{-}37a)$$

$$\nabla \times (\mathbf{H} - \mathbf{H}_0) = j\omega(\varepsilon\mathbf{E} - \varepsilon_0\mathbf{E}_0) \quad (7\text{-}37b)$$

Let us define the difference between the fields $\mathbf{E}$ and $\mathbf{E}_0$, and $\mathbf{H}$ and $\mathbf{H}_0$ as the *scattered* (disturbance) fields $\mathbf{E}^s$ and $\mathbf{H}^s$, that is,

$$\mathbf{E}^s = \mathbf{E} - \mathbf{E}_0 \Rightarrow \mathbf{E}_0 = \mathbf{E} - \mathbf{E}^s \quad (7\text{-}38a)$$

$$\mathbf{H}^s = \mathbf{H} - \mathbf{H}_0 \Rightarrow \mathbf{H}_0 = \mathbf{H} - \mathbf{H}^s \quad (7\text{-}38b)$$

$$\nabla \times \mathbf{E}^s = -j\omega\left[\mu\mathbf{H} - \mu_0(\mathbf{H} - \mathbf{H}^s)\right] = -j\omega(\mu - \mu_0)\mathbf{H} - j\omega\mu_0\mathbf{H}^s \quad (7\text{-}39a)$$

$$\nabla \times \mathbf{H}^s = j\omega\left[\varepsilon\mathbf{E} - \varepsilon_0(\mathbf{E} - \mathbf{E}^s)\right] = j\omega(\varepsilon - \varepsilon_0)\mathbf{E} + j\omega\varepsilon_0\mathbf{E}^s \quad (7\text{-}39b)$$

By defining volume equivalent electric $\mathbf{J}_{eq}$ and magnetic $\mathbf{M}_{eq}$ current densities

$$\mathbf{J}_{eq} = j\omega(\varepsilon - \varepsilon_0)\mathbf{E} \tag{7-40a}$$

$$\mathbf{M}_{eq} = j\omega(\mu - \mu_0)\mathbf{H} \tag{7-40b}$$

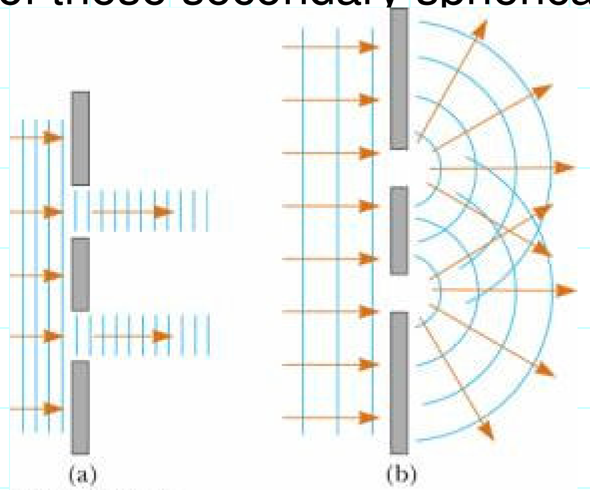
The electric and magnetic fields scattered by a material obstacle can be Generated by using equivalent electric $\mathbf{J}_{eq}$ and $\mathbf{M}_{eq}$ magnetic volume current Densities.

Volume equivalent current densities are most useful for finding the electric And magnetic fields scattered by a dielectric object

Surface Equivalence

- Actual sources are replaced by equivalent sources
- These fictitious sources are said to be equivalent within a region because they produce within that region the same fields as the actual sources
- This principle was formulated by Schelkunoff and is a more rigorous formulation of Huygens's principle:

" Each Point on a primary wavefront can be considered to be a new source of a secondary spherical wave and that a secondary wavefront can be considered as the envelop of these secondary spherical waves"



Surface Equivalence

" A field in a lossy region is uniquely specified by the sources within the region plus the tangential components of the electric field over the boundary, or the tangential components of the magnetic fields over the boundary, or the former Over part of the boundary and the latter over the rest of the boundary "

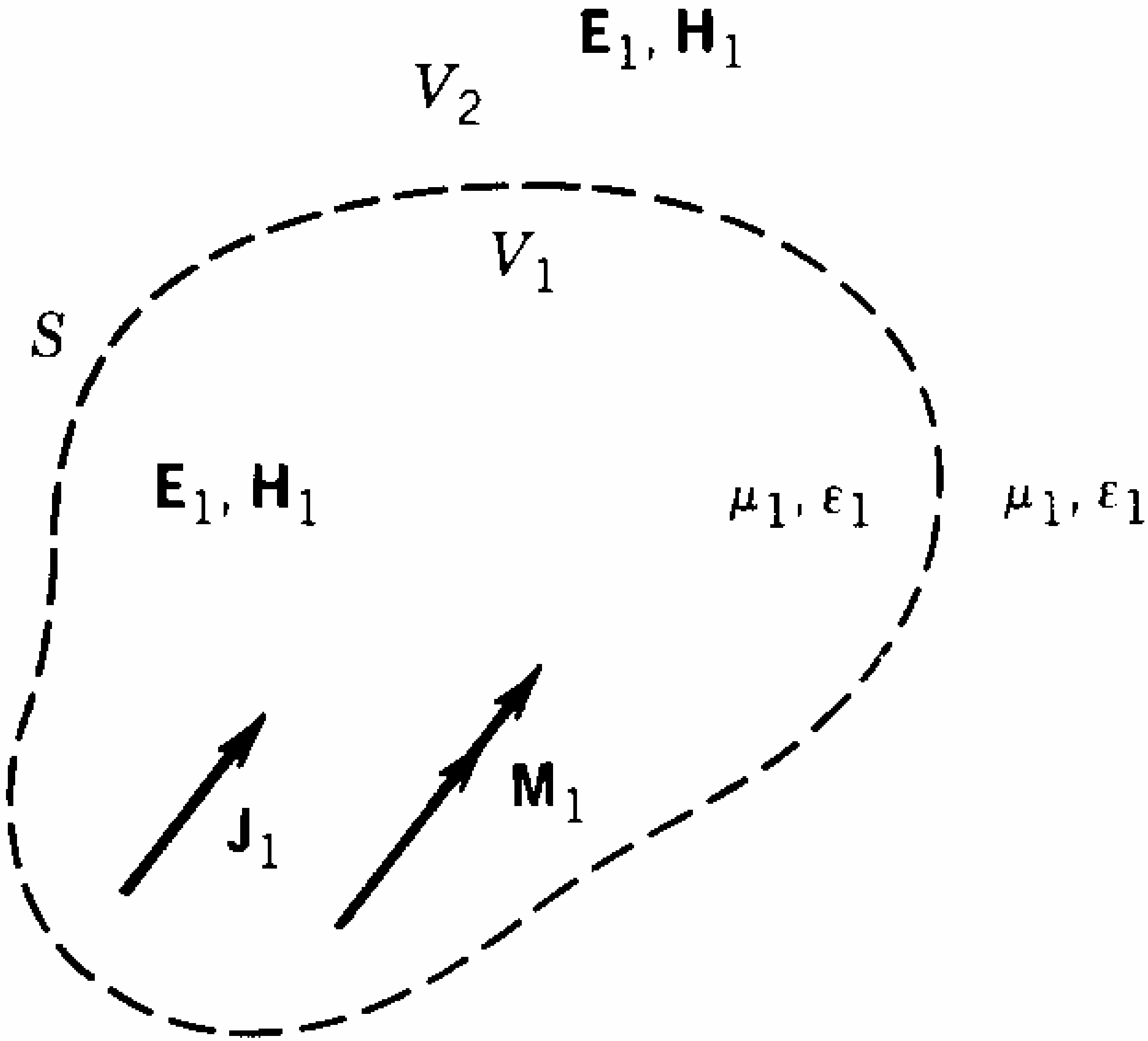
The fields in a lossless medium are considered to be the limit, as the losses go To zero., of the corresponding fields in lossy media.

If the tangential electric and magnetic field are known over a closed surface, the fields in the source-free region can be determined.

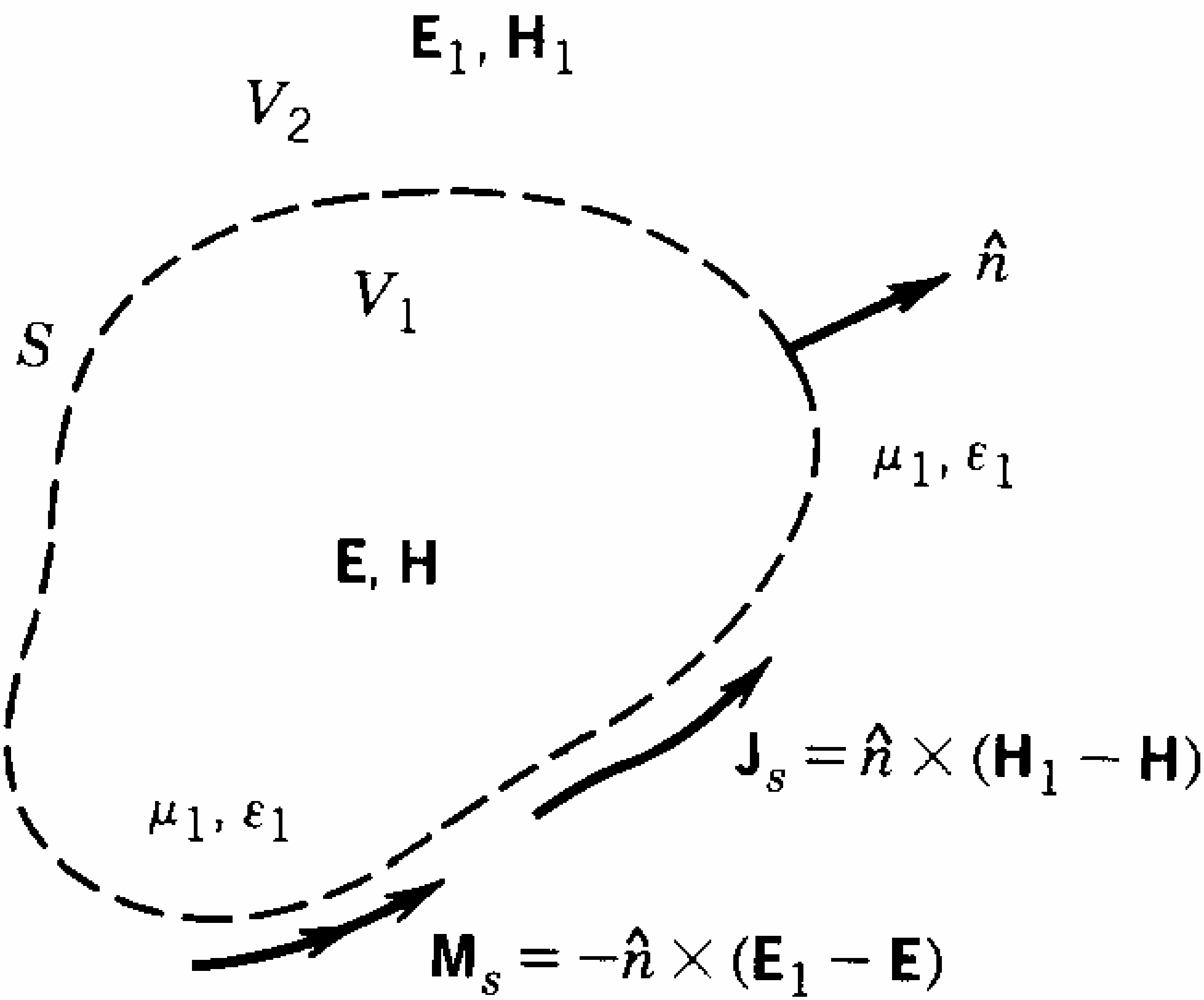
By the surface equivalence theorem, the fields outside an imaginary closed surface are obtained by placing, over the closed surface, suitable electric and magnetic current densities that satisfy the boundary conditions. The current densities are selected so that the fields inside the closed surface are zero and outside are equal to the radiation produced by the actual sources. Thus the technique can be used to obtain the fields radiated outside a closed surface by sources enclosed within

The degree of accuracy depends on the knowledge of the tangential components of the field over the closed surface

The surface equivalence theorem is developed by considering an actual radiating source, which is represented electrically by current densities $\mathbf{J}_1$ and $\mathbf{M}_1$, as



E1, H1
V2
V1
S
E1, H1
μ1, ε1
μ1, ε1
J1
M1


$\mathbf{E}_1, \mathbf{H}_1$
V_2
$\hat{n}$
V_1
S
μ_1, ϵ_1
$\mathbf{E}, \mathbf{H}$
$\mathbf{J}_s = \hat{n} \times (\mathbf{H}_1 - \mathbf{H})$
μ_1, ϵ_1
$\mathbf{M}_s = -\hat{n} \times (\mathbf{E}_1 - \mathbf{E})$

$$\mathbf{J}_s = \hat{n} \times (\mathbf{H}_1 - \mathbf{H})$$

$$\mathbf{M}_s = -\hat{n} \times (\mathbf{E}_1 - \mathbf{E})$$

Special case: Love'e equivalence principle

$$\mathbf{J}_s = \hat{n} \times (\mathbf{H}_1 - \mathbf{H})\big|_{\mathbf{H}=0} = \hat{n} \times \mathbf{H}_1$$

$$\mathbf{M}_s = -\hat{n} \times (\mathbf{E}_1 - \mathbf{E})\big|_{\mathbf{E}=0} = -\hat{n} \times \mathbf{E}_1$$

Love's Equivalent

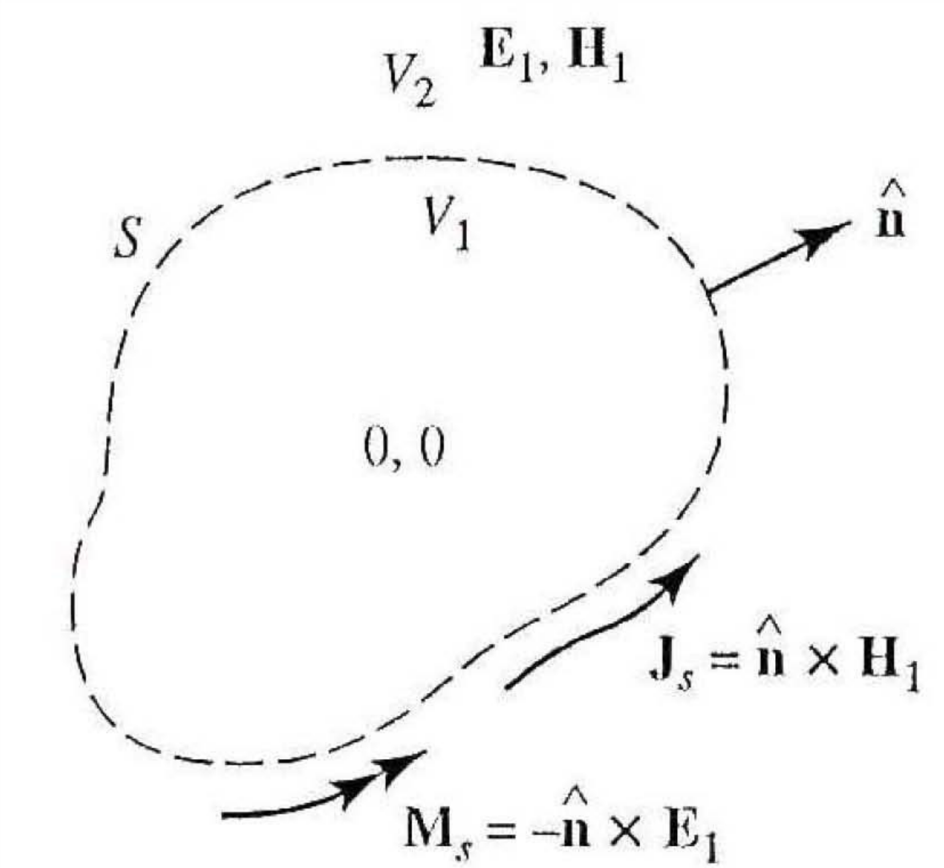


Fig. 12.3a

Electric Conductor Equivalent

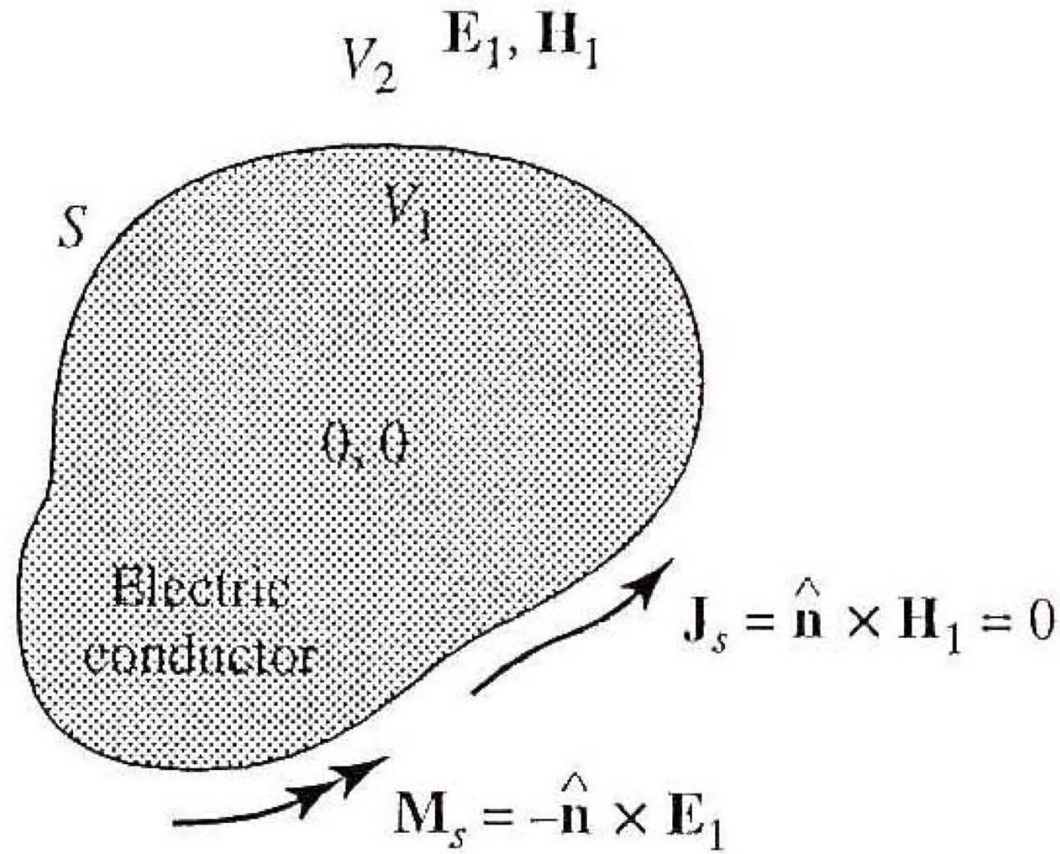


Fig. 12.3b

Magnetic Conductor Equivalent

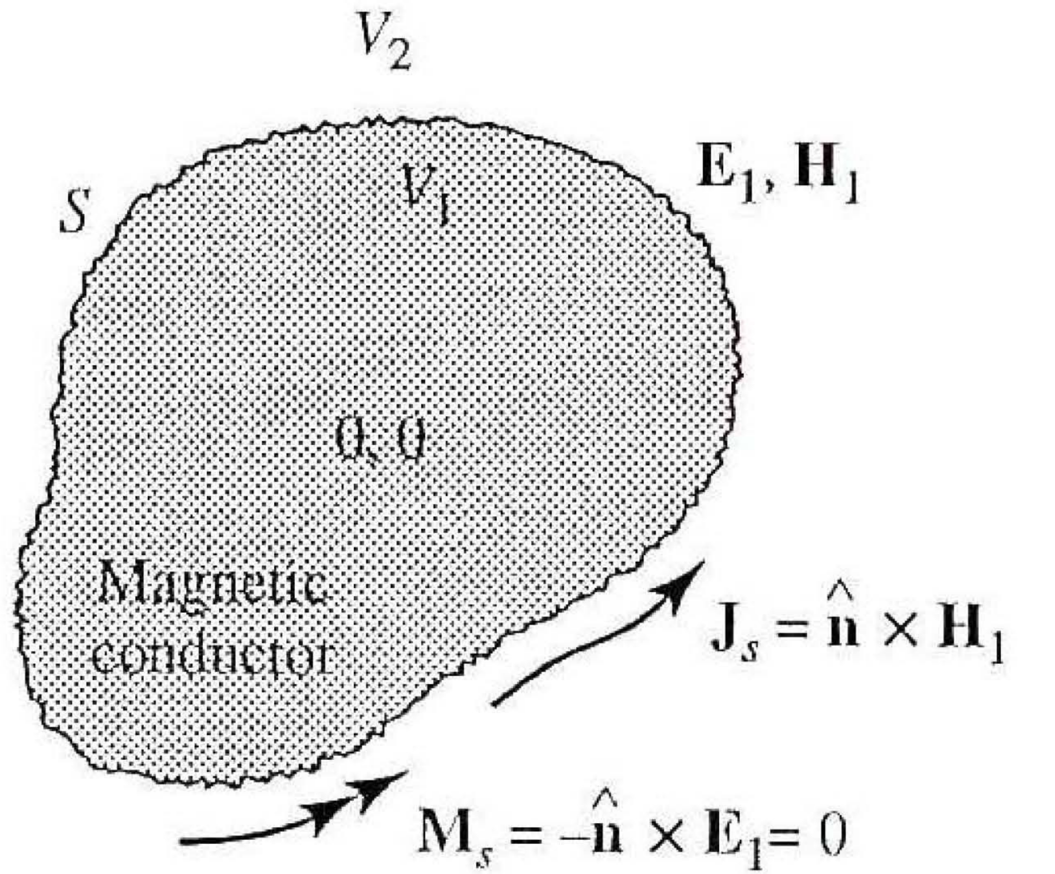


Fig. 12.3c

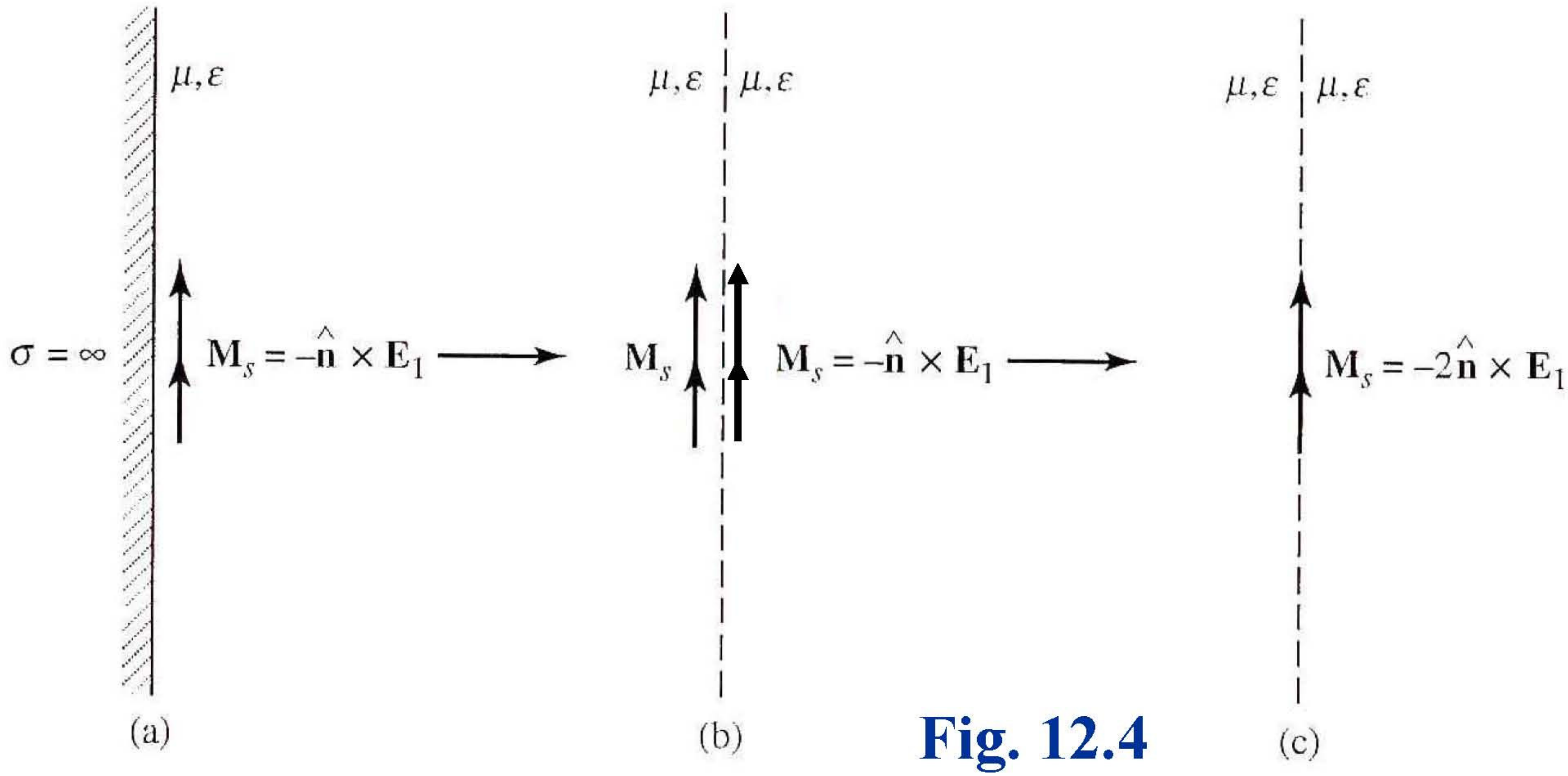


Fig. 12.4

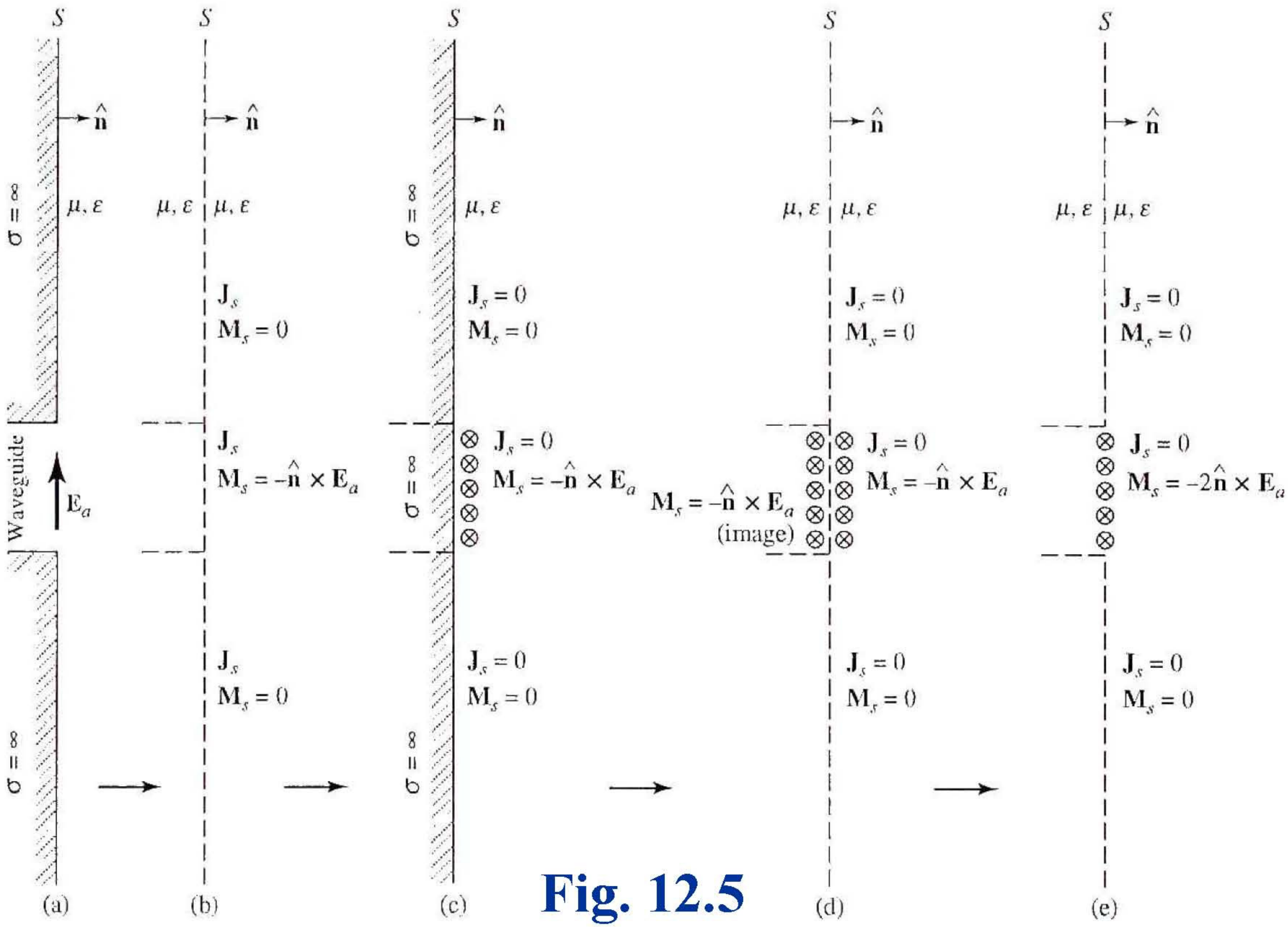

S
n̂
σ = ∞
μ, ε
Waveguide
E_a
J_s
M_s = 0
M_s = −n̂ × E_a
J_s = 0
M_s = −n̂ × E_a (image)
M_s = −2n̂ × E_a
(a)
(b)
(c)
(d)
(e)

Fig. 12.5

The following steps must be used to form an equivalent and solve an aperture problem.

1. Select an imaginary surface that encloses the actual sources (the aperture). The surface must be chosen judiciously so that the tangential components of the electric and/or the magnetic field are known, exactly or approximately, over its entire span. In many cases this surface is a flat plane extending to infinity.
2. Over the imaginary surface form equivalent current densities $\mathbf{J}_s, \mathbf{M}_s$ that take one of the following forms.
   - **a.** $\mathbf{J}_s$ and $\mathbf{M}_s$ over S assuming that the **E** and **H** fields within S are not zero.
   - **b.** $\mathbf{J}_s$ and $\mathbf{M}_s$ over S assuming that the **E** and **H** fields within S are zero (Love's theorem).
   - **c.** $\mathbf{M}_s$ over S ($\mathbf{J}_s = 0$) assuming that within S the medium is a perfect electric conductor.
   - **d.** $\mathbf{J}_s$ over S ($\mathbf{M}_s = 0$) assuming that within S the medium is a perfect magnetic conductor
3. Solve the equivalent problem. For forms **a** and **b**, equations 6-30 through 6-35a can be used. For form **c**, the problem of a magnetic current source next to a perfect electric conductor must be solved [(6-30) through (6-35a) cannot be used directly, because the current density does not radiate into an unbounded medium]. If the electric conductor is an infinite flat plane, the problem can be solved exactly by image theory. For form **d**, the problem of an electric current source next to a perfect magnetic conductor must be solved. Again (6-30) through (6-35a) cannot be used directly. If the magnetic conductor is an infinite flat plane, the problem can be solved exactly by image theory.

Two-Wire Transmission Line

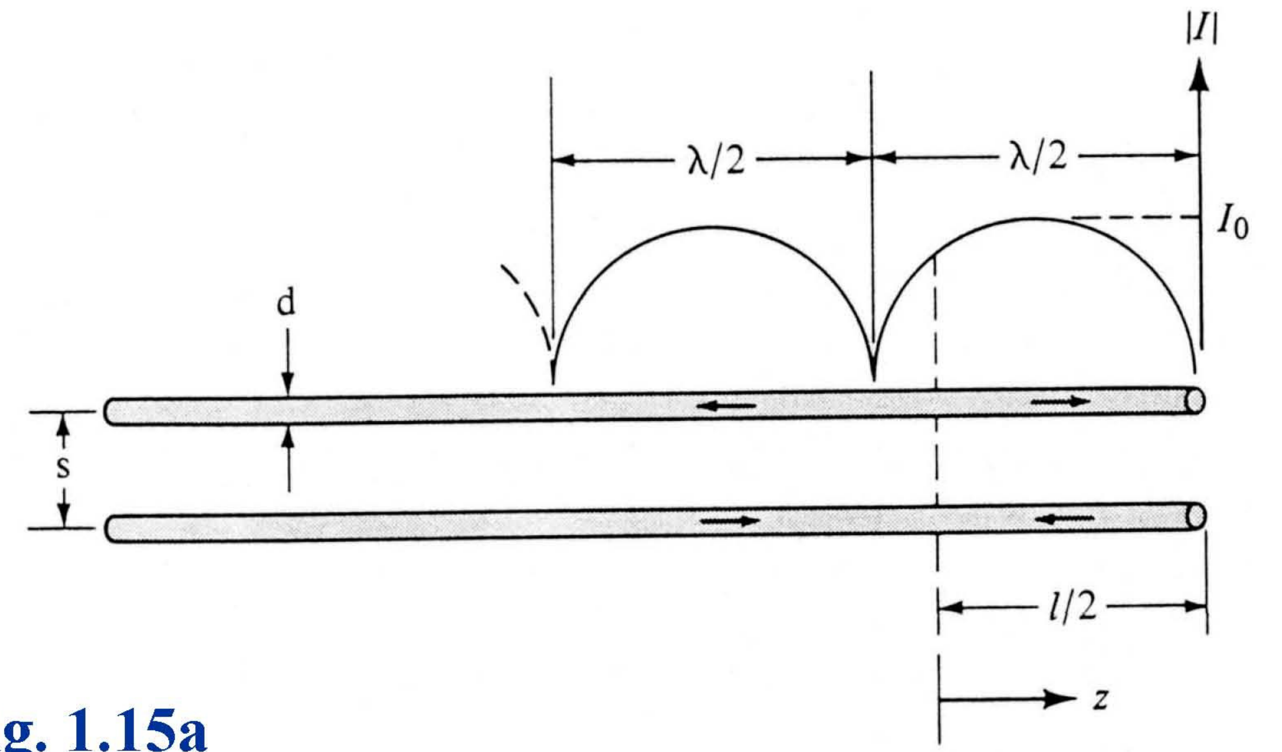


Fig. 1.15a

$\ell = \lambda / 2$

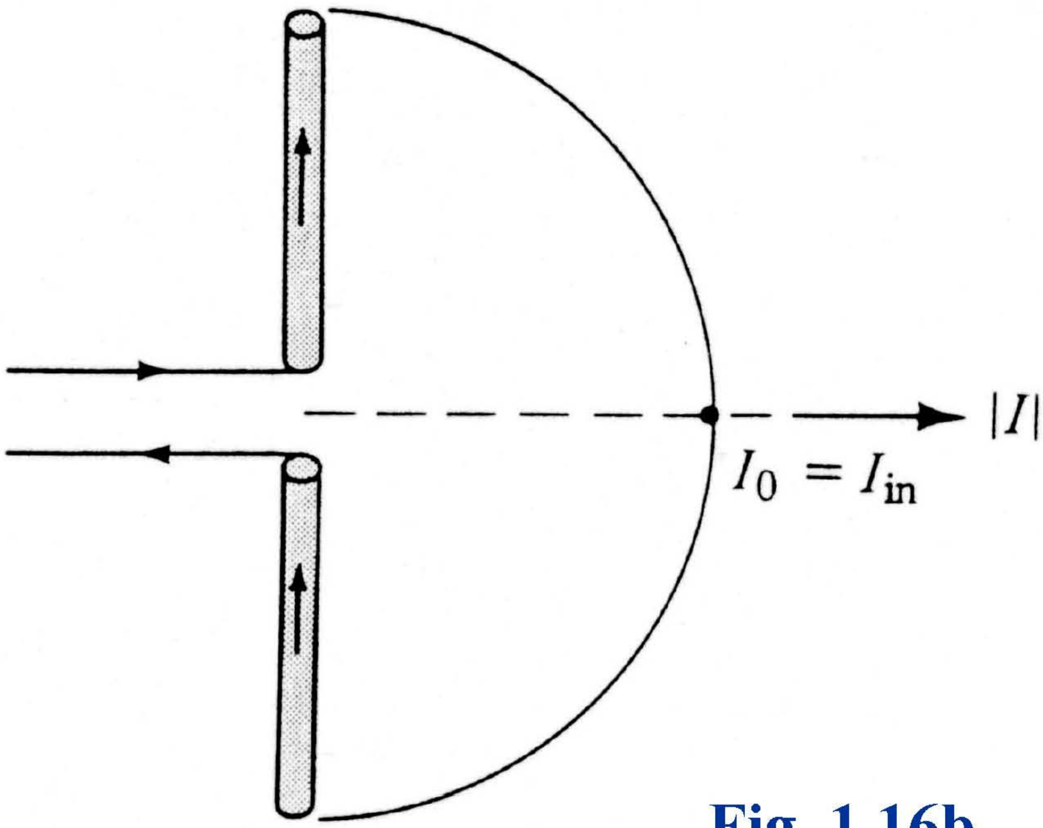


Fig. 1.16b

Image Theory

Vertical Polarization

Vertical Electric Dipole (VED)

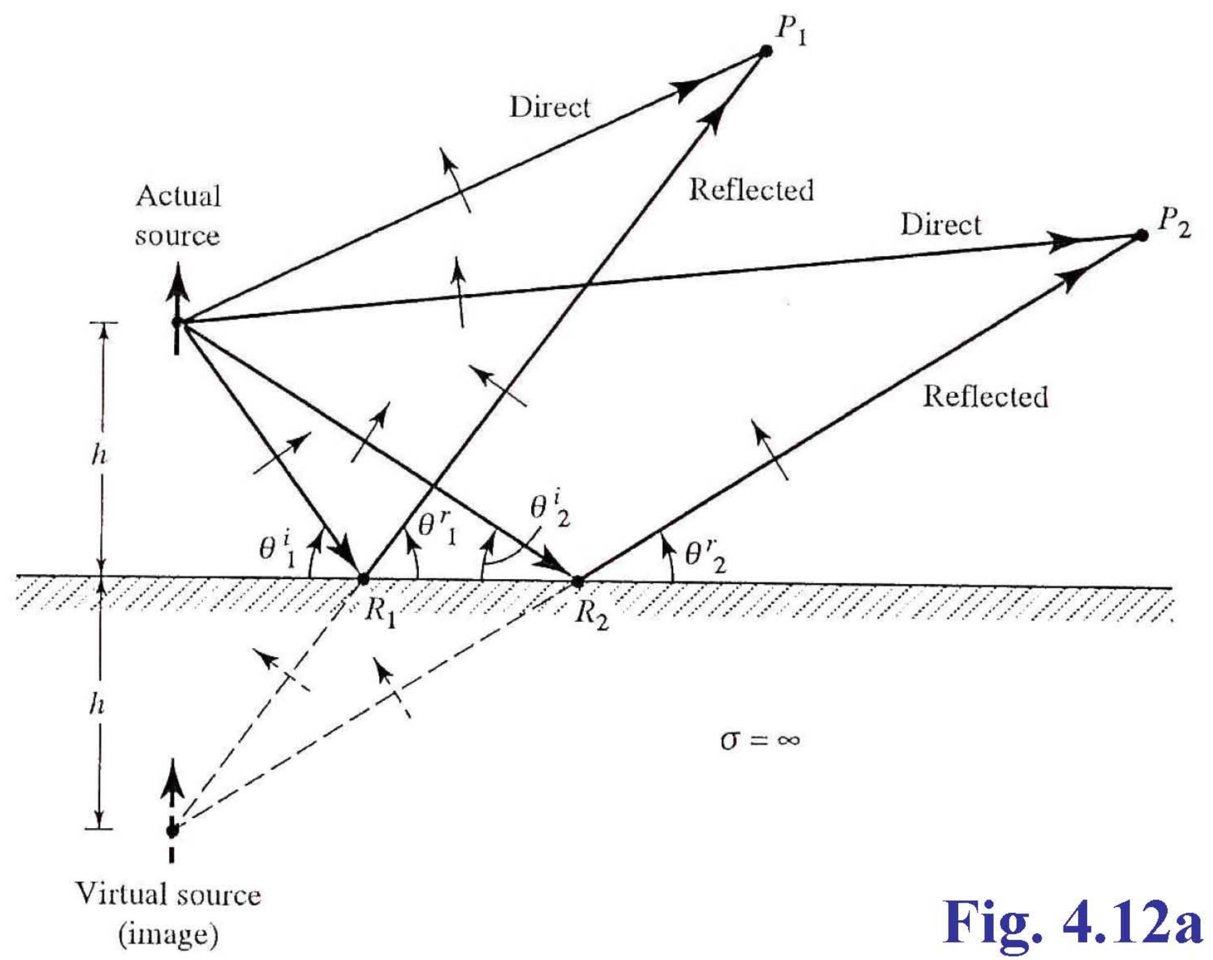


Fig. 4.12a

Field Components at Point of Reflection

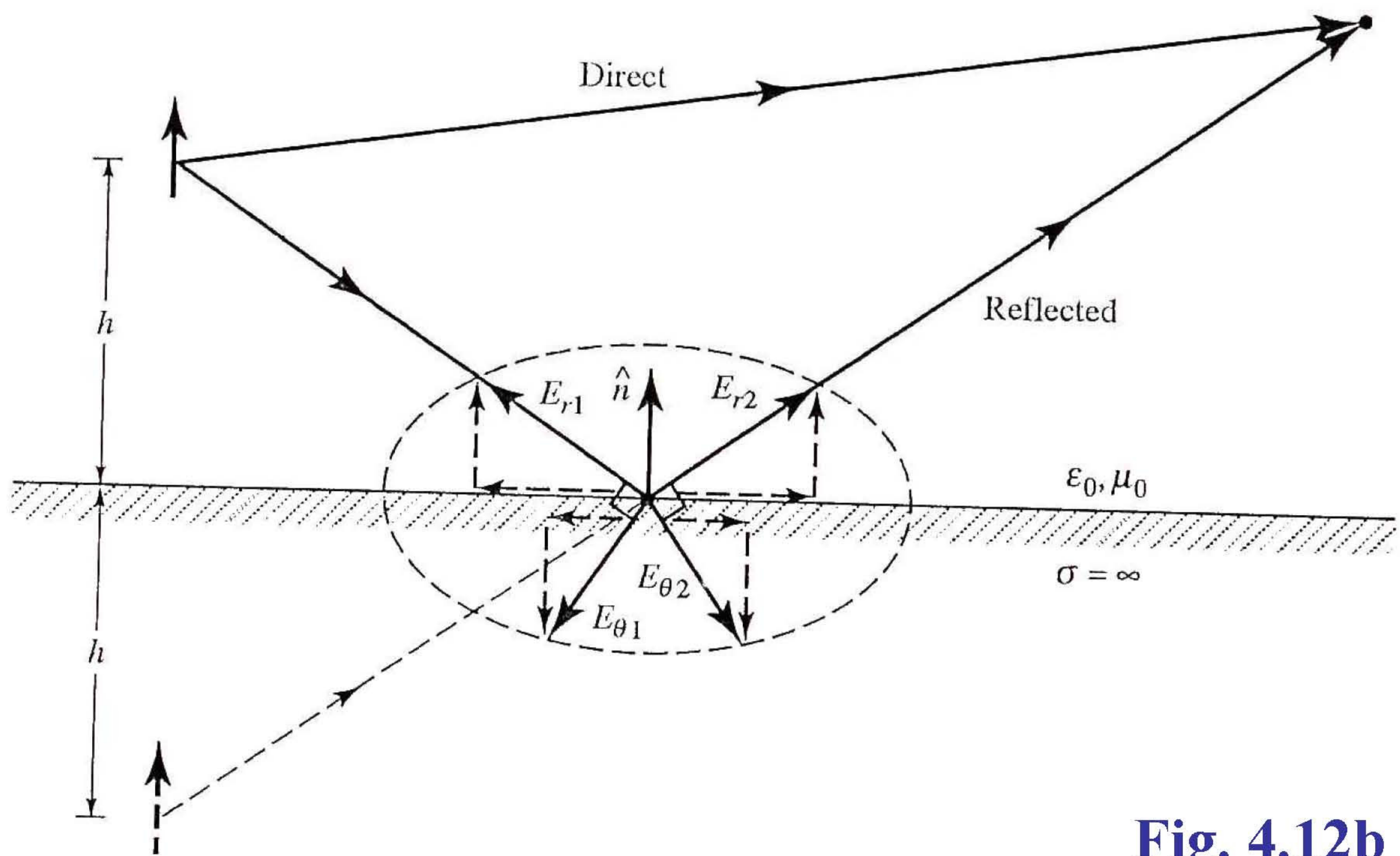
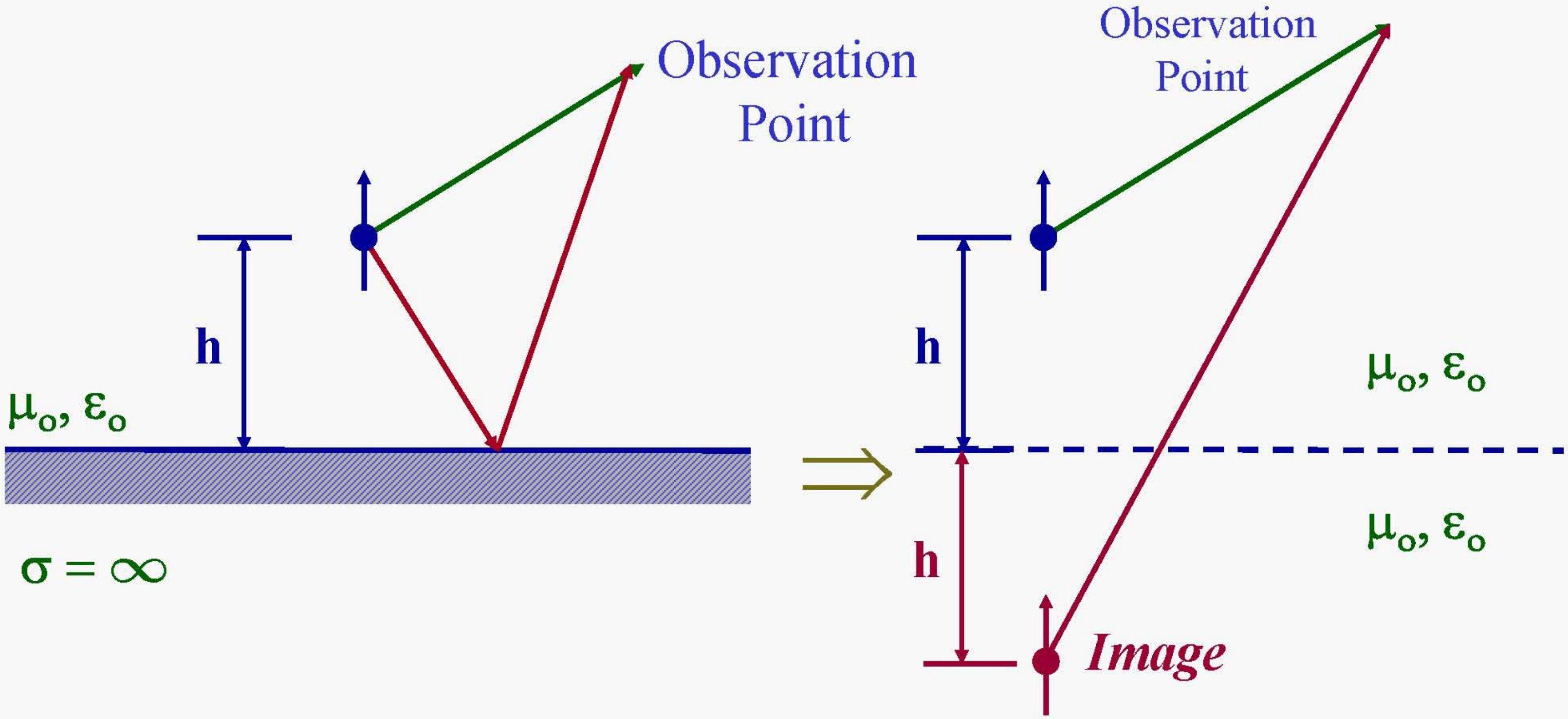


Fig. 4.12b

Actual and Equivalent Problems



Actual Problem

Equivalent Problem

Electric Conductor

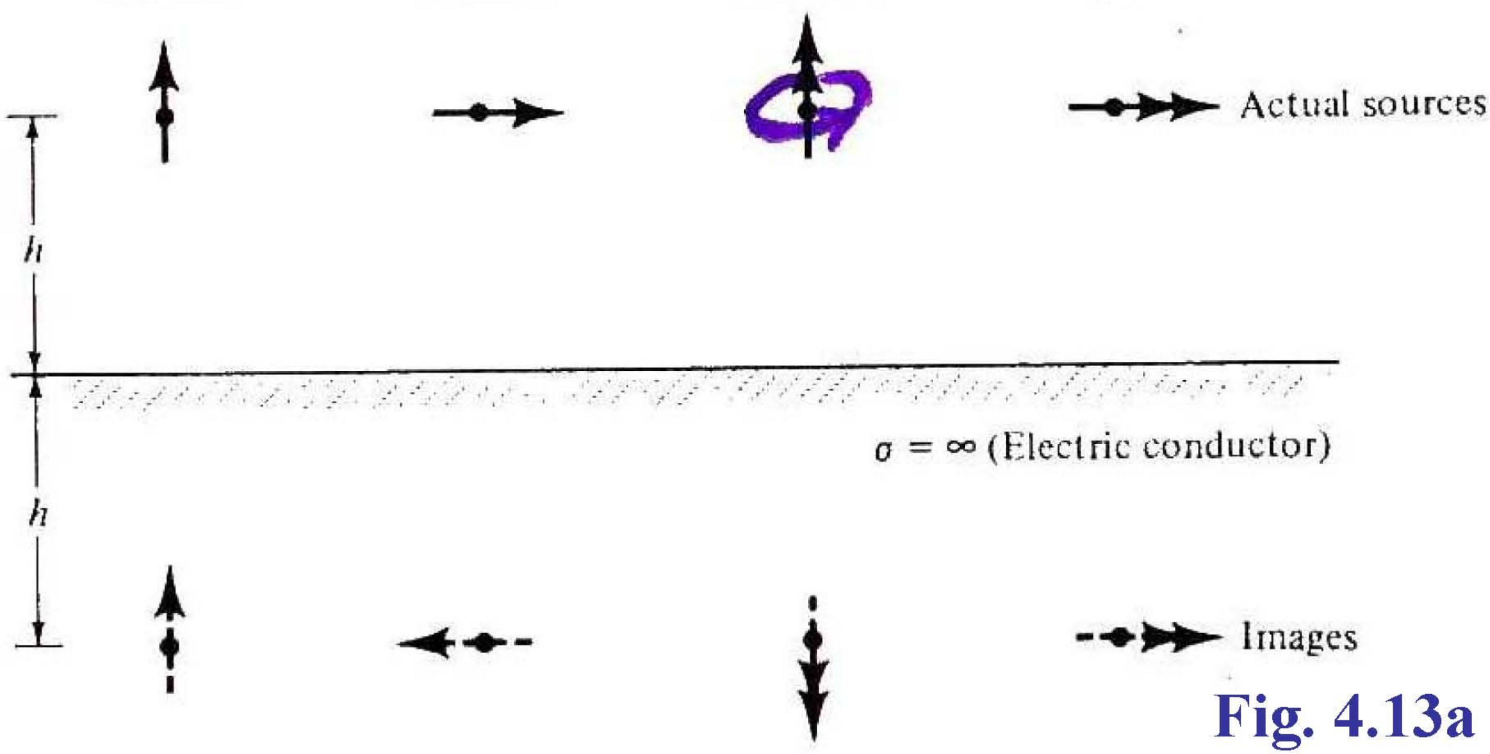


Fig. 4.13a

Magnetic Conductor

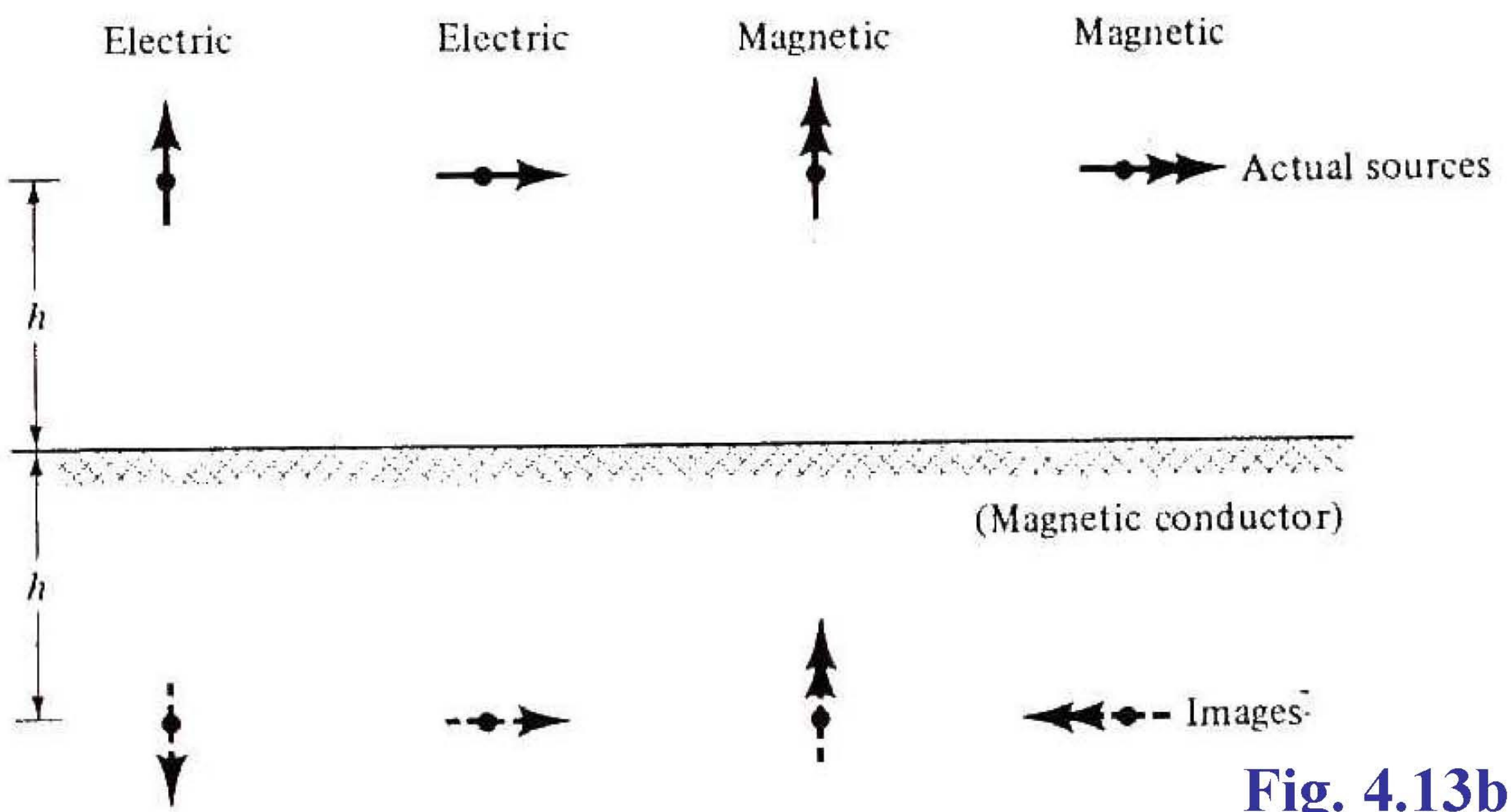
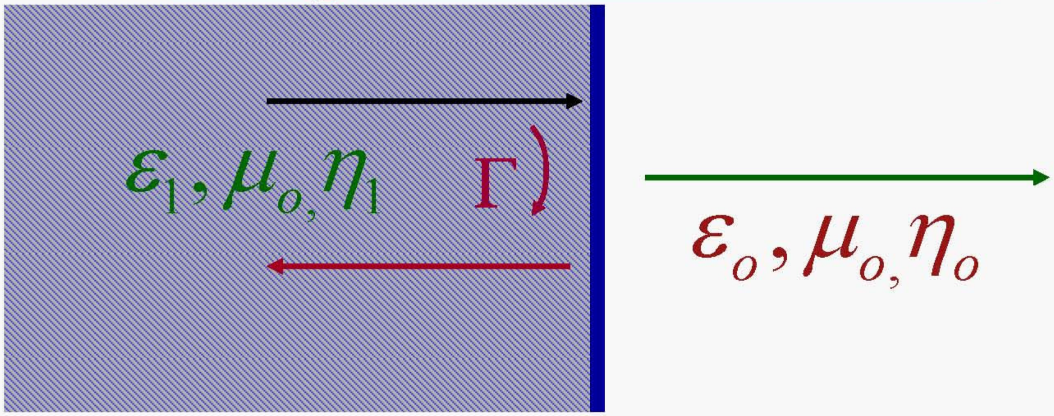


Fig. 4.13b

Magnetic Conductor



$$\Gamma = \frac{\eta_0 - \eta_1}{\eta_0 + \eta_1} = \frac{\sqrt{\frac{\mu_0}{\varepsilon_0}} - \sqrt{\frac{\mu_0}{\varepsilon_1}}}{\sqrt{\frac{\mu_0}{\varepsilon_0}} + \sqrt{\frac{\mu_0}{\varepsilon_1}}} = \frac{\sqrt{\frac{\mu_0}{\varepsilon_0}} - 1}{\sqrt{\frac{\mu_0}{\varepsilon_0}} + 1}$$

$$\Gamma = \frac{\sqrt{\varepsilon_r} - 1}{\sqrt{\varepsilon_r} + 1} \overset{\varepsilon_r \to \text{large}}{\simeq} +1 \qquad \text{(Open Circuit)}$$

Vertical Electric Dipole Above Ground Plane

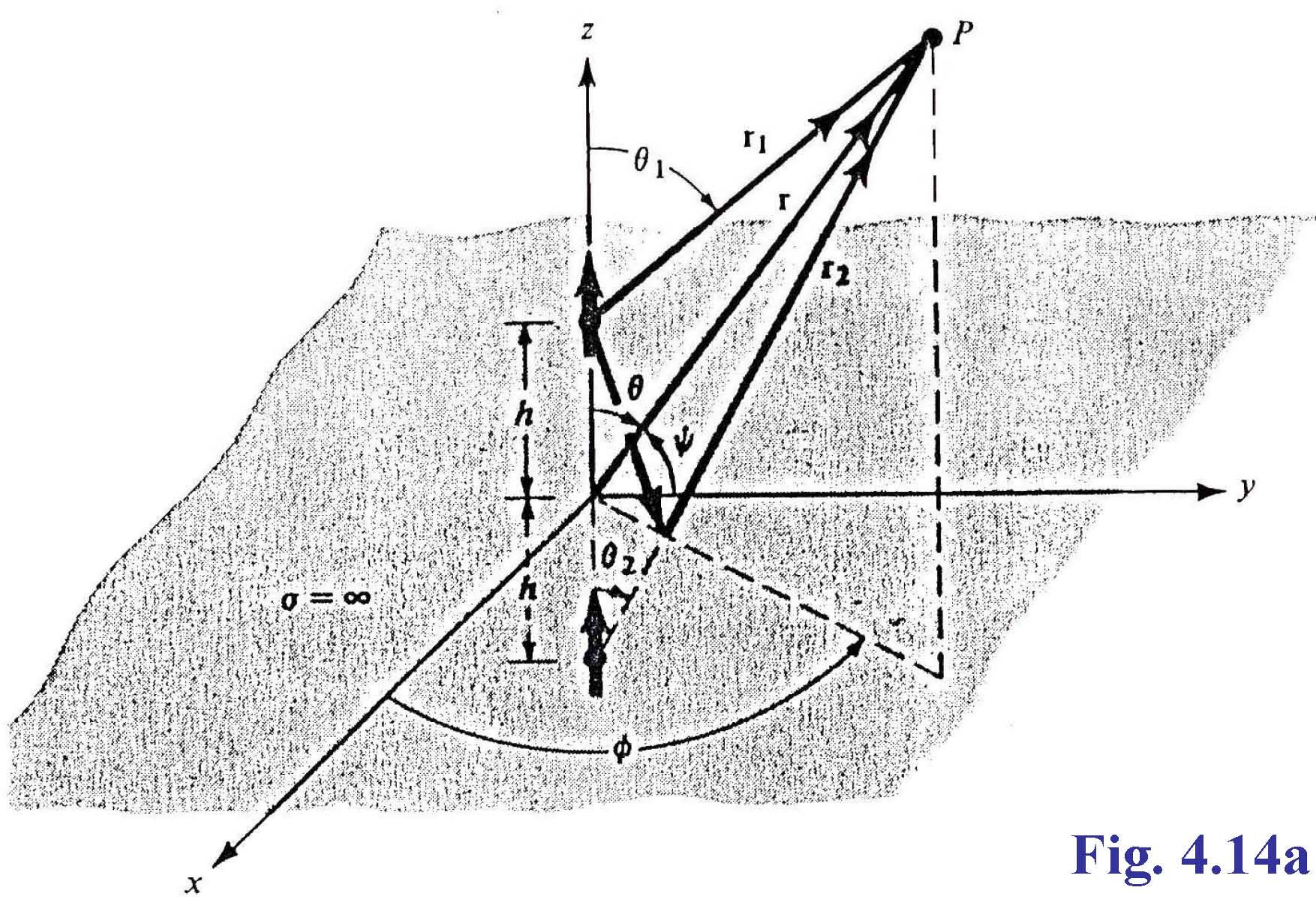


Fig. 4.14a

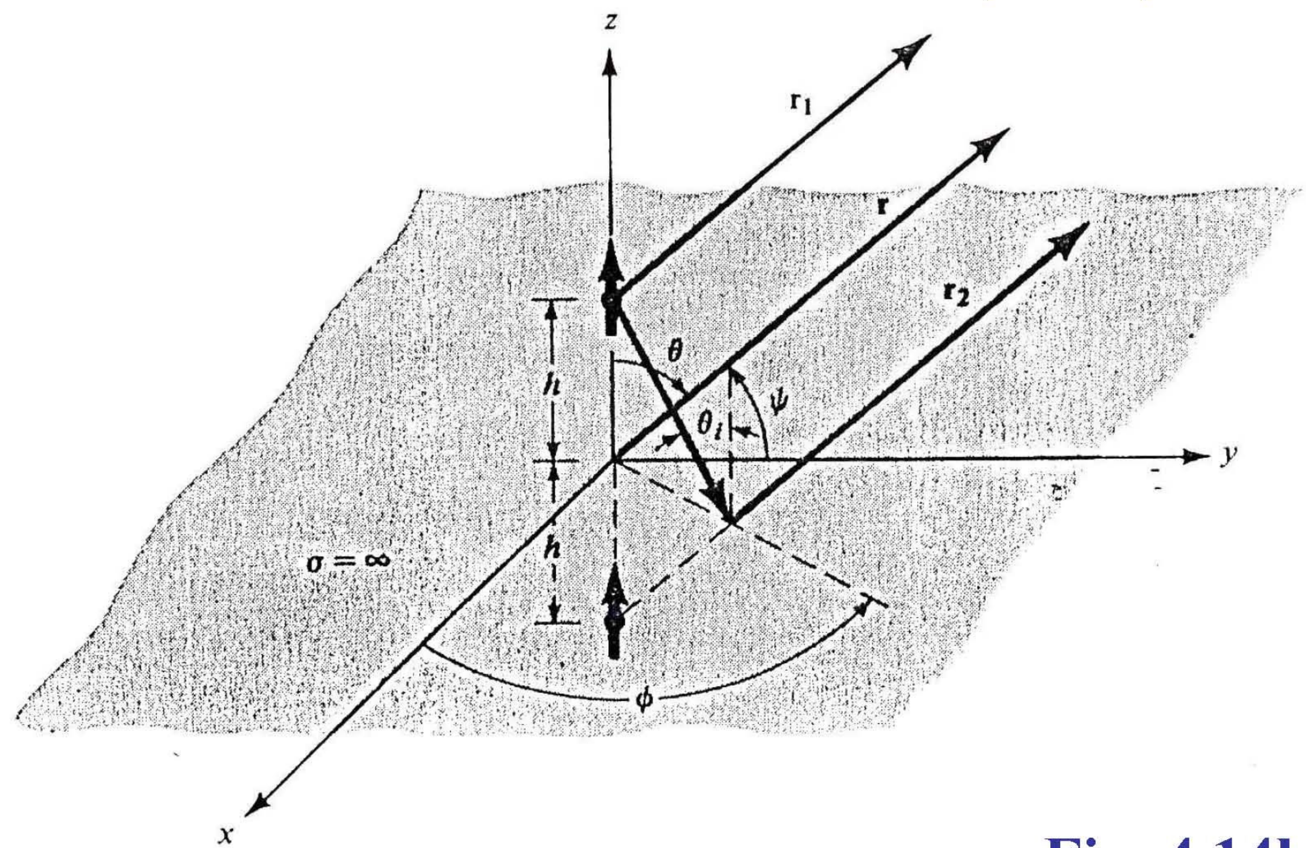
$$E_\theta^d = j\eta \frac{kI_o \ell e^{-jkr_1}}{4\pi r_1} \sin\theta_1 \qquad (4\text{-}94)$$

$$E_\theta^r = R_v \left\{ j\eta \frac{kI_o \ell e^{-jkr_2}}{4\pi r_2} \sin\theta_2 \right\} \qquad (4\text{-}95)$$

$$= +1 \left\{ j\eta \frac{kI_o e^{-jkr_2}}{4\pi r_2} \sin\theta_2 \right\} \qquad (4\text{-}95a)$$

$$E_\theta^t = E_\theta^d + E_\theta^r$$

Vertical Electric Dipole Above Infinite Perfect Electric Conductor (PEC)



(b) Far–field observations

Fig. 4.14b

$$r_1 = \left[r^2 + h^2 - 2rh\cos\theta \right]^{1/2} \qquad \text{(4-96a)}$$

$$\overbrace{\simeq}^{r>>h} \left[r^2 - 2rh\cos\theta \right]^{1/2}$$

$$\overbrace{\simeq}^{r>>h} r\left[1 - \frac{1}{r} 2h\cos\theta \right]^{1/2} \overbrace{\simeq}^{r>>h} r\left[1 - \frac{h}{r}\cos\theta \right]$$

$$r_1 \overbrace{\simeq}^{r>>h} r - h\cos \qquad \text{(4-97a)}$$

$$r_2 = \left[r^2 + h^2 - 2rh\cos(\pi - \theta) \right]^{1/2}$$

$$= \left[r^2 + h^2 + 2rh\cos\theta \right]^{1/2} \qquad \text{(4-96b)}$$

$$\overbrace{\simeq}^{r>>h} \left[r^2 + 2rh\cos\theta \right]^{1/2} = r\left[1 + \frac{1}{r} 2h\cos\theta \right]^{1/2}$$

$$r_2 \overbrace{\simeq}^{r>>h} r\left[1 + \frac{h}{r}\cos\theta \right] = r + h\cos\theta \qquad \text{(4-97b)}$$

Far-Field Approximations

$$\left.\begin{aligned} r_1 &= r - h\cos\theta \\ r_2 &= r + h\cos\theta \end{aligned}\right\} \text{ for phase terms} \qquad \text{(4-97a,b)}$$

$$r_1 \cong r_2 \cong r \;\} \text{ for amplitude terms} \qquad \text{(4-98)}$$

$$\theta_1 \cong \theta_2 \cong \theta$$

$$E_\theta = \underbrace{j\eta \frac{kI_o \ell e^{-jkr}}{4\pi r} \sin\theta}_{Element \quad Factor} \underbrace{\{2\cos(kh\cos\theta)\}}_{Array \quad Factor} \qquad z \geq 0$$

$$E_\theta = 0 \qquad z < 0$$

(4-99)

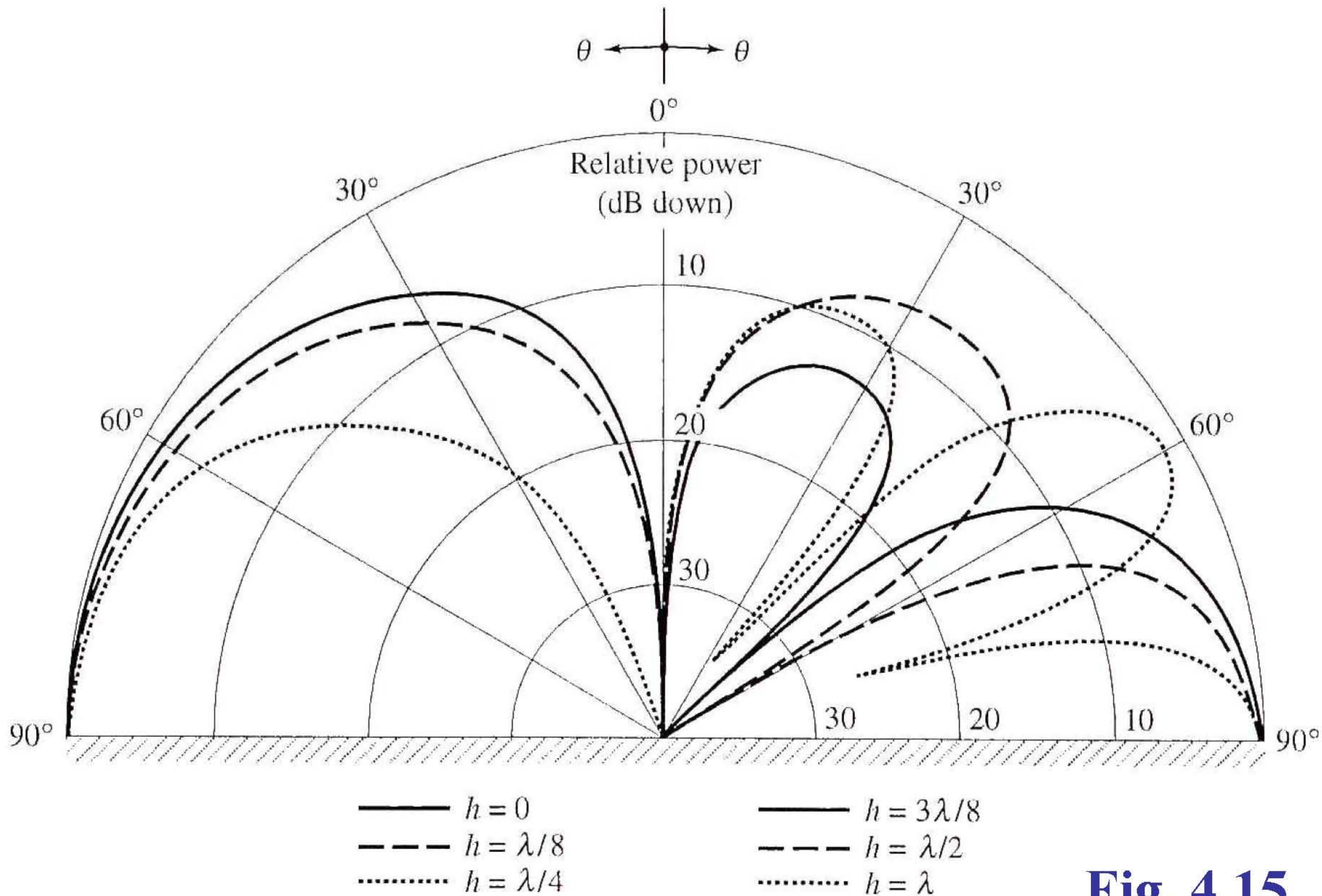
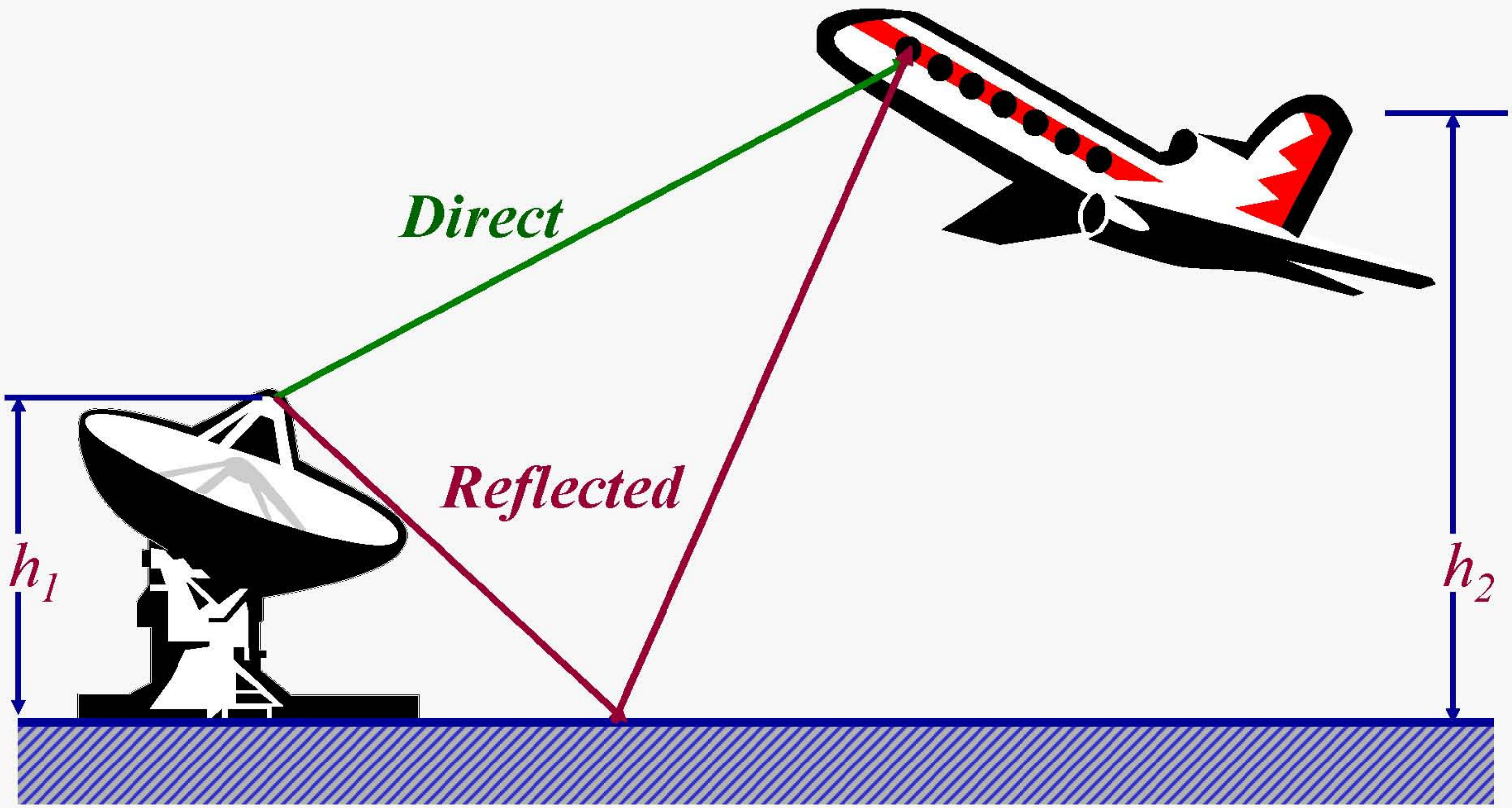
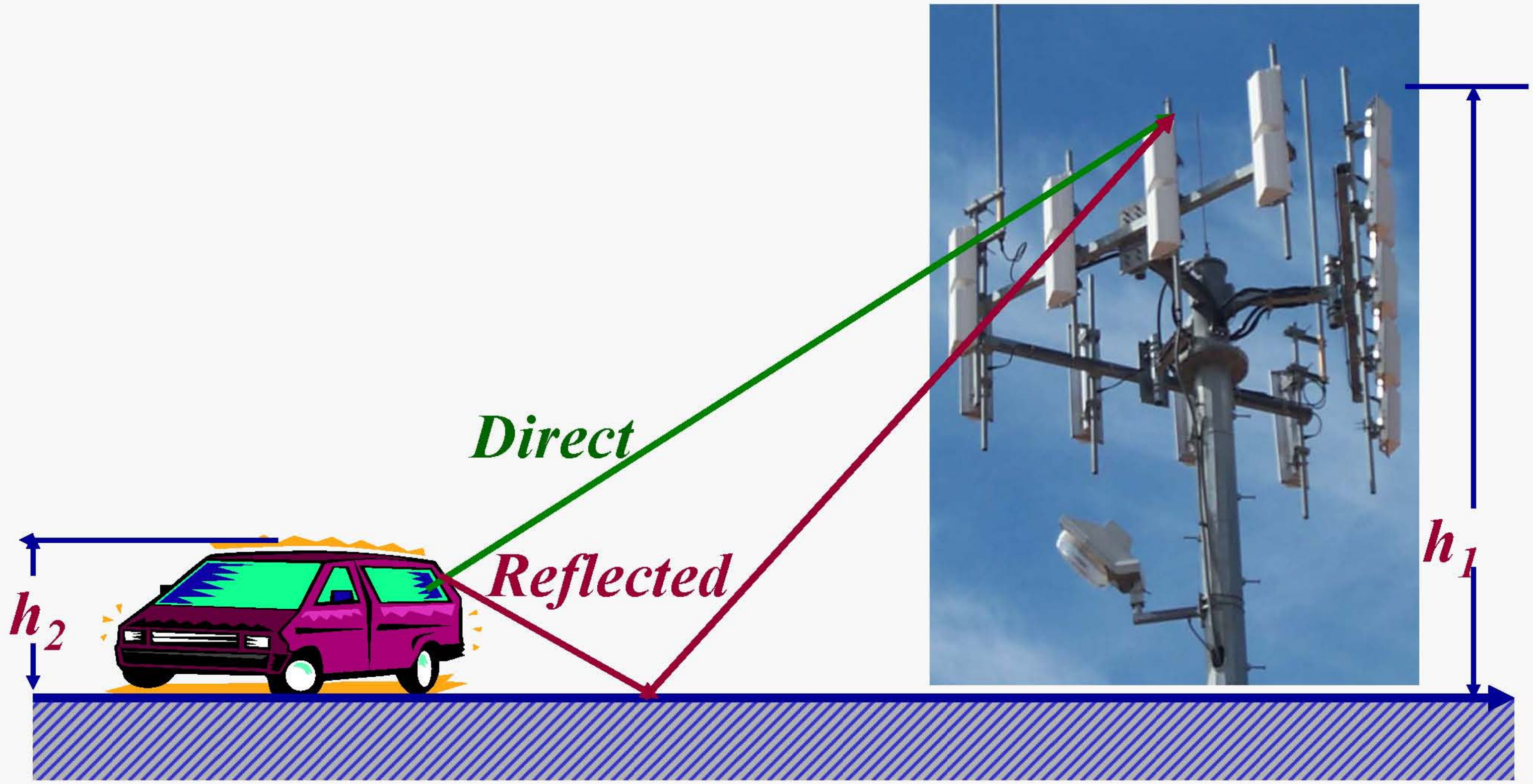

θ
θ
0°
Relative power
(dB down)
30°
30°
10
20
30
60°
60°
90°
90°
30
20
10
h = 0
h = λ/8
h = λ/4
h = 3λ/8
h = λ/2
h = λ

Fig. 4.15

Multipath: Direct and Reflected Rays



Multipath: Direct and Reflected Rays



Vertical Dipole

$$\text{Number of Lobes} \cong 2\left(\frac{h}{\lambda}\right)+1 \qquad (4\text{-}100)$$

$$h >> \lambda$$

$$P_{rad} = \oiint_S \underline{W}_{av} \cdot d\underline{s}$$

$$= \int_0^{2\pi} \int_0^{\pi/2} \underline{W}_{rad} \cdot \hat{a}_r r^2 \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi/2} \frac{2}{2\eta} |E|^2 r^2 \sin\theta \, d\theta \, d\phi$$

(4-101)

Effect of Imperfectly Conducting, Flat Earth

- To improve the radiation efficiency at these frequencies, radial wires or metallic disks are sometimes placed on the ground to simulate a perfectly conducting ground plane.

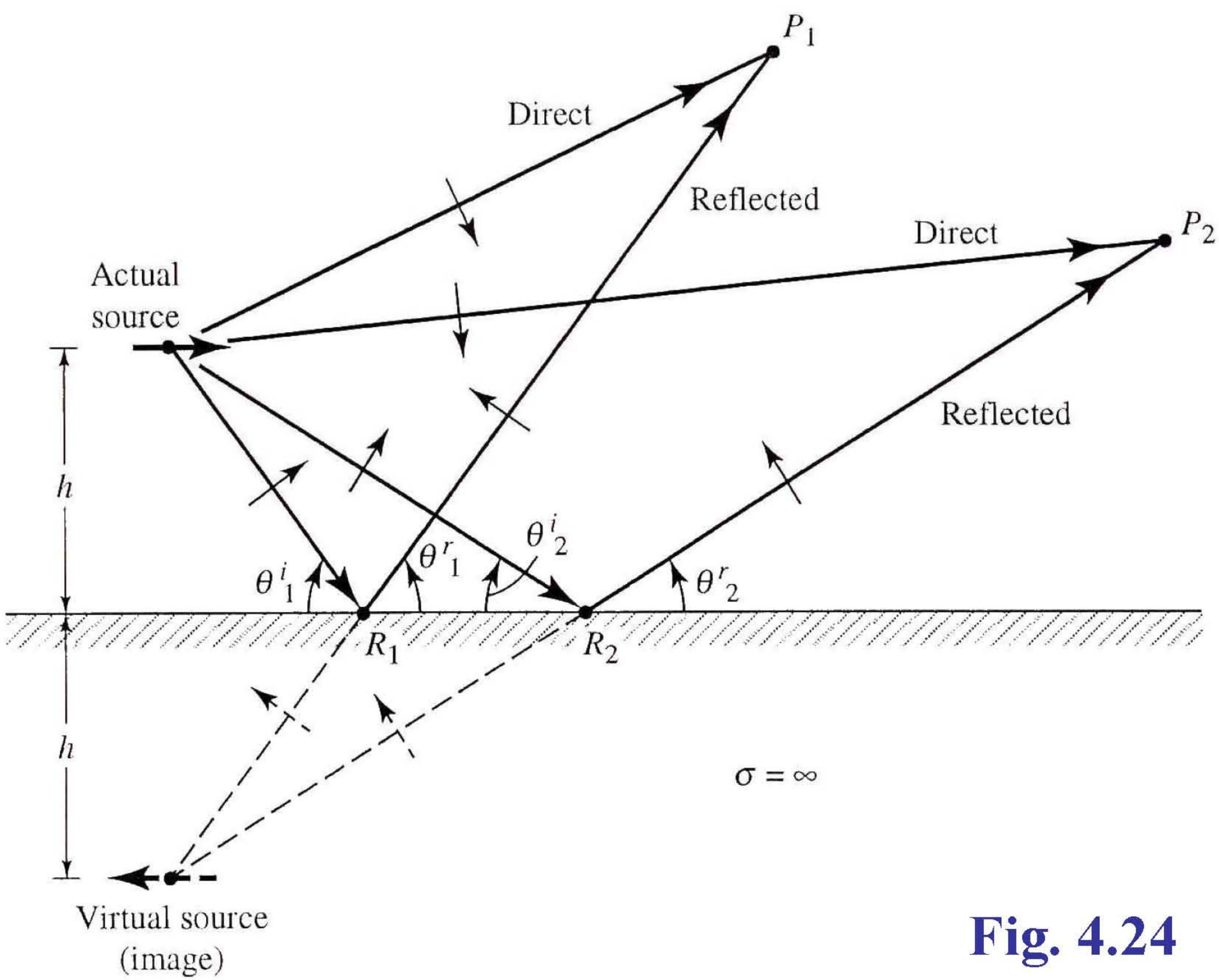
Examples of Antennas on Cellular and Cordless Telephones, Walkie-Talkies, and CB Radios

Fig. 4.22

Triangular Array Of Linear Dipoles For Wireless Mobile Communication Base Stations

Fig. 4.23

Horizontal Polarization



P_1
Direct
Reflected
Direct
P_2
Actual source
Reflected
h
θ^i_2
θ^r_1
θ^i_1
θ^r_2
R_1
R_2
h
$\sigma = \infty$
Virtual source (image)

Fig. 4.24

Horizontal Electric Dipole Above Ground Plane

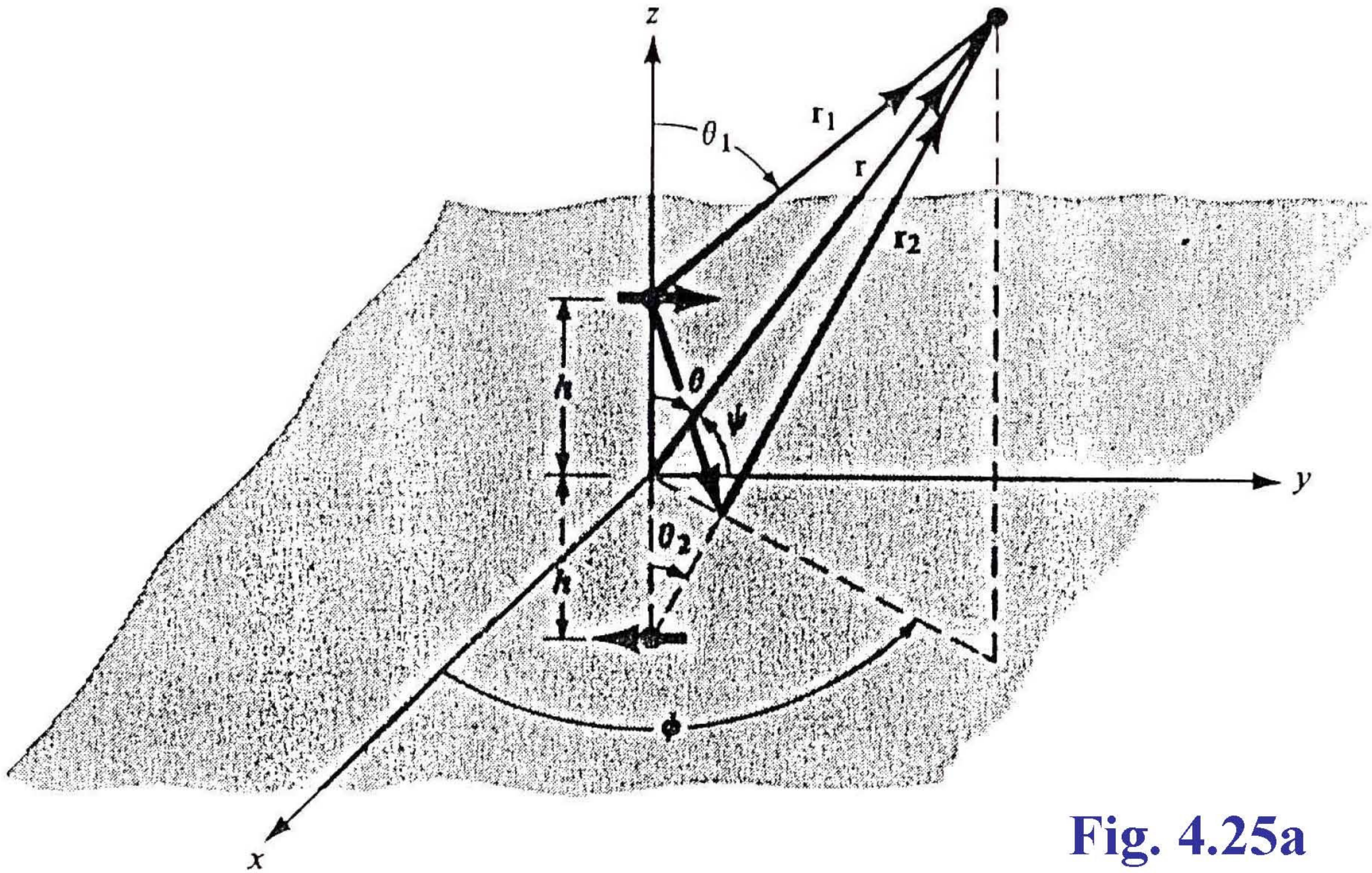


Fig. 4.25a

Horizontal Electric Dipole Above an Infinite Perfect Electric Conductor

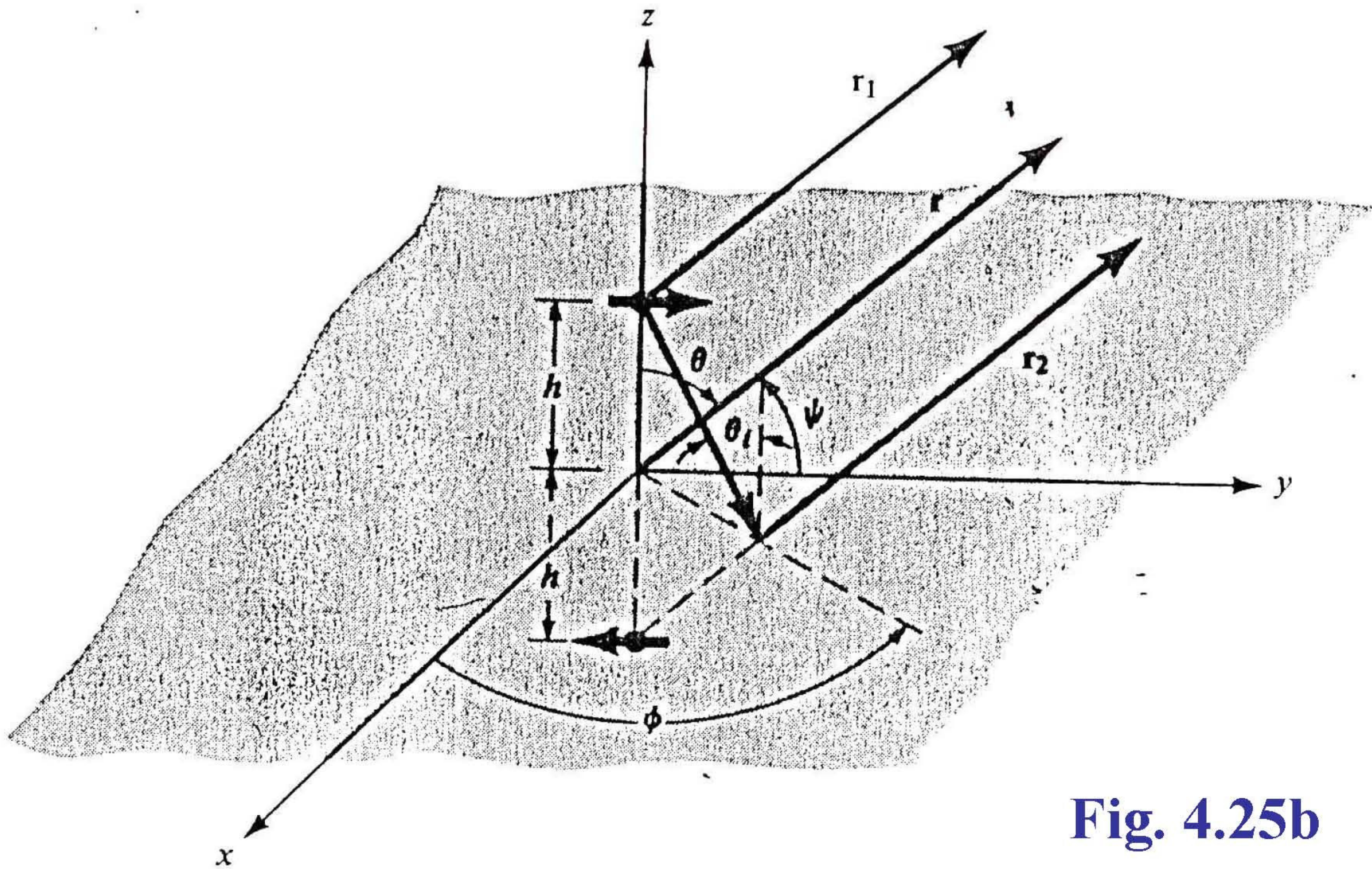


Fig. 4.25b

Lossy Surface

Modeling of the Effect of Earth on Antenna Systems

- An obstacle that is almost always present in an antenna system is the earth.
- The earth is NOT
  - perfectly conducting
  - flat

Effect of Imperfectly Conducting, Flat Earth

- The assumption of a flat earth is a good engineering approximation for observation angles greater than 3 degrees above the horizon.
- Antenna characteristics (especially radiation efficiency) at LF and MF (below 3 MHz) are profoundly and adversely affected by the lossy earth.

Effect of Imperfectly Conducting, Flat Earth

- To improve the radiation efficiency at these frequencies, radial wires or metallic disks are sometimes placed on the ground to simulate a perfectly conducting ground plane.

Constitutive Parameters of Ground

$$\varepsilon_0, \mu_0, \sigma = 0$$

$$\varepsilon_1, \mu_1, \sigma_1$$

Constitutive Parameters of Ground

$$\varepsilon_1, \mu_1, \sigma_1$$

or

$$\dot{\varepsilon}_1, \mu_1$$

Could also include magnetic conductivity, if necessary.

where

$$\dot{\varepsilon}_1 = \varepsilon_1^{'} - j\varepsilon_1^{''} = \varepsilon_1 - j\frac{\sigma_1}{\omega}$$

Effect of Imperfectly Conducting, Flat Earth

- For horizontal polarization:
  1. The phase of the reflection coefficient (R_h) is exactly 180° for PEC ground planes and near 180° for non-PEC ground planes.
- For vertical polarization:
  1. $R_v = +1$ for PEC ground plane, but is very different for non-PEC ground plane.
  2. For $\theta_i = 90°$, $R_v = -1$ for non-PEC ground plane. This causes a null in the pattern which does not occur for PEC ground plane.