Sample Midterm II with Solutions

Name ______________________________
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UNIVERSITY OF UTAH
ELECTRICAL & COMPUTER ENGINEERING DEPARTMENT

ANTENNA THEORY AND DESIGN

ECE 5324/6324

MIDTERM II

March 13, 2009

1. (25 points)

A half-wave dipole (antenna #1) is located at a distance of 0.25λ from a short circuited dipole (antenna #2) as sketched in Fig. 1:

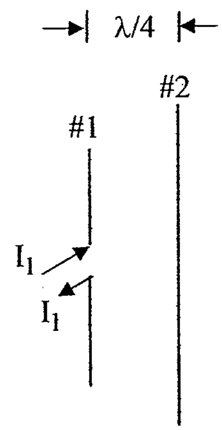


Fig. 1

It is given that $Z_{11} = 73 + j42$, $Z_{22} = 80 + j70$, and $Z_{12} = 38 - j30$ ohms.

pts

10 a. Calculate the current induced in antenna #2 in terms of current fed into antenna #1 i.e. I_2 / I_1 (magnitude and phase).

10 b. Calculate the feed point impedance Z_1 of antenna #1 which is now different from Z_{11}.

5 c. What would be the feed point impedance Z_1' of the driven antenna #1 if it were folded on itself to form a two-arm folded dipole of equal diameters for the two arms, respectively.

Midterm II – Solutions

March 13, 2009

1. a. $V_1 = I_1 Z_{11} + I_2 Z_{12}$ (1) $Z_{11} = 73 + j42$

$V_2 = 0 = I_1 Z_{12} + I_2 Z_{22}$ (2) $Z_{22} = 80 + j70$

$Z_{12} = 38 - j30$

From Eq. (2)

$$\frac{I_2}{I_1} = -\frac{Z_{12}}{Z_{22}} = \frac{-(38 - j30)}{80 + j70} = -0.455 \angle -38.29^\circ - 41.18^\circ$$

$$= 0.455 \angle -79.47^\circ + 180^\circ$$

$$= 0.455 \angle +100.53^\circ = -0.0832 + j0.447$$

b. From (1)

$$Z_1 = \frac{V_1}{I_1} = Z_{11} + 0.455\, e^{j100.53} (38 - j30)$$

$$e^{j100.53^\circ} \quad e^{-j38.29^\circ}$$

$$= 73 + j42 + 0.455 \times 48.41\, e^{+j62.24^\circ}$$

$$= 73 + j42 + 22.03\, e^{+j62.24^\circ}$$

$$= 73 + j42 + 10.26 + j19.49$$

$$= \boxed{83.26 + j61.49} = \boxed{103.50\, e^{+j36.43^\circ}}$$

c. $Z_1' = 4 Z_1 = 414\, e^{j36.43^\circ}$ or $4(83.26 + j61.49)$

$= 333.04 + j245.96\ \Omega$

2. (25 points)

pts

15 a. Design a **three** reactance network to match an antenna of impedance $Z_A = 60 + j30$ ohms to a two-wire transmission line of characteristic impedance $Z_o = 350$ ohms using a circuit arrangement shown in Fig. 2.

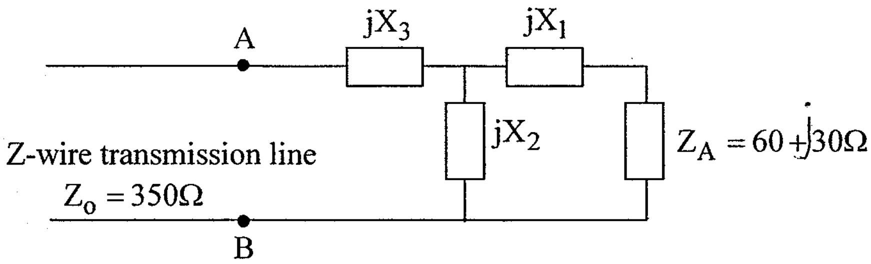


Fig. 2.

Select X_1 and X_2 such that

$$X_1 + X_2 = -X_A = -30\Omega$$

If possible, take X_1 to be a negative (capacitive) reactance.

10 b. For an antenna frequency of 400 MHz, calculate the values of three reactive elements, inductances and/or capacitances needed for your designed circuit.

2. Following a procedure similar to that of the Course writeup "A reactive Three-element Circuit for Antenna Matching", we can write

$$Z_{AB} = \frac{(60+j30+jX_1)jX_2}{60+j(30+X_1+X_2)} + jX_3 = 350+j0 \ \Omega \quad (1)$$

It is given that $X_1 + X_2 = -30$. Thus Eq. 1 can be written as

$$Z_{AB} = \frac{(60-jX_2)\,jX_2}{60} + jX_3 = 350+j0 \quad (2)$$

Equating real and imaginary parts on the two sides of Eq. (2), we can write

~~Equating imaginary parts~~ $\frac{\cancel{60}\,X_2}{\cancel{60}} + X_3 = 0$ $\boxed{X_3 = -X_2}$

Equating real parts

$$\frac{X_2^2}{60} = 350 \rightarrow X_2 = \pm 144.9 \quad (1)$$

For X_2, we can a capacitive reactance i.e. $X_2 = -144.9 = -\frac{1}{\omega C_2}$

$$C_2 = \frac{1}{\omega \times 144.9} = \frac{1}{2\pi \times 4 \times 10^8 \times 144.9} = \boxed{2.746\ pF}$$

Note that $X_1 + X_2 = -30$

$$X_1 = -30 - X_2 = -30 + 144.9 = 114.9 = \omega L_1$$

$$L_1 = \frac{114.9}{2\pi \times 4 \times 10^8} = \boxed{45.7\ nH}$$

$$X_3 = -X_2 = +144.9 = \omega L_3$$

$$\boxed{L_3 = 57.6\ nH}$$

If possible, take X_1 to be a negative (capacitive) reactance

Alternative Solution

From (1) on p. 2, we should take

$$\boxed{X_2 = +144.9}$$

Then $jX_1 = j(-30 - X_2) = -j174.9\ \Omega = \dfrac{-j}{\omega C_1}$

$$C_1 = \frac{\cancel{10^{12}}\ 10^4}{2\pi \times 4 \times \cancel{10^8} \times 174.9} = 2.27\ pF = 2270\ fF$$

$jX_2 = j144.9 = j\omega L_2$

$$L_2 = \frac{144.9 \times \cancel{10^9}\ 10}{2\pi \times 4 \times \cancel{10^8}} = 57.65\ nH$$

$$jX_3 = -jX_2 = -j144.9 = \frac{-j}{\omega C_3}$$

$$C_3 = \frac{1}{2\pi \times 4 \times 10^8 \times 144.9} = 2.746\ pF$$

3. (25 points)

pts

5 a. Write expressions for the radiated electric field and power density of a ten-element collinear antenna array of $\lambda/2$ elements mounted one above the other so that the **axis of the array is along the z-axis**. Assume that each of the antennas are fed with equal magnitude and in-phase currents and that the center-to-center spacing between the antennas is 0.8λ.

5 b. Calculate the direction/s of maximum radiation for this antenna array.

5 c. Calculate the main beam width or beam width between first nulls for the principal lobe of radiation of the antenna array.

5 d. Neglecting mutual impedance effects, calculate the gain of the antenna array relative to an isotropic radiator.

5 e. For a total radiated power of 10 KW, calculate the electric field strength created at a distance of 12 miles (19.2 km).

3. a. This is a z-oriented antenna array of ten half wave dipoles (3

From p. 24 of the class Notes

$N_z = 10$, $\alpha_z = 0$; $d_z = 0.8\lambda$, $\beta d_z = 1.6\pi$

$$|AF| = \frac{\sin(N_z \Psi_z/2)}{\sin(\Psi_z/2)} = \frac{\sin\{5(1.6\pi\cos\theta)\}}{\sin\{\frac{1}{2}(1.6\pi\cos\theta)\}} \quad (1)$$

From

$$E_\theta| = j\frac{60 I_m}{r} F(\theta) e^{-j\beta r} \hat{\theta}$$

for $\lambda/2$ dipole

$$\frac{\beta L}{2} = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \pi/2$$

$$\text{where } F(\theta) = \frac{\cos(\frac{\beta L}{2}\cos\theta) - \cos(\frac{\beta L}{2})}{\sin\theta}$$

$$= \frac{\cos(\pi/2 \cos\theta)}{\sin\theta}$$

power density

$$S = \frac{|E_\theta|^2}{2\eta}|AF|^2$$

b. For direction of maximum radiation $\theta = 90^\circ$

Thus the maximum power density is in the xy plane (plane perpendicular to the orientation of the array.

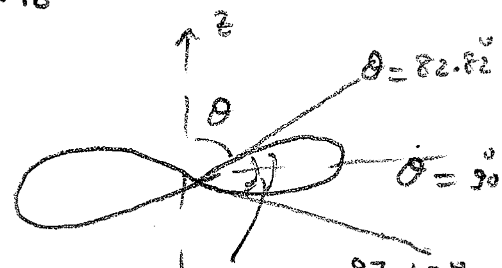
c. From Eq. (1) for directions of zero radiation

$$\frac{N_z \Psi_z}{2} = 8\pi\cos\theta_{FN} = \pm\pi$$

$$\theta_{FN} = \cos^{-1}\left(\frac{1}{8}\right) = 82.82^\circ$$

$$\rightarrow \cos^{-1}\left(-\frac{1}{8}\right) = 97.18^\circ$$

$$BWFN = 97.18^\circ - 82.82^\circ = 14.36^\circ$$



d. From Eq. (57) on p. 33 of Notes

$$G = D = D_0 N = 1.64 \times 10 = 16.4 \text{ (12.15 dBi)}$$

e. $$S_{max} = D S_0 = 16.4 \times \frac{P_{rad}}{4\pi r^2} = 16.4 \times \frac{10^{-2}}{4\pi(19.2\times 10^3)^2}$$

$$= 3.54 \times 10^{-5} \text{ W/m}^2$$

3. (Continued)

$$\frac{E_{max}\Big|^2_{rms}}{\eta} = S_{max} = 3.54 \times 10^{-5}$$

$$E_{max}\Big|_{rms} = \sqrt{120\pi \times 3.54 \times 10^{-5}} = 115.52 \ \frac{mV}{m}\Big|_{rms}$$

$$E_{max}\Big|_{peak} = 115.52\sqrt{2} \ \frac{mV}{m} = 162.2 \ \frac{mV}{m}\Big|_{peak}$$

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Score:

Problem 1 _________ of a possible 25 points

Problem 2 _________ of a possible 25 points

Problem 3 _________ of a possible 25 points

Total _________ of a possible 75 points