Homework 4.

3.4-9) frequency $f = 1200\,KHz$

number of turns $N = 1000$

$\mu_r = 50$

length $= 30mm = 0.03m$

width $= 8mm = 0.008m = d$

Radiation resistance
Radiation efficiency
Reactance
} → (7)

$\lambda = \frac{c}{f} = 250m$

$b = \frac{d}{2} = \frac{0.008}{2} = 0.004m$

$S = \pi b^2$

$= \pi \times (0.004)^2$

$S = 5.02 \times 10^{-5} m^2$

(2)

$L_T = \pi N d$

$= \pi (1000)(0.008)$

$\boxed{L_T = 25.13\,m}$

$R = \frac{L}{25}$

$= \frac{0.03}{(0.004)}$

$\boxed{R = 3.75}$

$D = 0.37 R^{-1.44}$

$= 0.37 (3.75)^{-1.44}$

$\boxed{D = 0.055}$

$\mu_{eff} = \frac{\mu_m}{1 + D(\mu_m - 1)}$

$= \frac{50}{1 + 0.055(50-1)}$

$\boxed{\mu_{eff} = 13.5}$

Radiation resistance $R_R = 31200 \left(\frac{N \mu_{eff} S}{\lambda^2} \right)^2$

$= 31200 \left(\frac{1000 \times 13.5 \times 5.02 \times 10^{-5}}{250^2} \right)^2$

$\boxed{R_R = 3.66\,\mu\Omega}$

Radiation efficiency $e_R = \frac{R_R}{R_A}$

$\boxed{R_A = R_R + R_W}$

$R_W = \frac{b}{a} R_S$

$R_S = \sqrt{\frac{2\pi f \mu_0}{2\sigma}}$

$= \sqrt{\frac{2\pi (1200 \times 10^3)(1.25 \times 10^{-6})}{2 (5.8 \times 10^7)}}$

$= \sqrt{\frac{9.42}{11.6 \times 10^7}} = 2.84 \times 10^{-4}\,\Omega$

$\boxed{R_S = 284 \times 10^{-4}\,\Omega}$

$2a = 0.255\text{mm}$

$a = 0.1275\text{mm}$

$R_w = \frac{b}{a} R_s (N)$

$= \frac{0.004}{0.1275 \times 10^{-3}} \cdot 2.814 \times 10^{-4} (1000)$

$R_w = 8.97\Omega$

Radiation efficiency $e_{ff} = \frac{R_{ff}}{R_{ff} + R_w} = \frac{R_{ff}}{R_A}$

$R_A = 8.97 + 3.66 \times 10^{-6}$

$R_A = 8.97$

$e_{ff} = \frac{R_{ff}}{R_A}$

$= \frac{3.66 \times 10^{-6}}{8.97}$

$e_{ff} = 4.08 \times 10^{-7}$

$L = N \mu_{eff} \mu_0 b \left(\ln\left(\frac{8b}{a}\right) - 2 \right)$

$= 1000 \times 13.5 \times 1.25 \times 10^{-6} \times 0.004 \left(\ln\left(\frac{8(0.004}{0.1275 \times 10^{-3}}\right) - 2 \right)$

$L = 2.37 \times 10^{-6} = 237\,\mu H$

$X = \omega L$

$= 2\pi f L$

$= 2\pi (1200 \times 10^3)(237 \times 10^{-6})$

$= 1794$

$= j\,1.79 K\Omega$

Total impedance will be

$Z = R_w + R_{rad} + j\omega L$

$= 8.77 + 3.66 + j\,1.79 K\Omega$

$\boxed{Z = 12.63 + j\,1794\,\Omega}$

$Q = 2\pi f \left(\frac{L}{R_{rad} + R_w}\right)$

$= 2\pi (1200 \times 10^3) \left(\frac{237\mu}{3.66 \times 10^{-6} + 8.97}\right)$

$\boxed{Q = 199.2}$

3.4-11) Frequency $f = 30\,MHz$

diameter $d = 2cm = 0.02m$

$L_1 = 0.5m, \quad L_2 = 0.5m$

a) Radiation resistance

b) Input impedance

c) Radiation efficiency

Radiation Resistance:

$$R_R = 31200 \left(\frac{NS}{\lambda^2}\right)^2$$

Given frequency $f = 30MHz$

So $\lambda = \frac{c}{f} = 9.9m \approx 10m$

$S = L_1 L_2$

$= 0.5 \times 0.5\,m^2$

$\boxed{S = 0.25\,m^2}$

Single turn: So $N = 1$

$$R_{s1} = 31200 \left(\frac{0.25}{100}\right)^2$$

$$\boxed{R_{s1} = 0.195\,\Omega}$$

Radiation Efficiency:-

$$\boxed{e_{s1} = \frac{R_{s1}}{R_A}}$$

where $R_A = R_{s1} + R_w$

$$R_s = \sqrt{\frac{2\pi f \mu_0}{2\sigma}}$$

$$= \sqrt{\frac{2\pi \times 30\times10^6 \times 1.25\times10^{-6}}{2\times5.8\times10^7}}$$

$$\boxed{R_s = 1.425\ m\Omega}$$

$$\boxed{R_w = \frac{2(l_1+l_2)R_s}{a\pi\alpha}}$$

Radius $a = \frac{0.02}{2}$

$= 0.01m$

$$= \frac{2(0.5+0.5)(1.425\times10^{-3})}{2\pi(0.01)}$$

$$= 0.045$$

$$\boxed{R_w = 0.045\,\Omega}$$

So $e_{s1} = \frac{R_{s1}}{R_A}$

$$e_{gr} = \frac{0.195}{0.195 + 0.045}$$

$$= 0.81$$

Efficiency $e_{gr} = 81\%$

Radiation efficiency $e_{gr} = 81\%$

Input Impedance $X = \omega L$

$$L = \frac{\mu}{\pi}\left[l_2 \cosh^{-1}\frac{l_1}{2a} + l_1 \cosh^{-1}\frac{l_2}{2a} \right]$$

$$L = \frac{1.257 \times 10^{-6}}{\pi}\left[0.5 \cosh^{-1}\frac{0.5}{2(0.01)} + 0.5 \cosh^{-1}\frac{0.5}{2(0.01)} \right]$$

$$L = 1.565\ \mu H$$

$$X = j\omega L$$

$$= j 2\pi \times 30 \times 10^6 \times 1.565 \times 10^{-6}$$

$$= j 294.9\ \Omega$$

So input impedance will be

$$Z = R_w + R_g + j\omega L$$

$$= 0.045 + 0.195 + j 294.9$$

$$Z = 0.24 + j\,294.9\ \Omega$$