Antenna Theory & Design

Homework (1)

(1) Calculate and compare the power density and electric field strength for $\theta = 90°$ for a distance of 20km from dipole antenna radiating 100W of power at 130MHz. Consider antennas of following length

a) $L = 0.03\lambda$ d) $L = 0.9\lambda$

b) $L = 0.15\lambda$ e) $L = 1.3\lambda$

c) $L = 0.5\lambda$

How does the electric field strength vary as antenna length is increased from 0.03λ to 1.3λ is by a factor of nearly 441?

Sol. From the table in page (7) of class notes

$\frac{L}{2}\lambda$	R_a
0.015	0.197Ω
0.075	4.6002Ω
0.25	73.128Ω
0.45	2227.2Ω
0.65	142.06Ω

$\boxed{\text{For } \frac{L}{2}\lambda = 0.015}$

$$I_a = \sqrt{\frac{2P_{rad}}{R_a}}$$

$$= \sqrt{\frac{2(100)}{0.197}}$$

$$\boxed{I_a = 31.862\,\Omega}$$

$$I_m = \frac{I_a}{\sin\left(\frac{\beta L}{2}\right)} = \frac{I_a}{\sin\left(\frac{\pi L}{\lambda}\right)}$$

Since frequency is 1.30MHz,

the value of $\lambda = 230.6$ mt

$$L = 0.03\lambda$$

$$= 0.03\,(230.6)$$

$$\boxed{L = 6.918\text{ mt}}$$

$$I_m = \frac{I_a}{\sin\left(\frac{\pi L}{\lambda}\right)}$$

$$= \frac{31.862}{\sin\left(\frac{180 \times 6.918}{230.6}\right)}$$

$$\boxed{I_m = 338.41\,A}$$

$$F(\theta) = \frac{\cos\left(\frac{\beta l}{2}\cos\theta\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin\theta}$$

$\theta = 90°$, $\beta = \frac{2\pi}{\lambda}$

$$F(\theta) = \frac{\cos\left(\frac{\pi L}{\lambda}\cos 90°\right) - \cos\left(\frac{\pi L}{\lambda}\right)}{\sin 90°}$$

$$= 1 - \cos\left(\frac{\pi L}{\lambda}\right)$$

$$= 1 - \cos\left(\frac{180 \times 6.918}{230.8}\right)$$

$$\boxed{F(\theta) = 0.0044}$$

$$E = \frac{60 I_m F(\theta)}{r}$$

$$= \frac{60 (338.7)(0.0044)}{20}$$

$$\boxed{E = 4.47 \ mV/m}$$

$$D = \frac{120}{R_a} \left. \frac{F^2(\theta)}{\sin^2\left(\frac{\pi L}{\lambda}\right)} \right|_{\theta = 90°}$$

Substituting all value in given eq.

we get $\boxed{D = 1.326}$

$$S_{max} = \left(\frac{P_{rad}}{4\pi r^2}\right) D \ \mu W/mt^2$$

$$= \frac{100}{4\pi(20\times20)} \ 1.326$$

$$\boxed{S_{max} = 0.029 \ \mu W/mt^2}$$

By doing similar calculations for

$\frac{L}{2}\lambda = 0.015$, $\frac{L}{2}\lambda = 0.075$, $\frac{L}{2}\lambda = 0.25$,

$\frac{L}{2}\lambda = 0.45$, $\frac{L}{2}\lambda = 0.65$ we get the following table

$\frac{L}{2}\lambda$	$R_a(\Omega)$	$I_a(A)$	$I_m(A)$	$F(\theta)\vert_{\theta=90^\circ}$	$E\left(\frac{mV}{m}\right)$	$S\left(\frac{\mu W}{m^2}\right)$	D
0.015	0.197	31.78	337.9	0.0044	4.46	0.027	1.326
0.075	4.60	6.59	11.15	0.1039	4.74	0.029	1.5009
0.25	73.12	1.65	1.64	1.0	4.9	0.032	1.6409
0.45	222.71	0.29	0.29	1.95	5.7	0.042	2.1521
0.65	142.1	1.19	1.19	1.58	6.91	0.064	3.25

2) Calculate also the Directivity D and gain for each of above antenna for the direction $\theta = 90°$. Express the results for calculated gains in dB

Sol From the calculations in problem ①

$L = 0.03\lambda$ $D = 1.326$

$L = 0.15\lambda$ $D = 1.5009$

$L = 0.5\lambda$ $D = 1.6409$

$L = 0.9\lambda$ $D = 2.1521$

$L = 1.3\lambda$ $D = 3.2539$

To find Gain we have to calculate R_{ohmic}

$$R_{ohmic} = \frac{R_s L}{4\pi a} \frac{1}{\sin^2\left(\frac{\beta L}{2}\right)} \left[1 - \frac{\sin(\beta L)}{(\beta L)}\right]$$

$$= \frac{R_s L}{4\pi a} \frac{1}{\sin^2\left(\frac{\pi L}{\lambda}\right)} \left[1 - \frac{\sin\left(\frac{2\pi L}{\lambda}\right)}{\left(\frac{2\pi L}{\lambda}\right)}\right]$$

So, to calculate R_s

$$\boxed{R_s = 1.988 \sqrt{\frac{f_{MHz}}{\sigma}}}$$

$f_{MHz} = 1.3MHz$

$\sigma = 3.5 \times 10^{7}\ S/m$

$$R_S = 1988\sqrt{\frac{1.3MHz}{3.5 \times 10^{7}\ S/m}}$$

$$\boxed{R_S = 0.3831\Omega}$$

$2a = 2.588$

for $L = 0.03\lambda$

$$R_{ohmic} = \frac{R_S L}{4\pi a}\frac{1}{\sin^2\left(\frac{\pi L}{\lambda}\right)}\left[1 - \frac{\sin\left(\frac{2\pi L}{\lambda}\right)}{\left(\frac{2\pi L}{\lambda}\right)}\right]$$

$$= \frac{0.3831 \times 6.927}{4\pi(1.29)}\frac{1}{\sin^2\left(\frac{180 \times 0.03\lambda}{\lambda}\right)}\left[1 - \frac{\sin\left(\frac{2\pi 0.03\lambda}{\lambda}\right)}{\frac{2\pi 0.03}{\lambda}}\right]$$

$R_{ohm} = 0.0018\Omega$

$$e_{cd} = \frac{R_a}{R_a + R_{ohmic}}$$

$$= \frac{0.1977}{0.1977 + 0.0018} = 0.991$$

$G = G_0 D$

$= 0.991 \times 1.326$

$= 1.315$

By doing the above calculations for

$L = 0.15\lambda$, $L = 0.5\lambda$, $L = 0.9\lambda$, $L = 1.3\lambda$.

we get the values of G as

L	D	G
0.03λ	1.3268	1.31
0.15λ	1.5009	1.51
0.5λ	1.6409	1.64
0.9λ	2.1521	2.15
1.3λ	3.25	3.25