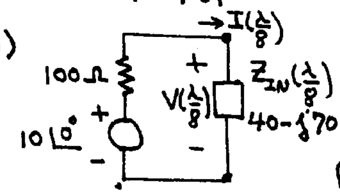
1.3) (a) $\Gamma_o = \dfrac{Z_L - Z_o}{Z_L + Z_o} = \dfrac{(100+j100)-50}{(100+j100)+50} = 0.62\angle 29.7^\circ$

$$Z_{IN}\left(\frac{\lambda}{8}\right) = 50\,\frac{(100+j100)\cos 45^\circ + j50\sin 45^\circ}{50\cos 45^\circ + j(100+j100)\sin 45^\circ} = 40 - j70\ \Omega$$

$$VSWR = \frac{1+|\Gamma_o|}{1-|\Gamma_o|} = \frac{1+0.62}{1-0.62} = 4.26$$

(b)



$$V\left(\frac{\lambda}{8}\right) = \frac{10\angle 0^\circ(40-j70)}{40-j70+100} = 5.15\angle -33.7^\circ$$

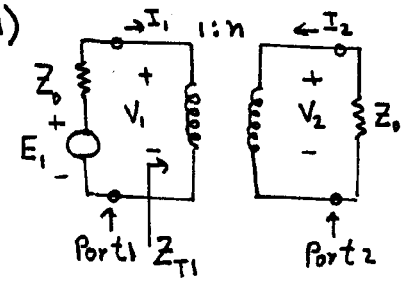
$$I\left(\frac{\lambda}{8}\right) = V\left(\frac{\lambda}{8}\right) \Big/ Z_{IN}\left(\frac{\lambda}{8}\right) = 0.064\angle 26.6^\circ$$

$$P\left(\frac{\lambda}{8}\right) = \frac{1}{2}\,\mathrm{Re}\left[V\left(\frac{\lambda}{8}\right) I^*\left(\frac{\lambda}{8}\right)\right] = 82 \text{ mW}$$

$$V\left(\frac{\lambda}{8}\right) = 5.15\angle -33.7 = A_1 e^{j\frac{\pi}{4}}\left[1 + 0.62\angle 29.7^\circ e^{-j\frac{\pi}{2}}\right]$$

$$\therefore\ A_1 = 3.643\angle -56.31^\circ$$

1.11)



IN THE Z_o SYSTEM SHOWN, THE PARAMETERS S_{11} AND S_{21} ARE CALCULATED AS FOLLOWS:

$$I_2 = \frac{I_1}{n}, \quad V_2 = V_1 n$$

$$Z_{T1} = \frac{V_1}{I_1} = \frac{V_2}{I_2 n^2} = \frac{Z_o}{n^2}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{Z_{T1}-Z_o}{Z_{T1}+Z_o} = \frac{Z_o/n^2 - Z_o}{\frac{Z_o}{n^2} + Z_o} = \frac{1-n^2}{1+n^2}$$

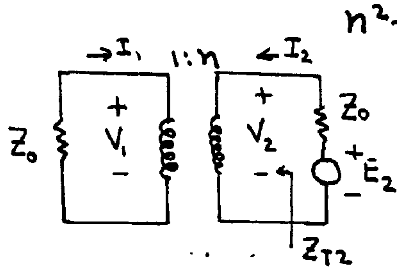
$$S_{21} = 2\sqrt{\frac{Z_{o2}}{Z_{o1}}}\,\frac{V_2}{E_1} = 2\frac{V_2}{E_1} \quad (1)$$

$$V_1 = \frac{E_1 Z_{T1}}{Z_{T1}+Z_o} = \frac{E_1 Z_o/n^2}{\frac{Z_o}{n^2}+Z_o} = \frac{E_1}{n^2+1}$$

$$V_2 = nV_1 = \frac{nE_1}{n^2+1} \quad \text{OR} \quad \frac{V_2}{E_1} = \frac{n}{n^2+1} \quad (2)$$

(2) INTO (1):

$$S_{21} = \frac{2n}{n^2+1}$$



THE PARAMETERS S_{22} AND S_{12} ARE CALCULATED AS FOLLOWS:

$$Z_{T2} = Z_o n^2$$

$$S_{22} = \frac{b_2}{a_2}\bigg|_{a_1=0} = \frac{Z_{T2}-Z_0}{Z_{T2}+Z_0} = \frac{n^2 Z_0 - Z_0}{n^2 Z_0 + Z_0} = \frac{n^2-1}{n^2+1}$$

$$S_{12} = 2\frac{V_1}{E_2} \quad (3)$$

$$V_2 = \frac{E_2 Z_{T2}}{Z_{T2}+Z_0} = \frac{E_2 Z_0 n^2}{Z_0 n^2 + Z_0} = \frac{E_2 n^2}{n^2+1}$$

$$V_1 = \frac{V_2}{n} = \frac{E_2 n}{n^2+1} \quad \text{OR} \quad \frac{V_1}{E_2} = \frac{n}{n^2+1} \quad (4)$$

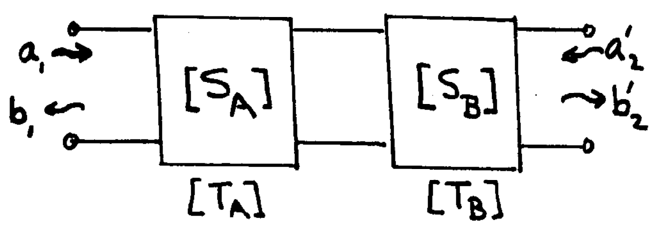
(4) INTO (3):

$$S_{12} = \frac{2n}{n^2+1}$$

AT PORTS 1' AND 2', WITH $\theta = \theta_1 = \theta_2$, WE OBTAIN:

$$[S'] = \begin{bmatrix} S_{11}e^{-j2\theta} & S_{12}e^{-j2\theta} \\ S_{21}e^{-j2\theta} & S_{22}e^{-j2\theta} \end{bmatrix} = e^{-j2\theta}\begin{bmatrix} \frac{1-n^2}{1+n^2} & \frac{2n}{n^2+1} \\ \frac{2n}{n^2+1} & \frac{n^2-1}{n^2+1} \end{bmatrix}$$

1.12)



[T] AND [S] PARAMETERS ARE RELATEB BY (1.4.11)

FROM (1.4.13):

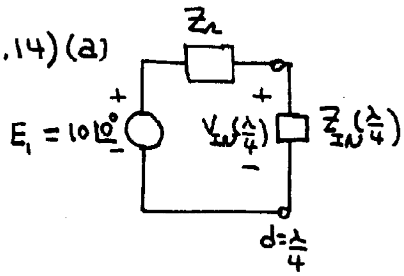
$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_{11}^A & T_{12}^A \\ T_{21}^A & T_{22}^A \end{bmatrix}\begin{bmatrix} T_{11}^B & T_{12}^B \\ T_{21}^B & T_{22}^B \end{bmatrix}\begin{bmatrix} b_2' \\ a_2' \end{bmatrix}$$

OVERALL T_{11} IS: $\quad T_{11} = T_{11}^A T_{11}^B + T_{12}^A T_{21}^B$

$$T_{11} = \frac{1}{S_{21}^A}\frac{1}{S_{21}^B} + \left(-\frac{S_{22}^A}{S_{21}^A}\right)\left(\frac{S_{11}^B}{S_{21}^B}\right)$$

SINCE $T_{11} = \frac{1}{S_{21}} = \frac{1 - S_{22}^A S_{11}^B}{S_{21}^A S_{21}^B} \Rightarrow S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{22}^A S_{11}^B}$

1.14) (a)



$$Z_{IN}(\tfrac{\lambda}{4}) = \frac{Z_o^2}{Z_L} = \frac{50^2}{50+j50} = 25-j25$$

NOTE: IF Z_ℓ IS GIVEN, THE VALUE OF Z_{IN} FOR MAXIMUM POWER IS $Z_{IN} = Z_\ell^*$. HOWEVER, IN THIS PROBLEM Z_{IN} IS GIVEN. HENCE, THE VALUE OF Z_ℓ FOR MAX. POWER DELIVERED TO Z_{IN} IS: $Z_\ell = -\mathrm{Im}[Z_{IN}]$

$$\therefore\ Z_\ell = j25$$

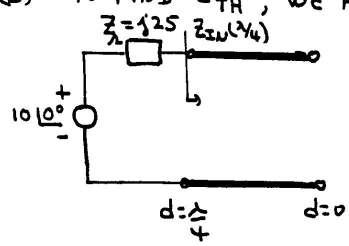
$$V_{IN}(\tfrac{\lambda}{4}) = \frac{10\angle 0° (25-j25)}{j25+(25-j25)} = 10-j10 = 14.14\angle -45°$$

$$I_{IN}(\tfrac{\lambda}{4}) = \frac{14.14\angle -45°}{25-j25} = 0.4$$

$$P_{IN} = \frac{1}{2}\mathrm{Re}[14.14\angle -45°\ (0.4)] = 2\ W$$

$$P_L = P_{IN} = 2\ W$$

(b) TO FIND E_{TH}, WE FIND THE OPEN CIRCUIT VOLTAGE AT $d=0$



$\Gamma_o = 1$, $V(d) = 2A\cos\beta d$

$$I(d) = j\frac{2A}{Z_o}\sin\beta d \quad (1)$$

$Z_{IN}(\frac{\lambda}{4}) = 0$ (A SHORT CIRCUIT)

$$\therefore I(\tfrac{\lambda}{4}) = \frac{10\angle 0°}{j25+0} = -j0.4 \quad (2)$$

FROM (1) AND (2): $-j0.4 = j\frac{2A}{50}\sin\frac{\pi}{2} \Rightarrow A = -10$

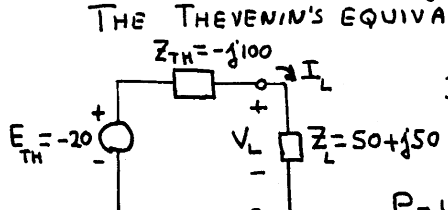
HENCE: $V(d) = 2(-10)\cos\beta d = -20\cos\beta d$

$$E_{TH} = V(0) = -20$$

TO FIND Z_{TH} WE SET $E_1 = 0$, THEN:

$$Z_{TH} = \frac{Z_o^2}{Z_\ell} = \frac{50^2}{j25} = -j100$$

THE THEVENIN'S EQUIVALENT CIRCUIT AT $d=0$ IS:



$$I_L = \frac{-20}{-j100+50+j50} = 0.283\angle -135°$$

$$V_L = 0.283\angle -135° (50+j50) = 20\angle -90°$$

$$P_L = \frac{1}{2}\mathrm{Re}[20\angle -90° (0.283\angle 135°)] = 2W$$

1.16) (a) $Z_{IN}(0) = Z_{IN}(d)\Big|_{d=\frac{\lambda}{8}} = 50\,\dfrac{(150+j150)+j50\tan 45^\circ}{50+j(150+j150)\tan 45^\circ} = 23 - j65\ \Omega$

(b) $a_1(0) = \dfrac{1}{2\sqrt{Z_o}}\left[V_1(0)+Z_o I_1(0)\right]$, $V_1(0) = E_1 - Z_o I_1(0)$

$$\therefore\ a_1(0) = \frac{E_1}{2\sqrt{Z_o}} = \frac{10}{2\sqrt{50}} = 0.707$$

$$a_1(\tfrac{\lambda}{8}) = a_1(0)\,e^{-j\pi/4} = 0.707\,\underline{\lfloor -45^\circ}$$

$$b_1(0) = \frac{1}{2\sqrt{50}}\left[V_1(0) - 50 I_1(0)\right] = \frac{1}{2\sqrt{50}}\left[10 - 50 I_1(0) - 50 I_1(0)\right] \quad (1)$$

$$I_1(0) = \frac{10\,\underline{\lfloor 0^\circ}}{50+23-j65} = 0.102\,\underline{\lfloor 41.68^\circ} \quad (2)$$

(2) INTO (1): $b_1(0) = \dfrac{1}{2\sqrt{50}}\left[10 - 2(50)(0.102\,\underline{\lfloor 41.68^\circ})\right] = 0.508\,\underline{\lfloor -70.65^\circ}$

$$b_1(\tfrac{\lambda}{8}) = b_1(0)\,e^{j\pi/4} = 0.508\,\underline{\lfloor -25.65^\circ}$$

SINCE $Z_2 = Z_o$, THE OUTPUT IS MATCHED. HENCE, $a_2(0) = 0$

(c) $V_1(0) = I_1(0)\,Z_{IN}(0) = 0.102\,\underline{\lfloor 41.68^\circ}\,(23-j65) = 7.05\,\underline{\lfloor -28.8^\circ}$

OR $V_1(0) = \sqrt{Z_o}\left[a_1(0)+b_1(0)\right] = \sqrt{50}\left[0.707 + 0.508\,\underline{\lfloor -70.65^\circ}\right] = 7.05\,\underline{\lfloor -28.8^\circ}$

$V_1(\tfrac{\lambda}{8}) = \sqrt{Z_o}\left[a_1(\tfrac{\lambda}{8})+b_1(\tfrac{\lambda}{8})\right] = \sqrt{50}\left[0.707\,\underline{\lfloor -45^\circ} + 0.508\,\underline{\lfloor -25.65^\circ}\right] = 8.47\,\underline{\lfloor -36.92^\circ}$

$I_1(\tfrac{\lambda}{8}) = \dfrac{8.47\,\underline{\lfloor -36.92^\circ}}{150+j150} = 0.04\,\underline{\lfloor -81.92^\circ}$

(d), (e) $P_1(0) = \frac{1}{2}\mathrm{Re}\left[V_1(0) I_1^*(0)\right] = 0.12\,\mathrm{W}$, $P_1(\tfrac{\lambda}{8}) = \frac{1}{2}\mathrm{Re}\left[V_1(\tfrac{\lambda}{8}) I_1^*(\tfrac{\lambda}{8})\right] = 0.12\,\mathrm{W}$

ALSO: $P_1(0) = P_1(\tfrac{\lambda}{8}) = \frac{1}{2}|a_1(0)|^2 - \frac{1}{2}|b_1(0)|^2 = \frac{1}{2}|a_1(\tfrac{\lambda}{8})|^2 - \frac{1}{2}|b_1(\tfrac{\lambda}{8})|^2 = \frac{1}{2}(0.707)^2 - \frac{1}{2}(0.508)^2 = 0.12\,\mathrm{W}$

(f) $S_{11}(\tfrac{\lambda}{8}) = \dfrac{Z_{T1} - Z_o}{Z_{T1} + Z_o} = \dfrac{150+j150-50}{150+j150+50} = 0.721\,\underline{\lfloor 19.44^\circ}$

$$S_{11}(0) = S_{11}(\tfrac{\lambda}{8})\,e^{-j2(\pi/4)} = 0.721\,\underline{\lfloor 19.44^\circ}\,(\underline{\lfloor -90^\circ}) = 0.721\,\underline{\lfloor -70.56^\circ}$$

(g) $(VSWR)_{in} = \dfrac{1+|S_{11}|}{1-|S_{11}|} = 6.17$, $(VSWR)_{out} = 1$

(h) $\lambda = \dfrac{3\,10^8}{10^9} = 0.3\,m$ (OR 30 cm) . $\ell = \dfrac{\lambda}{8} = \dfrac{30}{8} = 3.75\ cm$

(i) $b_2(\tfrac{\lambda}{8}) = S_{21}\,a_1(\tfrac{\lambda}{8}) + S_{22}\cancel{a_2(\tfrac{\lambda}{8})}^{0} = 3\,\underline{\lfloor 60^\circ}\,(0.707\,\underline{\lfloor -45^\circ}) = 2.12\,\underline{\lfloor 15^\circ}$

$$P_2(0) = \tfrac{1}{2}|b_2(0)|^2 = \tfrac{1}{2}|b_2(\tfrac{\lambda}{8})|^2 = \tfrac{1}{2}(2.12)^2 = 2.25\ \mathrm{W}$$

1.17) (a) $Z_{IN}(0) = Z_{IN}(d)\Big|_{d=\frac{\lambda}{4}} = 75\,\dfrac{(150+j150)+j75\tan 45°}{75+j(150+j150)\tan 45°} = 60-j105\ \Omega$

(b) THE VSWR IN THE $l_2 = \frac{\lambda}{4}$ LINE IS UNITY, SINCE THE LINE IS MATCHED.

IN THE $l_1 = \frac{\lambda}{8}$ LINE : $\Gamma_0 = \dfrac{(150+j150)-75}{(150+j150)+75} = 0.62\ \underline{|29.7°}$

$\therefore\ VSWR = \dfrac{1+0.62}{1-0.62} = 4.26$

(c) $V_1(0) = \dfrac{10\underline{|0°}\ Z_{IN}(0)}{Z_{IN}(0)+Z_1} = \dfrac{10(60-j105)}{60-j105+100} = 6.32\ \underline{|-26.98°}$

$I_1(0) = \dfrac{V_1(0)}{Z_{IN}(0)} = \dfrac{6.32\underline{|-26.98°}}{60-j105} = 0.0523\ \underline{|33.27°}$

$a_1(0) = \dfrac{1}{2\sqrt{75}}\left[V_1(0)+75\,I_1(0)\right] = 0.516\ \underline{|-4.6°}$

$b_1(0) = \dfrac{1}{2\sqrt{75}}\left[V_1(0)-75\,I_1(0)\right] = 0.32\ \underline{|-64.88°}$

$a_1(\frac{\lambda}{8}) = a_1(0)e^{-j\pi/4} = 0.516\ \underline{|-49.6°}$

$b_1(\frac{\lambda}{8}) = b_1(0)e^{j\pi/4} = 0.32\ \underline{|-19.8°}$

$a_2(0) = 0$

(d) $P_1(0) = \frac{1}{2}|a_1(0)|^2 - \frac{1}{2}|b_1(0)|^2 = \frac{1}{2}(0.516)^2 - \frac{1}{2}(0.32)^2 = 0.082\ W$

$P_1(\frac{\lambda}{8}) = \frac{1}{2}|a_1(\frac{\lambda}{8})|^2 - \frac{1}{2}|b_1(\frac{\lambda}{8})|^2 = \frac{1}{2}(0.516)^2 - \frac{1}{2}(0.32)^2 = 0.082\ W$

(e) $P_{AVS} = \dfrac{|E_1|^2}{8\,Re[Z_1]} = \dfrac{(10)^2}{8(100)} = 0.125\ W$

$\frac{1}{2}|a_1(0)|^2 = 0.133\ W$

SINCE $Z_1 \neq Z_0$, IT FOLLOWS THAT $P_{AVS} \neq \frac{1}{2}|a_1(0)|^2$

1.19) IN EXAMPLE 1.7.1: $V = 5.59\angle{-26.57^\circ}$, $a_p = 0.5$, $b_p = 0$

(a) $V_p^+ = \dfrac{E_s Z_s^*}{2R_s} = \dfrac{10(100-j50)}{2(100)} = 5.59\angle{-26.57^\circ}$

$V_p^- = V - V_p^+ = 5.59\angle{-26.57^\circ} - 5.59\angle{-26.57^\circ} = 0$

(b) $a_p = \dfrac{\sqrt{R_s}}{Z_s^*} V_p^+ = \dfrac{\sqrt{100}}{100-j50}(5.59\angle{-26.57^\circ}) = 0.5$

$b_p = \dfrac{\sqrt{R_s}}{Z_s} V_p^- = 0$

1.25) (a) FOR REAL $[Z_0]$ AND $[Y_0]$

$$[I] = [y][V]$$

$$[I^-] = [I^+] - [I]$$

$$[V^-][Y_0] = [V^+][Y_0] - [y]([V^+] + [V^-])$$

$$([Y_0] + [y])[V^-] = ([Y_0] - [y])[V^+]$$

$$[V^-] = ([Y_0] + [y])^{-1}([Y_0] - [y])[V^+]$$

$$= [S][V^+]$$

$$\therefore\ [S] = -([Y_0] + [y])^{-1}([y] - [Y_0]) \qquad (1)$$

$$[Y_0] - [y] = [Y_0][S] + [y][S]$$
$$[Y_0]([I]-[S]) = [y]([I]+[S])$$
$$[y] = [Y_0]([I]-[S])([I]+[S])^{-1}$$

(b) FROM (1):

$$[S] = -\left[\begin{bmatrix} Y_0 & 0 \\ 0 & Y_0 \end{bmatrix} + \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\right]^{-1} \left[\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} - \begin{bmatrix} Y_0 & 0 \\ 0 & Y_0 \end{bmatrix}\right]$$

$$[S] = -\begin{bmatrix} y_{11}+Y_0 & y_{12} \\ y_{21} & y_{22}+Y_0 \end{bmatrix}^{-1} \begin{bmatrix} y_{11}-Y_0 & y_{12} \\ y_{21} & y_{22}-Y_0 \end{bmatrix}$$

$$\begin{bmatrix} y_{11}+Y_0 & y_{12} \\ y_{21} & y_{22}+Y_0 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} y_{22}+Y_0 & -y_{12} \\ -y_{21} & y_{11}+Y_0 \end{bmatrix}}{(y_{11}+Y_0)(y_{22}+Y_0)-y_{12}y_{21}}$$

$$\therefore \quad [S] = -\frac{\begin{bmatrix} (y_{22}+Y_0)(y_{11}-Y_0) - y_{12}y_{21} & (y_{22}+Y_0)y_{12} - y_{12}(y_{22}-Y_0) \\ -y_{21}(y_{11}-Y_0) + y_{21}(y_{11}+Y_0) & -y_{12}y_{21} + (y_{11}+Y_0)(y_{22}-Y_0) \end{bmatrix}}{(y_{11}+Y_0)(y_{22}+Y_0) - y_{12}y_{21}}$$

$$[S] = \frac{1}{\Delta_2}\begin{bmatrix} (1-y'_{11})(1+y'_{22}) + y'_{12}y'_{21} & -2y'_{12} \\ -2y'_{21} & (1+y'_{11})(1-y'_{22}) + y'_{12}y'_{21} \end{bmatrix}$$

WHERE $\Delta_2 = (1+y'_{11})(1+y'_{22}) - y'_{12}y'_{21}$, $y'_{11} = y_{11}Z_0$, $y'_{12} = y_{12}Z_0$, $y'_{21} = y_{21}Z_0$, AND $y'_{22} = y_{22}Z_0$ ($Y_0 = 1/Z_0$).