eg 3.4.1 $S_{11} = 0.73\angle 175^\circ$ $S_{21} = 4.45\angle 65^\circ$ ①

$S_{12} = 0$ $S_{22} = 0.21\angle -80^\circ$

a) optimum terminations

$\Gamma_S = S_{11}^* = 0.73\angle -175^\circ$

$\Gamma_L = S_{22}^* = 0.21\angle 80^\circ$

Plot these on Smith chart

From the Smith chart we have

$z_s = 0.152 - j\,0.045$

$Z_s = 50\, z_s$

$z_L = 50(0.97 + j0.5)$

$Z_L = 50 z_L$

b) Calculate $G_{S,max}$, $G_{L,max}$ and $G_{TU,max}$

$$G_{S,max} = \frac{1}{1-|S_{11}|^2} = 2.141 \text{ or } 3.31\,dB$$

$$G_{L,max} = \frac{1}{1-|S_{22}|^2} = 1.046 \text{ or } 0.195\,dB$$

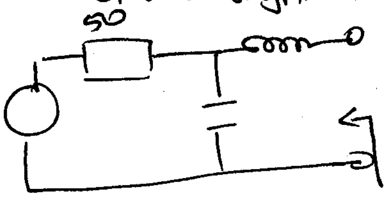
$$G_0 = |S_{21}|^2 = 19.8 \text{ or } 12.97\,dB$$

$$G_{TU,max}(dB) = 3.31 + 12.97 + 0.195 = 16.47\,dB$$

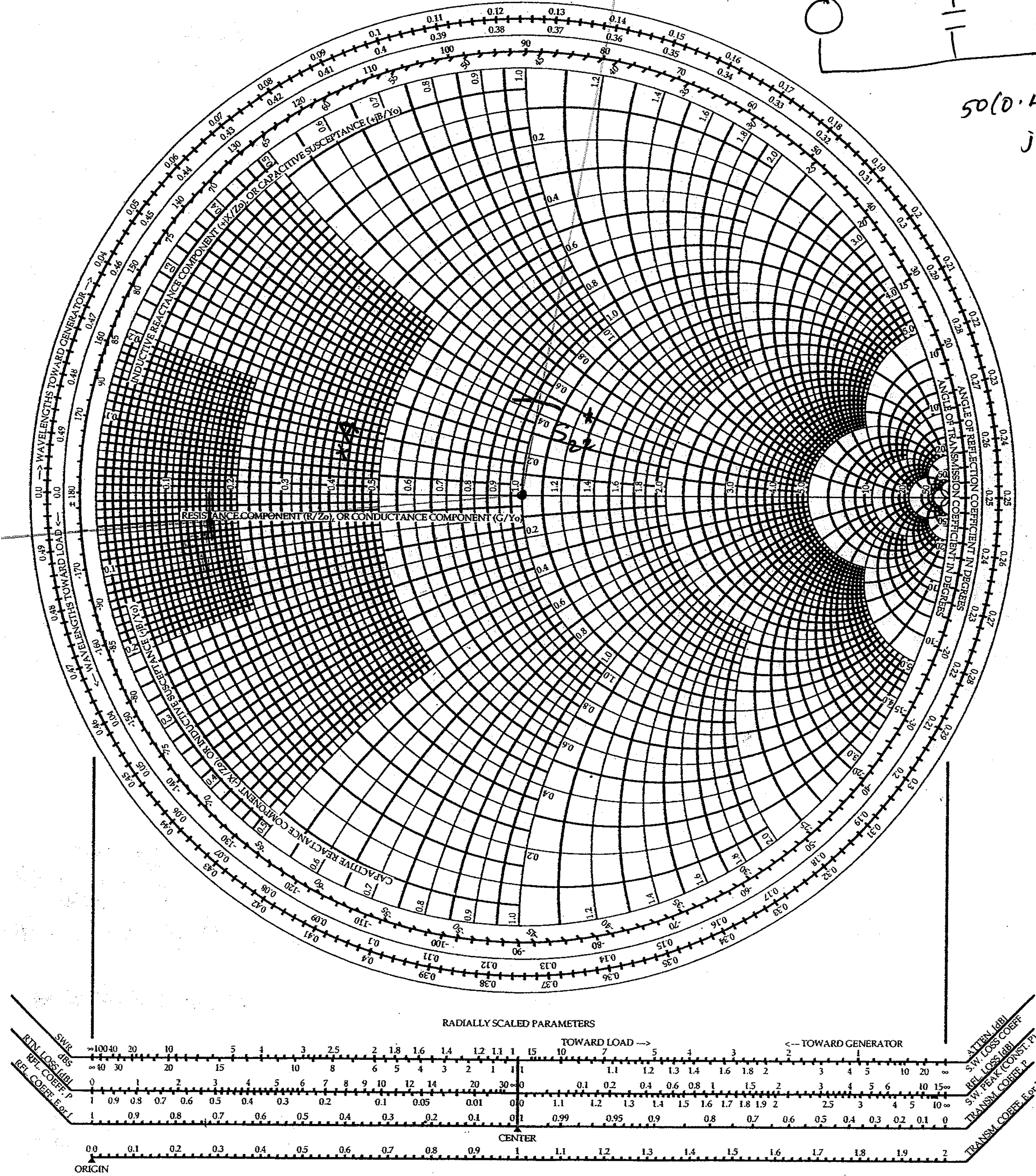
The Complete Smith Chart

Black Magic Design

Matching ckt design.



50(0.4+ j0.1)Ω



eg- For constant gain Os.

The Complete Smith Chart

Black Magic Design

1) Locate S_{ii}^* & draw a line from origin

2) Determine G_i where $0 < G_i < G_{i,max}$

Calculate $g_i = \frac{G_i}{G_{i,max}}$

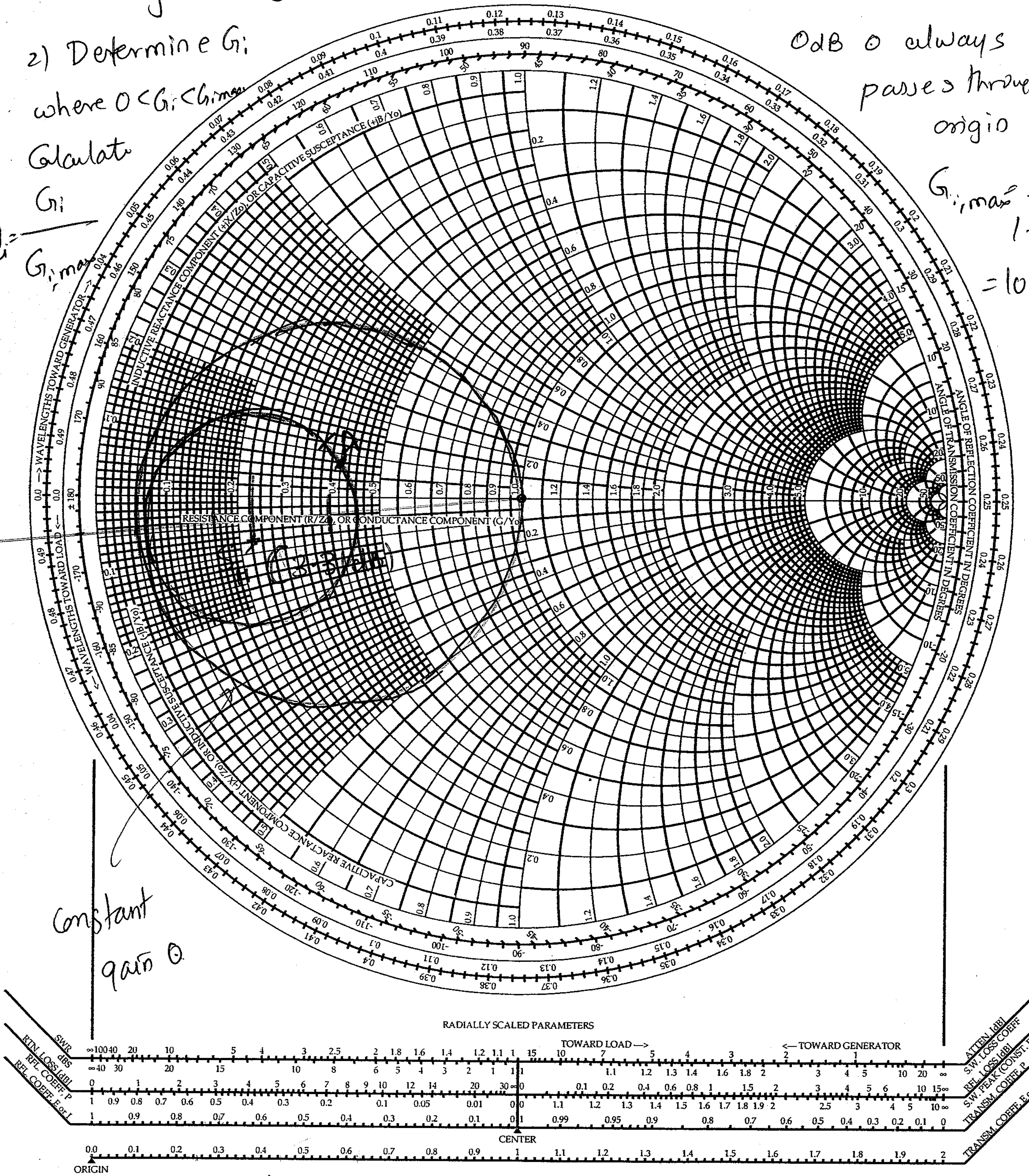
3) Determine C_{gi}

4) Determine r_{gi}

0dB O always passes through origin

$G_{i,max} = \frac{1}{1-|S_{ii}|^2}$

$= 10\log_{10} 1$



Constant gain O

eg $G_s(dB) = 2$ dB

$G_s = 1.59$

$g_s = 0.743$

$|C_{gs}| = 0.629$

$r_{gs} = 0.274$

c) Draw several G_S constant Os (See Smith chart)

a) Design i/p matching N/w

$\Gamma_L = 0.21\angle 80^\circ$

Any Γ_S along $G_S = 2dB$ provides constant gain. We choose point A & perform our matching

Potentially Unstable case $|S_{ii}| > 1$

$|S_{ii}| > 1$ and it is possible for a passive termination to produce an infinite value of G_i. The infinite values of G_i is produced by the critical value of Γ_i, called Γ_{ic}, given by

$$\Gamma_{i,c} = \frac{1}{S_{ii}} \qquad - (1)$$

Equation (1) states that the real part of the impedance associated with $\Gamma_{i,c}$ is equal to the magnitude of the negative resistance associated with S_{ii}. Therefore the total i/p & output loop resistance is zero, and oscillations will occur.

g_i defined as

$$g_i = G_i \left(1-|S_{ii}|^2\right) = \frac{1-|\Gamma_i|^2}{|1-S_{ii}\Gamma_i|^2} \left(1-|S_{ii}|^2\right)$$

g_i can attain -ve values because $|S_{ii}|>1$. the center of constant

G_i ○ is given by

$$C_{g_i} = \frac{g_i S_{ii}^*}{1-|S_{ii}|^2(1-g_i)}$$

and the radius is given by

$$r_{g_i} = \frac{\sqrt{1-g_i}\,\left(1-|S_{ii}|^2\right)}{1-|S_{ii}|^2(1-g_i)}$$

Gain G_i is infinite at $\Gamma_i = \Gamma_{i,c} = \frac{1}{S_{ii}}$. Since argument of C_{g_i} (i.e. $\angle S_{ii}^*$) is identical to argument of $\frac{1}{S_{ii}}$

it follows that center of the ○s are located along a line drawn from the origin to the point $1/S_{ii}$

The -ve resistance can be calculated using the Smith chart by locating the point $1/S_{ii}^*$ and interpreting the resistance ○s as being negative and the reactance ○s as labelled.

Plotting gain circles for Potentially unstable system

3.4.2

a) $|S_{11}| > 1$

-ve resistance

plot $\frac{1}{S_{11}^*} = 0.44\angle{-120^\circ}$

$Z_{in} = 50(0.5 - j0.46)$

$= -25 - j23\,\Omega$

$\Gamma_L = S_{22}^* = 0.6\angle 80^\circ$

The Complete Smith Chart

Black Magic Design

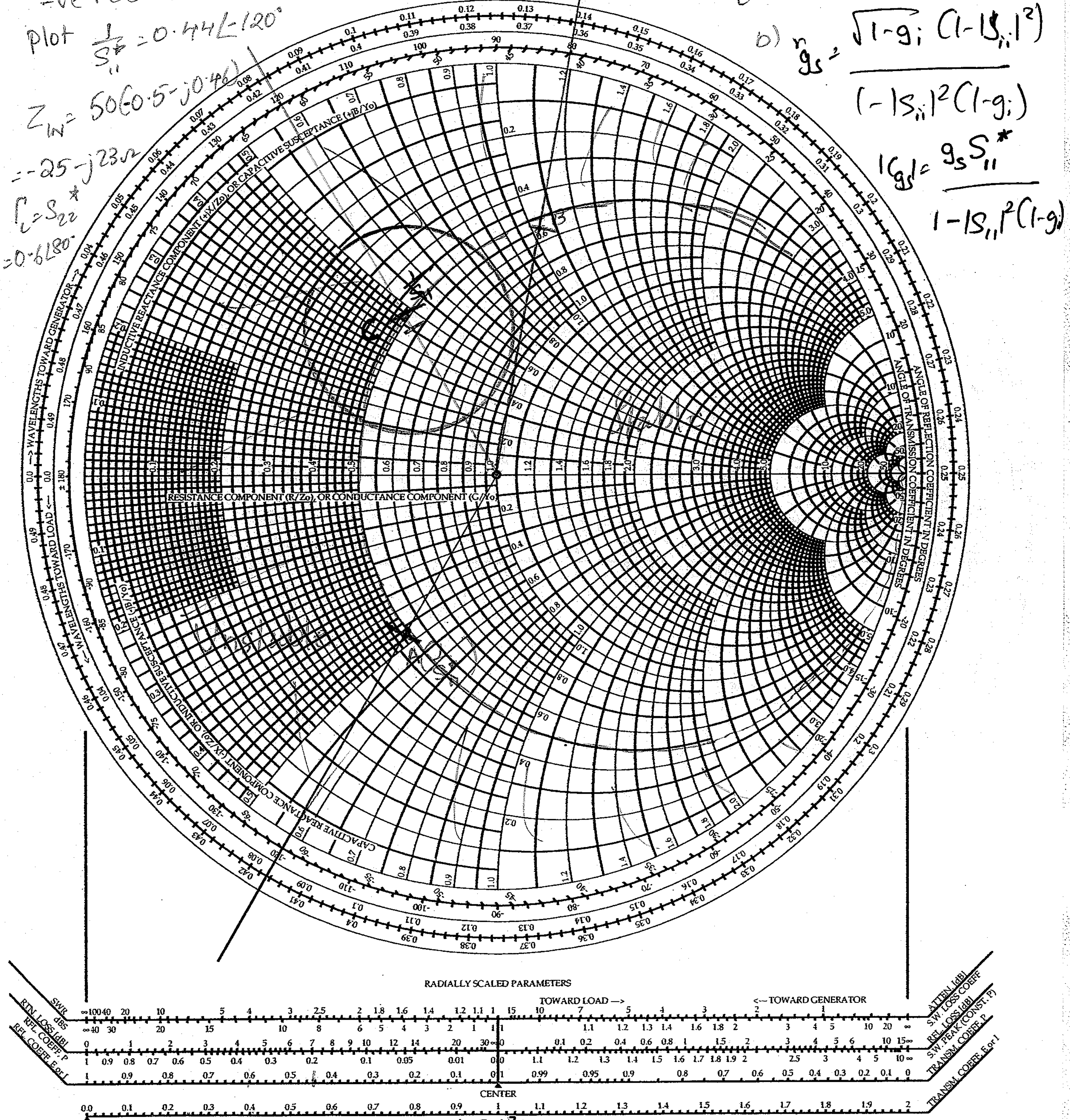
$\Gamma_L = 0.6\angle 80^\circ$

Plot this point (B)

$Z_L = 50(0.6 + j1.04)$

b) $g_s = \dfrac{\sqrt{1-g_i}\,(1-|S_{ii}|^2)}{(1-|S_{ii}|^2(1-g_i))}$

$|C_{gs}| = \dfrac{g_s S_{11}^*}{1-|S_{11}|^2(1-g_i)}$



$g_i = G_i(1-|S_{ii}|^2) = \dfrac{1-|\Gamma_i|^2}{|1-S_{ii}\Gamma_i|^2}(1-|S_{ii}|^2) = -13.123$

To prevent oscillations in the i/p & o/p Γ_i must be selected such that the real part of the termination impedance is larger than the magnitude of the negative resistance associated with the point $1/S_{ii}^*$.

When a -ve resistance occurs at the input, the stable region is that region where values of Γ_s produce a source impedance such that

$$Re(Z_s) > |Re(Z_{IN})|$$

$$Re(Z_L) > |Re(Z_{OUT})|$$

Unilateral Figure of Merit

What is the error involved for assuming $S_{12}=0$

$$\frac{G_T}{G_{TU}} = \frac{1}{|1-X|^2} \qquad X = \frac{S_{12} S_{21} \Gamma_S \Gamma_L}{(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)} \quad -(1)$$

$$G_T = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-\Gamma_{OUT}\Gamma_L|^2}$$

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

The ratio of transducer power gain to the unilateral transducer power gain is bounded by:

$$\frac{1}{(1+|X|)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-|X|)^2}$$

When $\Gamma_S = S_{11}^*$ & $\Gamma_L = S_{22}^*$, G_{TU} has a maximum value and, in this case, the maximum error introduced when using G_{TU} is bounded by

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}$$

$$U = \frac{|S_{12}|\,|S_{21}|\,|S_{11}|\,|S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$

is the unilateral figure of merit

$U \rightarrow$ varies with frequency.

Simultaneous Conjugate match (Bilateral case)

$S_{12} \neq 0$ unilateral assumption cannot be made.

The i/p and output reflection coefficients are given by

$$\Gamma_{IN} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \quad -(1)$$

$$\Gamma_{OUT} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S} \quad -(2)$$

The condition required to obtain maximum transducer power gain are.

$$\Gamma_S = \Gamma_{IN}^* \quad -(3)$$

$$\Gamma_L = \Gamma_{OUT}^* \quad -(4)$$

From (1), (2), (3), and (4) we get

$$\Gamma_S^* = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \quad -(5)$$

$$\Gamma_L^* = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S} \quad -(6)$$

From (5) & (6) we get

$$\Gamma_S = S_{11}^* + \frac{S_{12}^* S_{21}^*}{(1/\Gamma_L^*) - S_{22}^*} \quad -(7)$$

$$\Gamma_L^* = \frac{S_{22} - (S_{11}S_{22} - S_{12}S_{21})\Gamma_S}{1 - S_{11}\Gamma_S} = \frac{S_{22} - \Gamma_S \Delta}{1 - S_{11}\Gamma_S} \quad -(8)$$

Substituting (8) in (7) we get

$$\Gamma_S = S_{11}^* + \frac{S_{12}^* S_{21}^*}{[(1 - S_{11}\Gamma_S)/(S_{22} - \Gamma_S \Delta)] - S_{22}^*}$$

Solving this we get Γ_{MS} and Γ_{ML}, the values for conjugate match are obtained as.

$$\Gamma_{MS} = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \quad -(9)$$

$$\Gamma_{ML} = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \quad -(10)$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$C_1 = S_{11} - \Delta S_{22}^* \quad \& \quad C_2 = S_{22} - \Delta S_{11}^*$$

if $|B_1 / 2C_1| > 1$ and $B_1 > 0$ then $|\Gamma_{ms}| < 1$ when we use -ve sign in epn (9) and $|\Gamma_{ms}| > 1$ when we use +ve sign in epn (9)

if $|B_1 / 2C_1| > 1$ and $B_1 < 0$ then $|\Gamma_{ms}| > 1$ when we use -ve sign in epn (9) and $|\Gamma_{ms}| < 1$ when we use +ve sign in epn (9)

OBSERVATION: $|B_i / 2C_i| > 1$ implies that $|K| > 1$

Proof:

$$\left|\frac{B_i}{2C_i}\right| > 1 \Rightarrow \left|\frac{1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2}{2(S_{11} - S_{22}^* \Delta)}\right| > 1$$

or

$$\left|1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2\right| > 2|(S_{11} - S_{22}^* \Delta)|$$

Squaring both sides

$$\left|1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2\right|^2 > 4|(S_{11} - S_{22}^* \Delta)|^2$$

and

$$|S_{11} - S_{22}^* \Delta|^2 = (S_{11} - S_{22}^* \Delta)(S_{11}^* - S_{22} \Delta^*)$$
$$= |S_{12} S_{21}|^2 + (1 - |S_{22}|^2)(|S_{11}|^2 - |\Delta|^2)$$

Therefore

$$\left|1+|S_{11}|^2-|S_{22}|^2-|\Delta|^2\right|^2 > 4|S_{12}S_{21}|^2 + 4(1-|S_{22}|^2)(|S_{11}|^2-|\Delta|^2)$$

or

$$\left|1+|S_{11}|^2-|S_{22}|^2-|\Delta|^2\right|^2 - 4(1-|S_{22}|^2)(|S_{11}|^2-|\Delta|^2) > 4|S_{12}S_{21}|^2$$

or

$$(1-|S_{22}|^2-|S_{11}|^2+|\Delta|^2)^2 > 4|S_{12}S_{21}|^2$$

or

$$\left|1-|S_{22}|^2-|S_{11}|^2+|\Delta|^2\right| > 2|S_{12}S_{21}|$$

$\therefore$

$$\frac{\left|1-|S_{22}|^2-|S_{11}|^2+|\Delta|^2\right|}{2|S_{12}S_{21}|} > 1 \Rightarrow K > 1$$

If

$|K|>1$ ~~and B_i~~ with K positive, one solution of Γ_{MS} and Γ_{ML} has a magnitude less than 1, and the other solution has a magnitude greater than 1.

For $K>1$ and $B_i>0$, the solutions with the minus sign have magnitude less than 1.

Also $|\Delta|<1$ implies that $B_1>0$ and $B_2>0$ ∴ −ve sign must be used in equation for Γ_{MS} and Γ_{ML}

The maximum transducer power gain, under simultaneous conjugate match is obtained from G_T with $\Gamma_S = \Gamma_{IN}^* = \Gamma_{MS}$ and $\Gamma_L = \Gamma_{OUT}^* = \Gamma_{ML}$

$$G_{T,max} = \frac{1}{1-|\Gamma_{MS}|^2} |S_{21}|^2 \frac{1-|\Gamma_{ML}|^2}{|1-S_{22}\Gamma_{ML}|^2} \quad \text{--(1)}$$

Substituting Γ_{MS}, Γ_{ML} ~~and~~ into (1) it can be shown that $G_{T,max}$ can be expressed in the form

$$G_{T,max} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2-1}\right)$$

Since under ~~st~~ simultaneous conjugate match condition $G_T = G_P = G_A$ it follows that $G_{T,max} = G_{P,max} = G_{A,max}$.

The maximum stable gain is defined as the value of $G_{T,max}$ when $K=1$

$$G_{MSG} = \frac{|S_{21}|}{|S_{12}|}$$

For a potentially unstable transistor, G_{msG} is the figure of merit.

A simultaneous conjugate match does not exist for $K<1$. In a potentially unstable two-port network with $K>1$ but $|\Delta|>1$ solutions to Γ_{ms} and Γ_{mL} using the plus sign produce $|\Gamma_{ms}|<1$ and $|\Gamma_{mL}|<1$. In such a case the values of Γ_{ms} and Γ_{mL} ~~given by~~ using the +ve sign result is a minimum value of G_T, and i/p & o/p VSWR are unity. Substituting these values of Γ_{ms} and Γ_{mL} into equation of $G_{T,max}$, the minium value of G_T is given by

$$G_{T,min} = \frac{|S_{21}|}{|S_{12}|}\left(K + \sqrt{K^2-1}\right)$$