Signal flow graph analysis

→ A convenient technique to represent and analyze the transmission and reflection of waves in a microwave amplifier

→ Once the signal flow graph is developed, relations between the variables can be obtained using Mason's rule.

→ The flow graph technique permits expressions, such as power gains and voltage gains of complex microwave amplifiers, to be derived easily

Flow graph rules:

→ Each variable is designated as a node

→ The S parameters and reflection coefficients are represented by branches

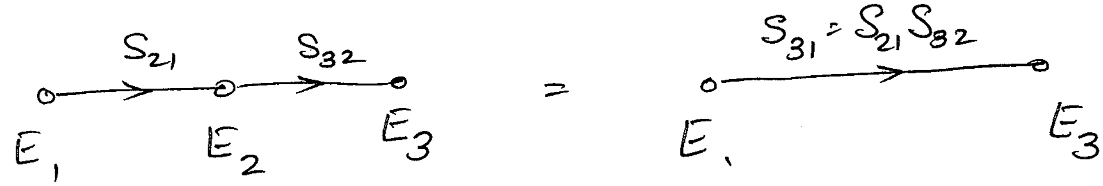
→ Branches enter dependent variable nodes and emanate from independent variable nodes. The independent variable nodes are the incident waves, and the reflected waves are dependent variables nodes.

→ A node is equal to the sum of the branches entering it:

Rules for solving S-parameters

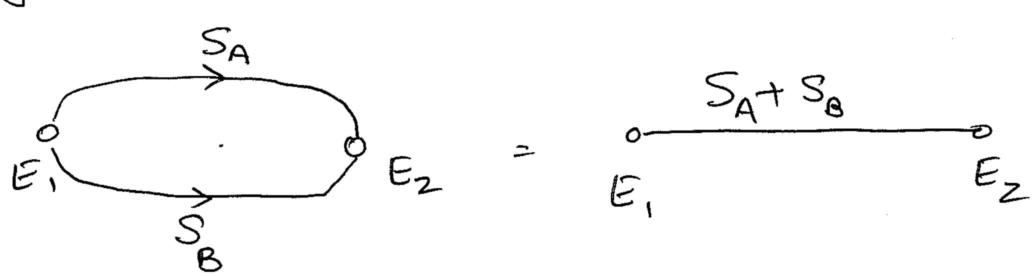
Rule 1 :-

Two branches, whose common node has only one incoming and one outgoing branch, may be combined to form a single brach whose coefficient is the product of the coefficients of the original branches. Thus a common node is eliminated



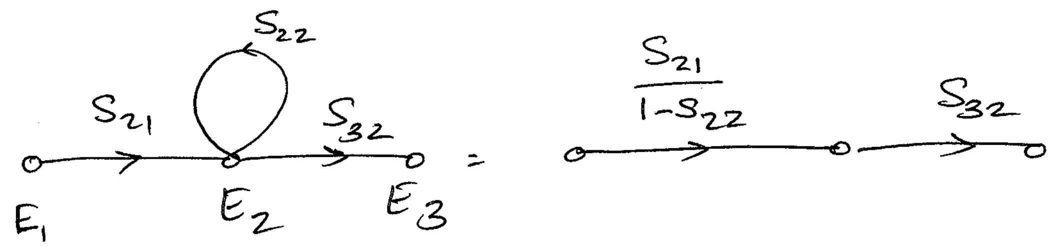
Rule 2 :-

Two branches pointing from a common node to another common node may be combined into a single branch whose coefficient is the sum of the coefficients of the original branches.



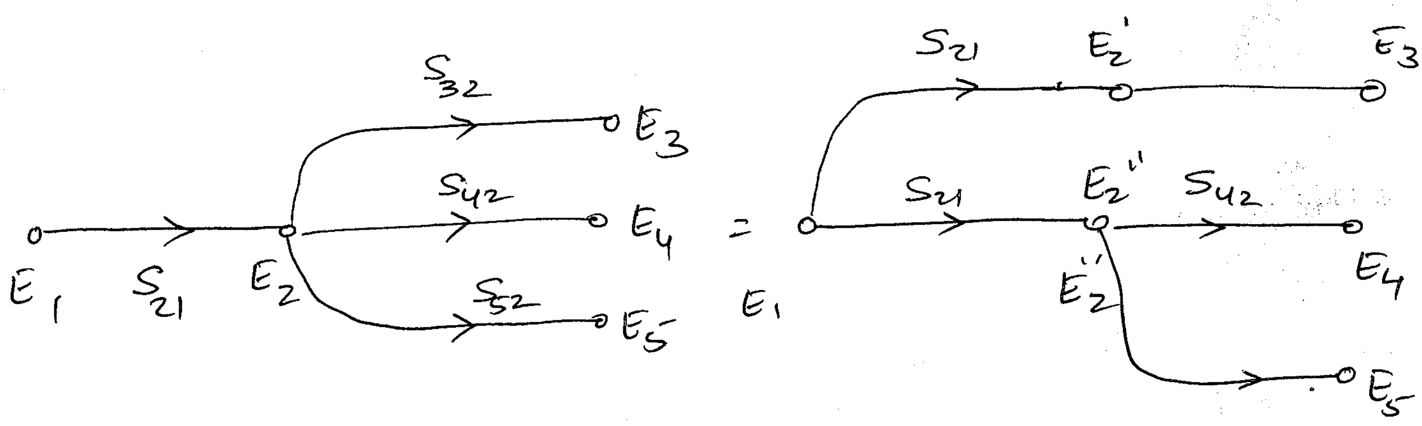
Rule 3 :-

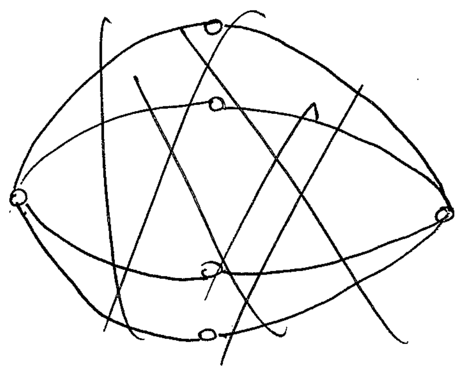
When node N posses a self loop of coefficient, the self loop may be eliminated by dividing the coefficient of every other branch entering node N to $1-S_{nn}$

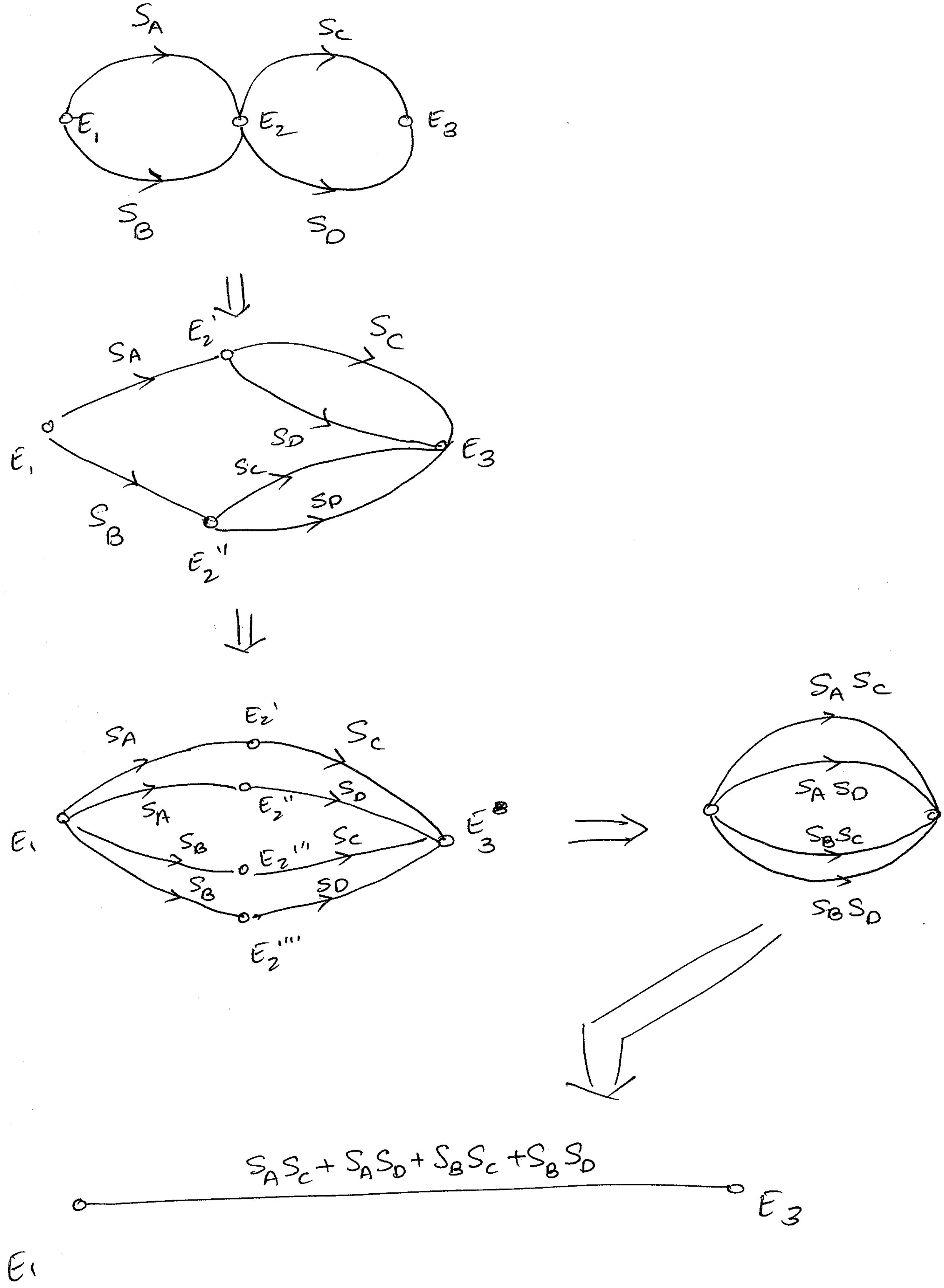


Rule 4:-

A node may be duplicated as long as the resulting flow graph contains, once and only once, each combination of separate input and output branches that connect to the original node.







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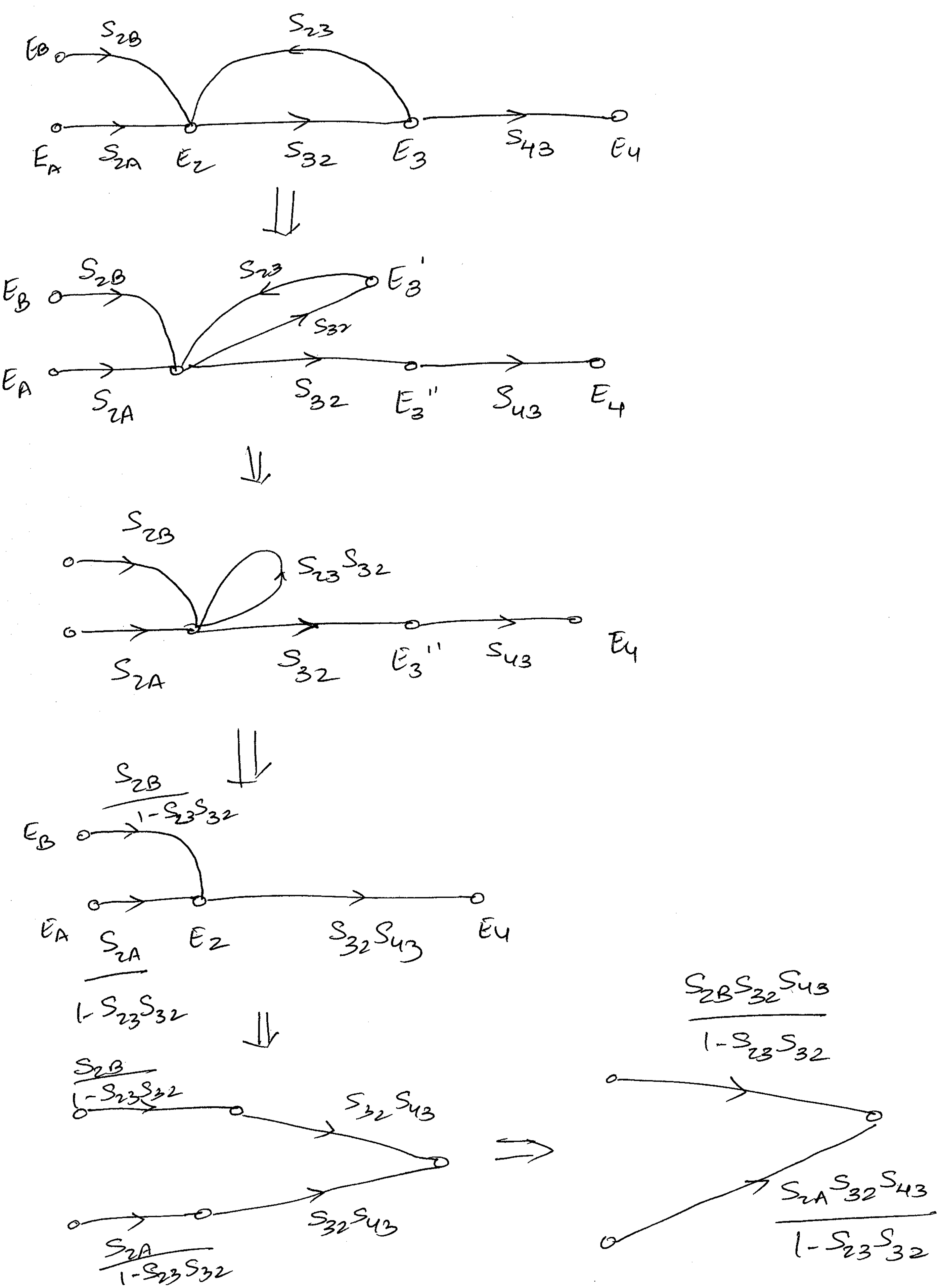
S_{2B}
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E_B
E_A
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E_2
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E_3
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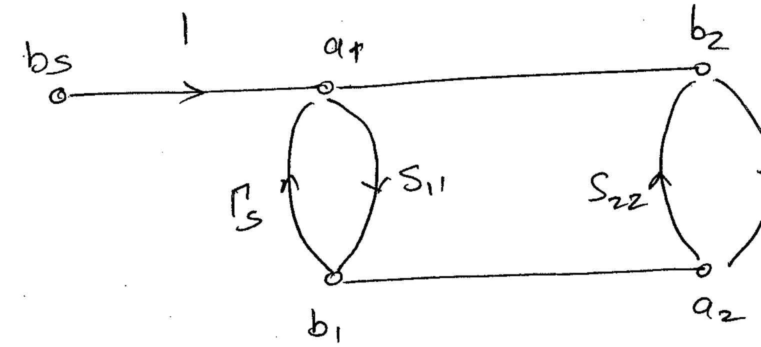
E_4
E_3'
E_3''
$S_{23}S_{32}$
$\frac{S_{2B}}{1-S_{23}S_{32}}$
$\frac{S_{2A}}{1-S_{23}S_{32}}$
$S_{32}S_{43}$
$\frac{S_{2B}S_{32}S_{43}}{1-S_{23}S_{32}}$
$\frac{S_{2A}S_{32}S_{43}}{1-S_{23}S_{32}}$

eg of complete ckt



Mason's Rule.

$$T = \frac{P_1\left[1 - \sum L(1)^{(1)} + \sum L(2)^{(1)} - \cdots\right] + P_2\left[1 - \sum L(1)^{(2)} + \cdots\right] + \cdots}{1 - \sum L(1) + \sum L(2) - \sum L(3) + \cdots}$$

P_1, P_2 - are different paths connecting the dependent and independent variables whose transfer function T is to be determined

A path is defined as a set of consecutive, codirectional branches along which no node is encountered more than once as we move in the graph from the independent to the dependent node.

For the example

$P_1 = S_{11}$ $\quad P_2 = S_{21}\Gamma_L S_{12}$

$\sum L(1)$ is sum of all first-order loops. A first order loop is defined as the product of the branches encountered in a round trip as we move from a node in the direction of the arrows back to that original node.

In our example :- $S_{11}\Gamma_S$, $S_{21}\Gamma_L S_{12}\Gamma_S$ & $S_{22}\Gamma_L$

$\Sigma L(2)$ is sum of all second order loops. A second order loop is defined as the product of any two nontouching first order loops.

In our example:- $S_{11}\Gamma_S$ & $S_{22}\Gamma_L$ do not touch, therefore, the product $S_{11}\Gamma_S S_{22}\Gamma_L$ is a second-order loop.

Term $\Sigma L(1)^{(P)}$ is the sum of all first-order loops that do not touch the path P between the independent and dependent variables. For path 1 $P_1 = S_{11}$ we find $\Sigma L(1)^{(1)} = \Gamma_L S_{22}$ and for path $P_2 = S_{21}\Gamma_L S_{12}$ $\Sigma L(1)^{(2)} = 0$

For the transfer function $\frac{b_1}{b_S}$ we have found that

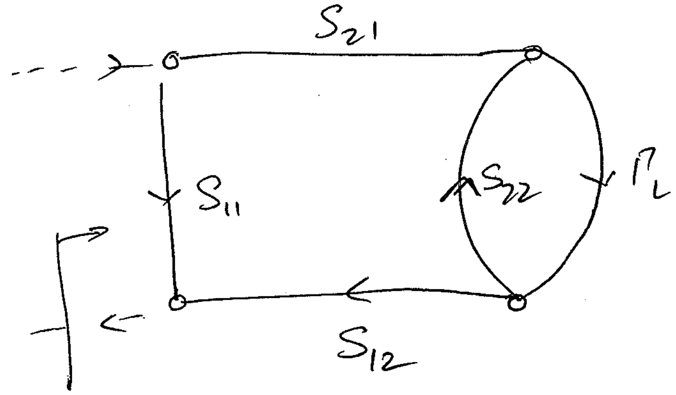
$$P_1 = S_{11} \quad P_2 = S_{21}\Gamma_L S_{12} \quad \Sigma L(1) = S_{11}\Gamma_S + S_{22}\Gamma_L + S_{21}\Gamma_L S_{12}\Gamma_S$$

$$\Sigma L(2) = S_{11}\Gamma_S S_{22}\Gamma_L \quad \& \quad \Sigma L(1)^{(1)} = \Gamma_L S_{22}$$

$$\therefore \frac{b_1}{b_S} = \frac{S_{11}(1-\Gamma_L S_{22}) + S_{21}\Gamma_L S_{12}}{1-(S_{11}\Gamma_S + S_{22}\Gamma_L + S_{21}\Gamma_L S_{12}\Gamma_S) + S_{11}\Gamma_S S_{22}\Gamma_L}$$

Application of Signal flow graph

→ Calculation of input reflection coefficient, Γ_{IN}



$\Gamma_{IN} = \frac{b_1}{a_1}$

$P_1 = S_{11}$ $P_2 = S_{21}\Gamma_L S_{12}$ $\Sigma L(1) = S_{22}\Gamma_L$ $\Sigma L(1)^{(1)} = S_{22}\Gamma_L$

Using Mason's Rule.

$$\Gamma_{IN} = \frac{S_{11}(1 - S_{22}\Gamma_L) + S_{21}\Gamma_L S_{12}}{1 - S_{22}\Gamma_L}$$

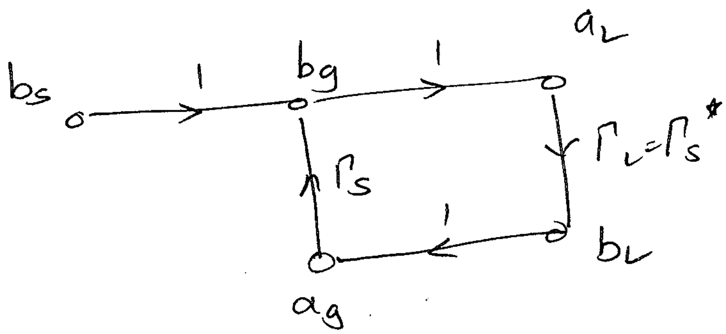
$$= S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

If $\Gamma_L = 0$ $\Gamma_{IN} = S_{11}$

If there is no transmission from o/p to i/p $(S_{12} = 0)$

$\Gamma_{IN} = S_{11}$. When $S_{12} = 0$, device is called a unilateral device

The signal flow graph can be drawn as:-



$$P_{AVS} = \frac{1}{2}|b_g|^2 - \frac{1}{2}|a_g|^2$$

$$b_g = b_s + b_g \Gamma_s \Gamma_s^*$$

$$a_g = b_g \Gamma_s^*$$

$$b_g = \frac{b_s}{1-|\Gamma_s|^2} \qquad a_g = \frac{b_s \Gamma_s^*}{1-|\Gamma_s|^2}$$

$$P_{AVS} = \frac{\frac{1}{2}|b_s|^2}{1-|\Gamma_s|^2}$$

The transducer gain:-

$$G_T = \frac{P_L}{P_{AVS}} = \frac{|b_2|^2}{|b_s|^2}\left(1-|\Gamma_L|^2\right)\left(1-|\Gamma_s|^2\right)$$

$$\frac{b_2}{b_s} = \frac{S_{21}}{1-(S_{11}\Gamma_s + S_{22}\Gamma_L + S_{21}\Gamma_L S_{12}\Gamma_s) + S_{11}\Gamma_s S_{22}\Gamma_L}$$

$$= \frac{S_{21}}{(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L) \div S_{12}S_{21}\Gamma_L\Gamma_S}$$

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|(\cancel{1}-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_L\Gamma_S|^2}$$

$$G_T = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{IN}\Gamma_S|^2} \, |S_{21}|^2 \, \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

<u>OR</u>

$$G_T = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} \, |S_{21}|^2 \, \frac{1-|\Gamma_L|^2}{|1-\Gamma_{OUT}\Gamma_L|^2}$$

Power gain G_P

$$G_P = \frac{P_L}{P_{IN}} = \frac{\left|\frac{b_2}{b_S}\right|^2 (1-|\Gamma_L|^2)}{\left|\frac{a_1}{b_S}\right|^2 (1-|\Gamma_{IN}|^2)}$$

$$G_P = \frac{1}{1-|\Gamma_{IN}|^2} \, |S_{21}|^2 \, \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$P_{AVN} = P_L \Big|_{\Gamma_L = \Gamma_{OUT}^*} = \left[\frac{1}{2} |b_2|^2 - \frac{1}{2} |a_2|^2 \right] \Big|_{\Gamma_L = \Gamma_{OUT}^*}$$

$$= \left[\frac{1}{2} |b_2|^2 (1 - |\Gamma_L|^2) \right] \Big|_{\Gamma_L = \Gamma_{OUT}^*} = \frac{1}{2} |b_2|^2 (1 - |\Gamma_{OUT}|^2)$$

$$G_A = \frac{P_{AVN}}{P_{AVS}} = \frac{|b_2|^2}{|b_S|^2} (1 - |\Gamma_{OUT}|^2)(1 - |\Gamma_S|^2)$$

$$\frac{b_2}{b_S} = \frac{S_{21}}{(1 - S_{11}\Gamma_S)(1 - S_{22}\Gamma_L) - S_{21}S_{12}\Gamma_L\Gamma_S}$$

$$= \frac{S_{21}}{(1 - S_{11}\Gamma_S)(1 - |\Gamma_{OUT}|^2)}$$

$$G_A = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1}{1 - |\Gamma_{OUT}|^2}$$

$$A_V = \frac{a_2 + b_2}{a_1 + b_1}$$

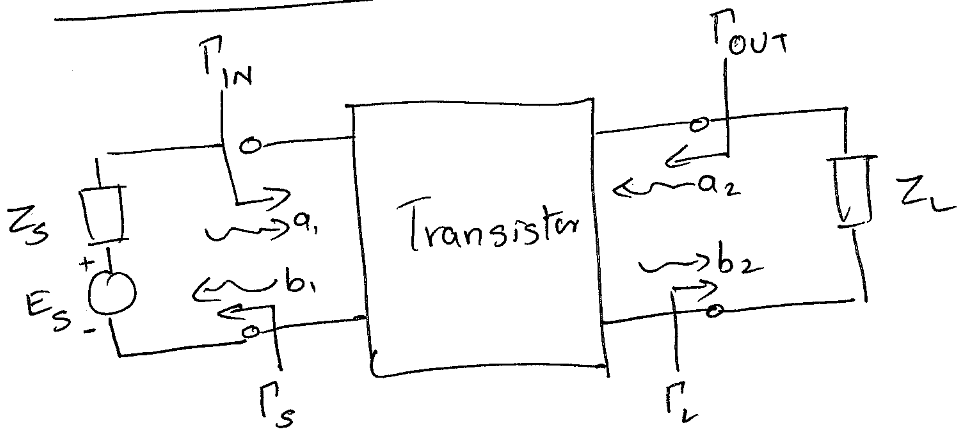
Dividing by b_s gives

$$A_v = \frac{a_2/b_s + b_2/b_s}{a_1/b_s + b_1/b_s}$$

$$A_v = \frac{S_{21}\,(1+\Gamma_L)}{(1-S_{22}\Gamma_L) + S_{11}(1-S_{22}\Gamma_L) + S_{21}\Gamma_L S_{12}}$$

Lecture - 6

Power- Gain Expressions



$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \qquad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

I/p reflection coefficient

$$\Gamma_{IN} = \frac{b_1}{a_1} \qquad - (1)$$

$$a_2 = \Gamma_L b_2 \qquad - (2)$$

$$b_1 = S_{11} a_1 + S_{12} a_2 \qquad - (3)$$

$$b_2 = S_{21} a_1 + S_{22} a_2 \qquad - (4)$$

Substitute (2) in (4)

$$b_2 = S_{21} a_1 + S_{22} \Gamma_L b_2$$

$$b_2 (1 - S_{22}\Gamma_L) = S_{21} a_1$$

$$b_2 = \frac{S_{21} a_1}{(1 - S_{22}\Gamma_L)} \quad - (5)$$

Substitute (5) & (2) into (3)

$$b_1 = S_{11} a_1 + \frac{S_{12} S_{21} a_1 \Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{IN} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L} \quad - (6)$$

To find $\Gamma_{OUT} = \left.\frac{b_2}{a_2}\right|_{E_S = 0}$

Since $E_S = 0$

$$a_1 = \Gamma_S b_1 \quad - (7)$$

Substitute (7) in eqn (3)

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_1 = \Gamma_S b_1 S_{11} + S_{12} a_2$$

$$b_1 = \frac{S_{12}}{1 - \Gamma_S S_{11}} a_2 \quad - (8)$$

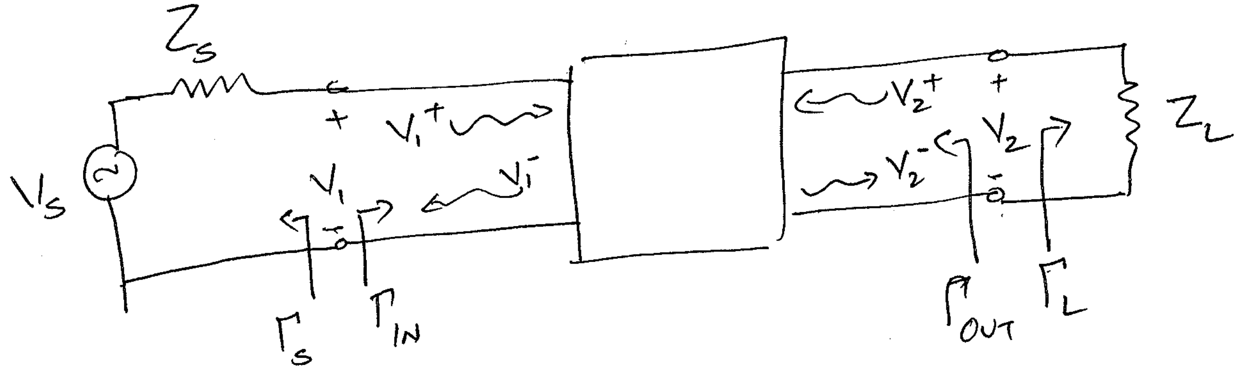
Substituting (8) & (7) in (4)

$$b_2 = \frac{S_{21}\, S_{12}\, \Gamma_S}{1-\Gamma_S S_{11}}\, a_2 + S_{22}\, a_2$$

$$\Gamma_{OUT} = \left.\frac{b_2}{a_2}\right|_{E_S=0} = S_{22} + \frac{S_{21}\, S_{12}\, \Gamma_S}{1 - S_{11}\Gamma_S} \quad -(9)$$

The power delivered to the i/p port

$$P_{IN} = \frac{1}{2}|a_1|^2 - \frac{1}{2}|b_1|^2 = \frac{1}{2}|a_1|^2\left(1 - |\Gamma_{IN}|^2\right) \quad -(10)$$



$$V_1 = V_{IN} = V_S \frac{Z_{IN}}{Z_{IN}+Z_S} = V_1^+ + V_1^- = V_1^+(1+\Gamma_{IN}) \quad -(11)$$

$$Z_{IN} = Z_0 \frac{1+\Gamma_{IN}}{1-\Gamma_{IN}} \quad -(12)$$

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

$$Z_S = Z_0 \frac{1+\Gamma_S}{1-\Gamma_S} \qquad - (13)$$

Combining (11) (12) & (13)

$$V_1^+ (1+\Gamma_{IN}) = \frac{V_S \, Z_0 \left(\frac{1+\Gamma_{IN}}{1-\Gamma_{IN}} \right)}{\left(\frac{1+\Gamma_{IN}}{1-\Gamma_{IN}} \right) Z_0 + Z_0 \left(\frac{1+\Gamma_S}{1-\Gamma_S} \right)}$$

$$V_1^+ = \frac{\frac{V_S Z_0}{1-\Gamma_{IN}} (1-\Gamma_{IN})(1-\Gamma_S)}{\left[(1+\Gamma_{IN})(1-\Gamma_S) + (1+\Gamma_S)(1-\Gamma_{IN}) \right] Z_0}$$

$$= \frac{V_S Z_0 (1-\Gamma_S)}{\left[1+\Gamma_{IN} - \Gamma_S - \Gamma_{IN}\Gamma_S + 1 + \Gamma_S - \Gamma_{IN} - \Gamma_S \Gamma_{IN} \right] Z_0}$$

$$V_1^+ = \frac{V_s}{2}\frac{(1-\Gamma_s)}{(1-\Gamma_s\Gamma_{IN})} \quad -(14)$$

$$P_{IN} = \frac{1}{2Z_0}|V_1^+|^2(1-|\Gamma_{IN}|^2) = \frac{|V_s|^2}{8Z_0}\frac{|1-\Gamma_s|^2}{|1-\Gamma_s\Gamma_{IN}|^2}(1-|\Gamma_{IN}|^2)$$

Power delivered to the load.

$$|b_2|^2 - |a_2|^2 = \frac{|V_2^-|^2}{2Z_0}(1-|\Gamma_L|^2) \quad -(15)$$

From epn (5)

$$\frac{b_2}{a_1} = \frac{S_{21}}{1-S_{22}\Gamma_L} = \frac{V_2^-}{V_1^+}$$

$$V_2^- = S_{21}V_1^+ + S_{22}\Gamma_L V_2^-$$

$$V_2^-[1-S_{22}\Gamma_L] = S_{21}V_1^+$$

$$V_2^- = \frac{S_{21}}{(1-S_{22}\Gamma_L)}V_1^+$$

Power delivered to the load

$$P_L = \frac{|V_1^+|^2}{2Z_0}\frac{|S_{21}|^2}{|1-S_{22}\Gamma_L|^2}(1-|\Gamma_L|^2) \quad -(16)$$

$$P_L = \frac{|V_s|^2}{8Z_0} \frac{|S_{21}|^2 (1-|\Gamma_L|^2) |1-\Gamma_s|^2}{|1-S_{22}\Gamma_L|^2 \, |1-\Gamma_s \Gamma_{IN}|^2} \quad -(17)$$

Power gain

$$G = \frac{P_L}{P_{IN}} = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)}{(1-|\Gamma_{IN}|^2) |1-S_{22}\Gamma_L|^2} \quad -(18)$$

Power available from source

$$P_{avs} = P_{IN}\Big|_{\Gamma_{IN}=\Gamma_s^*} = \frac{|V_s|^2}{8Z_0} \frac{|1-\Gamma_s|^2}{(1-|\Gamma_s|^2)} \quad -(19)$$

Power available from the network

$$P_{avn} = P_L\Big|_{\Gamma_L=\Gamma_{OUT}^*} = \frac{|V_s|^2}{8Z_0} \left. \frac{|S_{21}|^2 (1-|\Gamma_{OUT}|^2) |1-\Gamma_s|^2}{|1-S_{22}\Gamma_{OUT}^*|^2 \, |1-\Gamma_s \Gamma_{IN}|^2} \right|_{\Gamma_L=\Gamma_{OUT}^*} \quad -(20)$$

When $\Gamma_L = \Gamma_{OUT}^*$

$$|1-\Gamma_S\Gamma_{IN}|^2 \Big|_{\Gamma_L=\Gamma_{OUT}^*} = \frac{|1-S_{11}\Gamma_S|^2 (1-|\Gamma_{OUT}|^2)}{(1-S_{22}\Gamma_{OUT}^*|^2} \quad -(21)$$

which reduces (21) to

$$P_{avn} = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2 |1-\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{OUT}|^2)} \quad -(22)$$

Available power gain

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{OUT}|^2)} \quad -(23)$$

The transducer power gain

$$G_T = \frac{P_L}{P_{AVS}} = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{IN}|^2 |1-S_{22}\Gamma_L|^2} \quad -(24)$$

when i/p & o/p are matched for zero reflection

$G_T = |S_{21}|^2$

The unilateral transducer power gain G_{TU}.

$S_{12} = 0$

$$G_{TU} = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-S_{11}\Gamma_S|^2 \, |1-S_{22}\Gamma_L|^2} \quad -(25)$$

Two additional factors

$$\frac{P_{IN}}{P_{AVS}} = M_S = \frac{(1-|\Gamma_S|^2)(1-|\Gamma_{IN}|^2)}{|1-\Gamma_S\Gamma_{IN}|^2}$$ - Source mismatch factor.

if $\Gamma_{IN} = \Gamma_S^*$ $M_S = 1$

$$\frac{P_L}{P_{AVN}} = M_L = \frac{(1-|\Gamma_L|^2)(1-|\Gamma_{OUT}|^2)}{|1-\Gamma_{OUT}\Gamma_L|^2}$$ - Load mismatch factor.

Relation between S_p & S parameters are provided in pape 193

VSWR Calculations

We know that

$$P_L = P_{AVS}\left(1 - |\Gamma_o|^2\right)$$

$|\Gamma_o|$ provides a measure of what portion of P_{AVS} is delivered to the load.

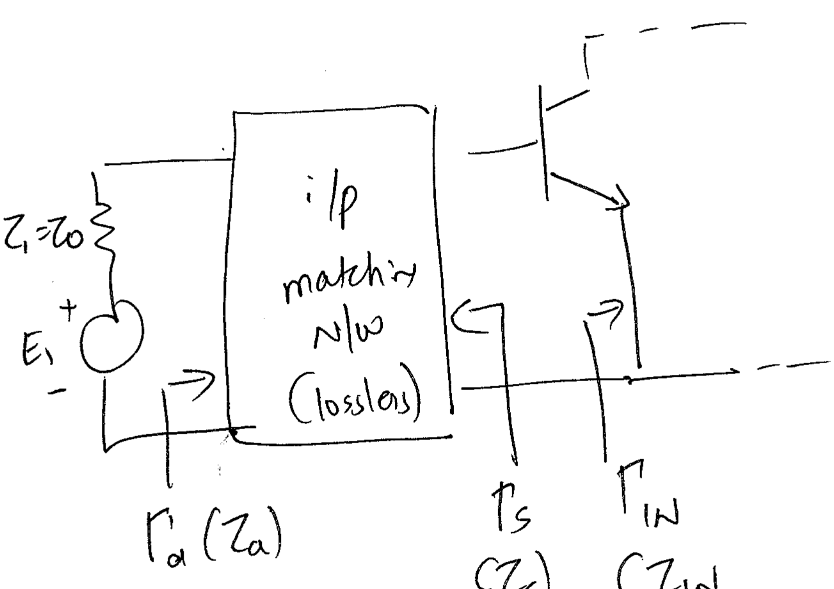
if VSWR = 1 $\quad |\Gamma_o| = 0 \quad P_L = P_{AVS}$

VSWR = 1.5 $\quad |\Gamma_o| = 0.2 \quad \dfrac{P_L}{P_{AVS}} = (1 - 0.04)$

$|\Gamma_o|^2 = 0.04 \rightarrow$ 4% of incident power is reflected by the load.

VSWR = 2 $\quad |\Gamma_o| = \frac{1}{3} \quad |\Gamma_o|^2 = 0.11$

11% of incident power is reflected by the load.



Reflection coefficient at i/p of lossless matching N/w normalized by Z_o is Γ_a

$$(VSWR)_{IN} = \frac{1+|\Gamma_a|}{1-|\Gamma_a|}$$

$$\Gamma_a = \frac{Z_a - Z_o}{Z_a + Z_o}$$

$$P_{IN} = P_{AVS}\,(1-|\Gamma_a|^2) \qquad -(1)$$

Also

$$P_{IN} = P_{AVS}\, M_S \qquad -(2)$$

$$\therefore\; M_S = (1-|\Gamma_a|^2)$$

$$|\Gamma_a| = \sqrt{1-M_S} \qquad -(3)$$

$$M_S = \frac{(1-|\Gamma_S|^2)(1-|\Gamma_{IN}|^2)}{|1-\Gamma_S\Gamma_{IN}|^2}$$

$$|\Gamma_a| = \sqrt{\frac{1-(1-|\Gamma_S|^2)(1-|\Gamma_{IN}|^2)}{(|1-\Gamma_S\Gamma_{IN}|)^2}} = \left|\frac{\Gamma_{IN}-\Gamma_S^*}{1-\Gamma_{IN}\Gamma_S}\right|$$

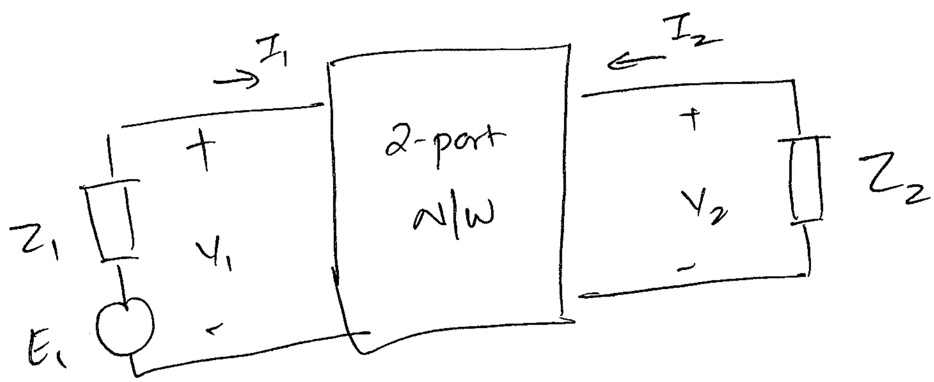
$$(VSWR)_{OUT} = \frac{1+|\Gamma_b|}{1-|\Gamma_b|}$$

$$\Gamma_b = \frac{Z_b - Z_0}{Z_b + Z_0}$$

$$|\Gamma_b| = \sqrt{1 - M_L}$$

$$|\Gamma_b| = \left| \frac{\Gamma_{OUT} - \Gamma_L^*}{1 - \Gamma_{OUT}\Gamma_L^*} \right|$$

Power gain expressions for S_p parameters.



$$G_T = \frac{P_L}{P_{AVS}} = |S_{p21}|^2$$

Power gain G_p

$$G_p = G_T \frac{P_{AVS}}{P_{IN}}$$

$$P_{IN} = P_{AVS}\left(1 - |S_{p11}|^2\right)$$

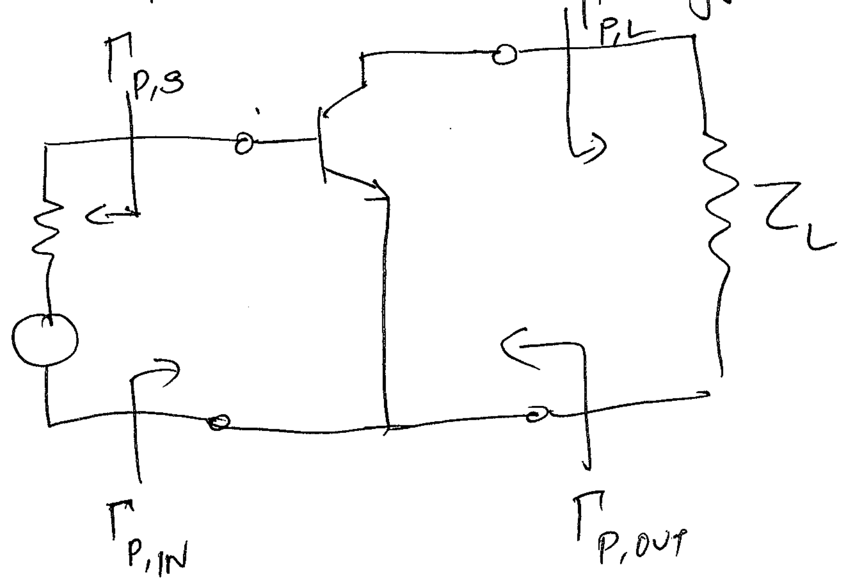
$$G_p = \frac{|S_{p21}|^2}{1 - |S_{p11}|^2}$$

$$G_A = \frac{G_T}{M_L} = \frac{G_T}{P_L} P_{AVN}$$

$$P_L = P_{AVN}\left(1 - |S_{p22}|^2\right)$$

$$G_A = \frac{|S_{p21}|^2}{1 - |S_{p22}|^2}$$

VSWR from power reflection coefficients



$$P_{IN} = P_{AVS}(1-|\Gamma_a|^2) = P_{AVS} M_S$$

At i/p transistor we can write

$$P_{IN} = P_{AVS}(1-|\Gamma_{P,IN}|^2) = P_{AVS} M_S'$$

$$\Gamma_{P,IN} = \frac{Z_{IN} - Z_S^*}{Z_{IN} + Z_S}$$

$$M_S' = 1 - |\Gamma_{P,IN}|^2$$

$$(VSWR)_{IN} = \frac{1 + |\Gamma_{P,IN}|}{1 - |\Gamma_{P,IN}|}$$

$$(VSWR)_{OUT} = \frac{1 + |\Gamma_{P,OUT}|}{1 - |\Gamma_{P,OUT}|}$$

$$\Gamma_{P,OUT} = \frac{Z_{OUT} - Z_L^*}{Z_{OUT} + Z_L}$$