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Microwave II

Lecture 1

Scattering Parameters

A set of parameters that are very useful in the microwave range are the scattering parameters. These parameters are defined in terms of traveling waves and completely characterize the behavior of a two-port Network

Impedance, Admittance, Hybrid and ABCD matrices

At low frequency a 2-port N/w can be represented as a impedance matrix, admittance matrix, hybrid matrix or a chain ~~of~~ or ABCD matrix

We can define these matrices as:

Z parameters

$$V_1 = Z_{11} i_1 + Z_{12} i_2$$
$$V_2 = Z_{21} i_1 + Z_{22} i_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Y parameters

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

h parameters

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

ABCD parameters

$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -i_2 \end{bmatrix}$$

These N/ws are really useful at low frequencies because the parameters are readily measurable using short and open circuit tests at the terminal of the two port Network.

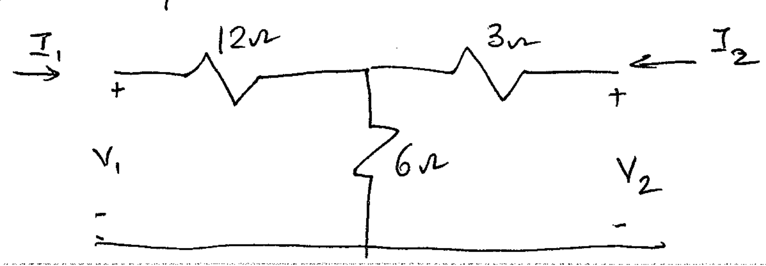
For example,

$$z_{11} = \left. \frac{v_1}{i_1} \right|_{i_2 = 0}$$

is measured with an ac open circuit at port 2 (i.e. $i_2 = 0$)

example

Consider a 2 port Network as shown in the figure below. Find the impedance parameters



Solution

Step 1

Calculate V_1 assuming port 1 is the source & port 2 is open ckt

$$V_1 = (12+6)\,I_1 = 18 I_1 \quad \& \quad V_2 = 6 I_1$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 18\,\Omega \qquad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 6\,\Omega$$

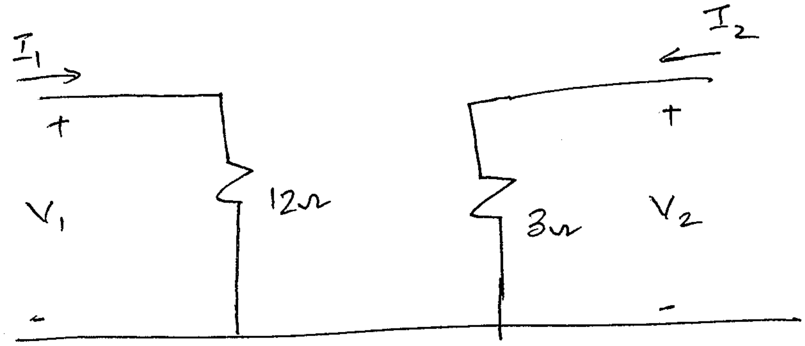
Similarly when a source is at port 2 and port 1 is open circuit

$$V_2 = (6+3)\,I_2 = 9 I_2 \qquad \& \quad V_1 = 6 I_2$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 9\,\Omega \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 6\,\Omega$$

$$\begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix}$$

Example



Soln

$$\begin{bmatrix} 12 & 0 \\ 0 & 3 \end{bmatrix}$$

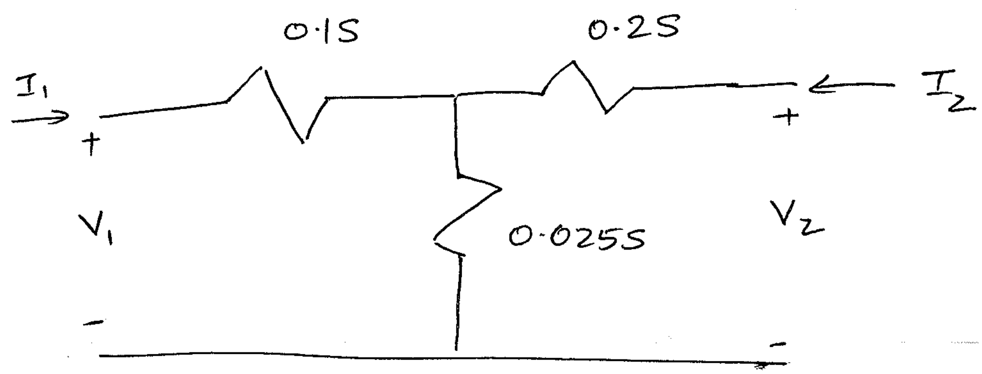
Admittance Parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

example



$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2 = 0}$$

$$I_1 = \frac{0.1\,(0.2 + 0.025)}{0.1 + 0.2 + 0.025} V_1 = \frac{0.0225}{0.325} V_1$$

$$\cancel{Y_{11}} = \left. \frac{I_1}{V_1} \right|_{V_2 = 0} = \frac{0.0225}{0.325}$$

If voltage across 0.2S is V_N, then

$$V_N = \frac{I_1}{0.2 + 0.025} = \frac{V_1}{3.25}$$

$$I_2 = -0.2V_N = \frac{-0.2}{3.25}V_1$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-0.2}{3.25} = -0.0615S$$

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$$I_2 = \frac{0.025}{0.325}V_2$$

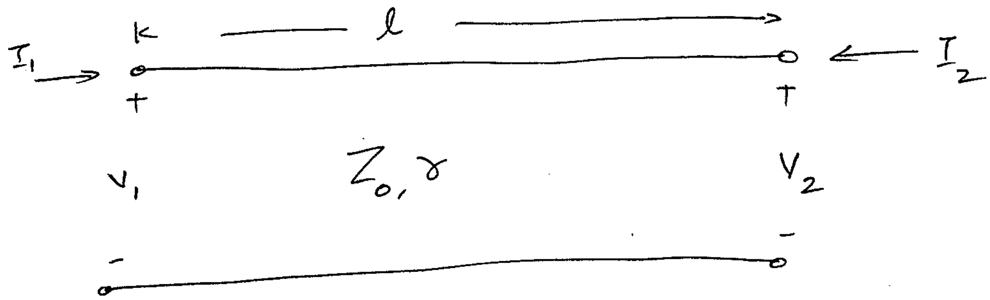
$$I_1 = \frac{-0.2}{3.25}V_2$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{0.2}{3.25} = -0.0615S$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{0.025}{0.325} = 0.0769S$$

example 2:

Find admittance parameters of a transmission line of length l



Soln

→ This circuit is symmetrical because interchanging port 1 and port 2 does not affect it. Hence Y_{22} must be equal to Y_{11}.

→ This circuit is reciprocal because if voltage V at port 1 produces a short circuit current I at port 2, voltage V at port 2 will produce current I at port 1. $(Y_{12} = Y_{21})$

→ Assume a source is connected at port 1 when the other port has a short circuit. If V_{in} is the incident voltage at port 1, it will appear as $V_{in}e^{-\gamma l}$ at port 2. Since the reflection coefficient of a short circuit is equal to -1, reflected voltage at this port is 180° out of phase with incident voltage.

Therefore reflected voltage at the port 1 ~~is 180°~~ is

$-V_{in}e^{-2\gamma l}$

$$V_1 = V_{in} - V_{in} e^{-2\gamma l}$$

$$V_2 = 0$$

$$I_1 = \frac{V_{in}}{Z_0}\left(1 + e^{-2\gamma l}\right)$$

$$I_2 = -\frac{2V_{in}}{Z_0} e^{-\gamma l}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{Z_0 \tanh \gamma l}$$

$$Y_{21} = \frac{-1}{Z_0 \sinh \gamma l}$$

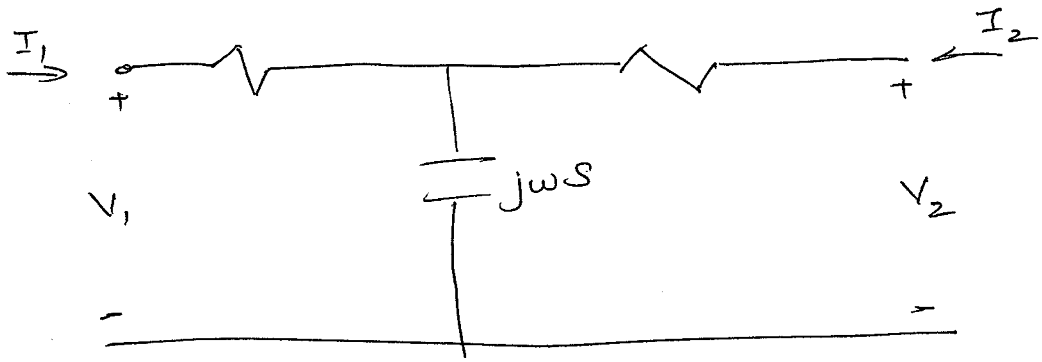
Transmission Parameters (elements of chain matrix)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

example



Soln

With a source at port 1 while port 2 has short ckt

$$V_1 = \left(1 + \frac{1}{1+j\omega}\right) I_1 = \frac{2+j\omega}{1+j\omega} I_1$$

$$I_2 = -\frac{\frac{1}{j\omega}}{\frac{1}{j\omega}+1} = -\frac{1}{1+j\omega} I_1$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 2+j\omega$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 1+j\omega$$

When source connected at port 1 while port 2 is open

$$V_1 = \left(1 + \frac{1}{j\omega}\right) I_1 = \frac{1+j\omega}{j\omega} I_1 \quad \& \quad V_2 = \frac{1}{j\omega} I_1$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 + j\omega$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = j\omega$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+j\omega & 2+j\omega \\ j\omega & 1+j\omega \end{bmatrix}$$

example 2

Find transmission parameters of transmission line

$$V_1 = V_{in} - V_{in} e^{-2\gamma l}$$

$$V_2 = 0$$

$$I_1 = \frac{V_{in}}{Z_0}\left(1 + e^{-2\gamma l}\right)$$

$$I_2 = -\frac{2V_{in}}{Z_0} e^{-\gamma l}$$

Therefore

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = Z_0 \sinh \gamma l$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = \frac{1 + e^{-2\gamma l}}{2e^{-\gamma l}} = \cosh \gamma l$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \cosh \gamma l$$

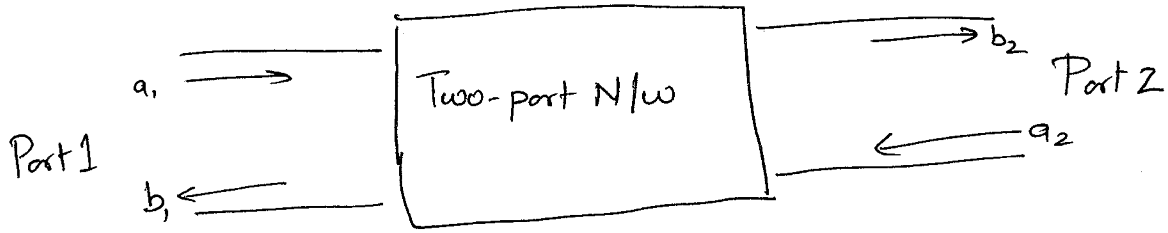
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{Z_0}\sinh \gamma l$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_0 \sinh \gamma l \\ \frac{1}{Z_0}\sinh \gamma l & \cosh \gamma l \end{bmatrix}$$

Scattering Parameters

Z-parameters are useful in analyzing series circuits while Y-parameters are useful in analyzing parallel connected circuits. Similarly, transmission parameters are useful for chain or cascade circuits.

However, the characterization procedure of these parameters requires an open or short circuit at the other port. This extreme reflection makes it very difficult to determine to determine the parameters of a network at radio and microwave frequencies. Therefore, a new representation based on travelling waves is defined. This is known as scattering matrix of the network. Elements of the matrix are called scattering parameters.



$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \quad \&$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}$$

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$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} \quad \&$$

$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$

S_{ii} is the reflection coefficient Γ_i at i^{th} port when the other port is matched terminated.

$S_{ij} \rightarrow$ forward transmission coefficient of the j^{th} port if i is greater than j, whereas it represents the reverse transmission coefficient if i is less than j with the other port terminated by a matched load.

The steady state total voltage and current at the i^{th} port is given as.

$$V_i = \underbrace{V_i^{in}}_{\text{incident}} + \underbrace{V_i^{ref}}_{\text{reflected}}$$

$$I_i = \frac{1}{Z_{0i}}\left(V_i^{in} - V_i^{ref}\right)$$

$$V_i^{in} = \frac{1}{2}\left(V_i + Z_{0i} I_i\right)$$

$$V_i^{ref} = \frac{1}{2}\left(V_i - Z_{0i} I_i\right)$$

Assuming both of the ports to be lossless so that Z_{0i} is a real quantity, the average power incident at the i^{th} port is:

$$P_i^{in} = \frac{1}{2} \mathrm{Re}\left[V_i^{in} (I_i^{in})^*\right] = \frac{1}{2} \mathrm{Re}\left[V_i^{in} \left(\frac{V_i^{in}}{Z_{0i}}\right)^*\right] = \frac{1}{2Z_{0i}} |V_i^{in}|^2$$

average power reflected from i^{th} port is,

$$P_i^{ref} = \frac{1}{2} Re\left[V_i^{ref}(I_i^{ref})^*\right] = \frac{1}{2} Re\left[V_i^{ref}\left(\frac{V_i^{ref}}{Z_{0i}}\right)^*\right] = \frac{1}{2Z_{0i}}|V_i^{ref}|^2$$

$$a_i = \frac{V_i^{in}}{\sqrt{2Z_{0i}}} = \frac{1}{2}\left[\frac{V_i + Z_{0i}I_i}{\sqrt{2Z_{0i}}}\right] = \frac{1}{2\sqrt{2}}\left(\frac{V_i}{\sqrt{Z_{0i}}} + \sqrt{Z_{0i}}\,I_i\right)$$

$$b_i = \frac{V_i^{ref}}{\sqrt{2Z_{0i}}} = \frac{1}{2}\left[\frac{V_i - Z_{0i}I_i}{\sqrt{2Z_{0i}}}\right] = \frac{1}{2\sqrt{2}}\left(\frac{V_i}{\sqrt{Z_{0i}}} - \sqrt{Z_{0i}}\,I_i\right)$$

~~Row~~ The a_i and b_i are defined in such a way that the squares of the magnitudes represent the power flowing in respective directions.

Power available from the source, P_{avs}, at port 1 is

$$P_{avs} = |a_1|^2$$

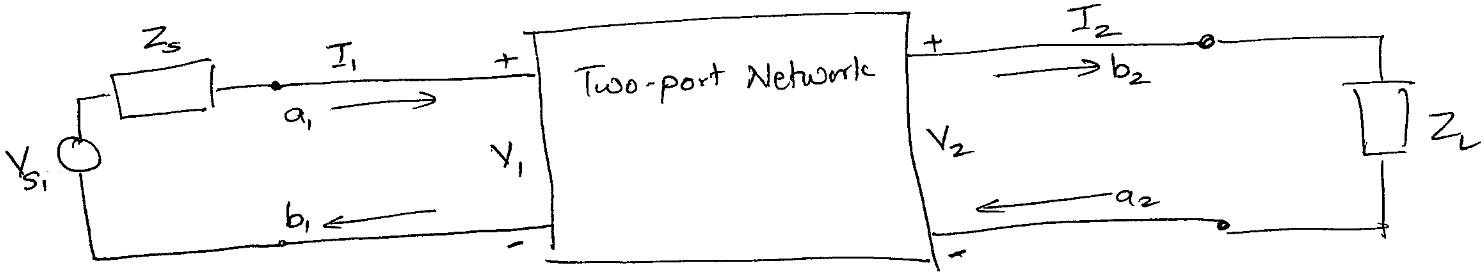
power reflected from port 1, P_{ref}, is

$$P_{ref} = |b_1|^2$$

Power delivered to the port, P_d, is

$$P_d = P_{avs} - P_{ref} = |a_1|^2 - |b_1|^2$$

Consider the following ckt



$$b_1 = S_{11} a_1 + S_{12} a_2 \qquad -(1)$$

$$b_2 = S_{21} a_1 + S_{22} a_2 \qquad -(2)$$

$$\Gamma_L = \frac{Z_L - Z_{02}}{Z_L + Z_{02}} = \frac{a_2}{b_2} \qquad -(3)$$

$$\Gamma_S = \frac{Z_S - Z_{01}}{Z_S + Z_{01}} = \frac{a_1}{b_1} \qquad -(4)$$

The input and output reflection coefficients are

$$\Gamma_1 = \frac{Z_1 - Z_{01}}{Z_1 + Z_{01}} = \frac{b_1}{a_1} \quad -(5) \qquad \Gamma_2 = \frac{Z_2 - Z_{02}}{Z_2 + Z_{02}} = \frac{b_2}{a_2} \quad -(6)$$

$$\frac{b_1}{a_1} = \Gamma_1 = \frac{Z_1 - Z_{01}}{Z_1 + Z_{01}} = S_{11} + S_{12} \frac{a_2}{a_1} \qquad -(7)$$

[Substituting (5) in (1) after dividing (1) by a_1]

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$$\frac{b_2}{a_2} = S_{22} + S_{21}\frac{a_1}{a_2} = \frac{1}{\Gamma_L} \Rightarrow \frac{a_1}{a_2} = \frac{1 - S_{22}\Gamma_L}{S_{21}\Gamma_L} \qquad -(8)$$

From (7) & (8)

$$\Gamma_1 = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \qquad -(9)$$

If a matched load is terminating port 2, $\Gamma_L = 0$

we get

$$\Gamma_1 = S_{11} \qquad -(10)$$

From (2) and (4) we get

$$\frac{b_2}{a_2} = \Gamma_2 = \frac{Z_2 - Z_{02}}{Z_2 + Z_{02}} = S_{22} + S_{21}\frac{a_1}{a_2} \qquad -(11)$$

&

$$\frac{b_1}{a_1} = S_{11} + S_{12}\frac{a_2}{a_1} = \frac{1}{\Gamma_S} \Rightarrow \frac{a_2}{a_1} = \frac{1 - S_{11}\Gamma_S}{S_{12}\Gamma_S} \qquad -(12)$$

From (11) & (12) we get

$$\Gamma_2 = S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{11}\Gamma_S} \qquad -(13)$$

If port 1 is matched and $\Gamma_s = 0$. Therefore

$$\Gamma_2 = S_{22}$$

S_{11} and S_{22} can be found by evaluating the reflection coefficients at respective ports while the other port is matched terminated.

Lets determine, S_{21} and S_{12}

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \quad -(14)$$

Now a_2 can be found as

$$a_2 = \frac{1}{2}\left[\frac{V_2 + Z_{02} I_2}{\sqrt{2Z_{02}}}\right] \quad -(15)$$

forcing a_2 to 0, we get

$$V_2 = -Z_{02} I_2 \quad -(16)$$

Substituting (16) in the equation for b_2 we get

$$b_2 = \frac{1}{2}\left(\frac{V_2 - Z_{01} I_1}{\sqrt{2Z_{01}}}\right) = \frac{V_{s1}}{2\sqrt{2Z_{01}}} \quad -(17)$$

$$S_{21} = \frac{b_2}{a_1}\bigg|_{a_2=0} = \frac{-\sqrt{Z_{02}}\, I_2/\sqrt{2}}{V_{s_1}/2\sqrt{2Z_{01}}} = \frac{2V_2}{V_{s_1}}\sqrt{\frac{Z_{01}}{Z_{02}}} \quad -(18)$$

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$$S_{12} = \frac{b_1}{a_2}\bigg|_{a_1=0} = \frac{2V_1}{V_{s_2}}\sqrt{\frac{Z_{02}}{Z_{01}}} \quad -(19)$$

An analysis of S-parameters indicates that

$$|S_{11}|^2 = \frac{|b_1|^2}{|a_1|^2}\bigg|_{a_2=0} = \frac{P_{avs} - P_d}{P_{avs}}. \quad -(20)$$

P_{avs} is power available at ~~port 2~~ source.

P_d - Power delivered to port 1

These two powers will be equal if the source impedance is conjugate of Z_1; that is, the source is matched with port 1

$$|S_{21}|^2 = \frac{Z_{02}(I_2/\sqrt{2})^2}{(\frac{1}{4}Z_{01})(V_{s_1}/\sqrt{2})^2} = \frac{Z_{02}(I_2/\sqrt{2})^2}{\frac{1}{2}\left[(\frac{1}{2Z_{01}})(\frac{V_{s_1}}{\sqrt{2}})^2\right]} = \frac{P_{AVN}}{P_{avs}} \quad -(21)$$

P_{avN} - power available at port 2
It will be equal to power delivered to a load that is matched to the port

The power ratio of (21) may be called the transducer power gain

It can be found that $|S_{22}|^2$ represents the ratio of power reflected from port 2 to power available from the source at port 2, while port 1 is terminated by a matched load Z_s and $|S_{12}|^2$ represents a reverse transducer power gain