

One Port Negative-Resistance Oscillators

$$Z_{IN}(A, \omega) = R_{IN}(A, \omega) + j X_{IN}(A, \omega)$$

$A \rightarrow$ amplitude of $i(t)$

$$R_{IN}(A, \omega) < 0$$

The oscillator is constructed by connecting the device to a passive load impedance

$$Z_L(\omega) = R_L(\omega) + j X_L(\omega)$$

The one-port N/w is stable if:

$$\text{Re}[Z_{IN}(A, \omega) + Z_L(\omega)] > 0$$

For oscillation to occur, the loop gain must be unity

$$\Gamma_{IN}(j\omega) \Gamma_L(j\omega) = 1$$

At the amplitude $A = A_0$ & frequency $\omega = \omega_0$, the N/w ② will oscillate when

$$\Gamma_{IN}(A_0, \omega_0) \Gamma_L(\omega_0) = 1 \quad - (1)$$

$$\Gamma_{IN}(A_0, \omega_0) = \frac{Z_{IN}(A_0, \omega_0) - Z_0}{Z_{IN}(A_0, \omega_0) + Z_0} \quad - (2)$$

$$\Gamma_L(\omega_0) = \frac{Z_L(\omega_0) - Z_0}{Z_L(\omega_0) + Z_0} \quad - (3)$$

From the three equations we get

$$Z_{IN}(A_0, \omega_0) + Z_L(\omega_0) = 0$$

Equating real & imaginary part

$$R_{IN}(A_0, \omega_0) + R_L(\omega_0) = 0$$

$$X_{IN}(A_0, \omega_0) + X_L(\omega_0) = 0$$

The device is defined to be unstable over some

frequency range $\omega_1 < \omega < \omega_2$ if $R_{IN}(A, \omega) < 0$

The one-port N/w is unstable for some ω in the range if the net resistance of the network is negative - i.e

$$R_{IN}(A, \omega) > R_L(\omega)$$

- (A)

Under proper conditions, a growing sinusoidal current will flow through the circuit i.e. at the start of oscillations, when amplitude A is small (A) must be satisfied. This is expressed in the form

$$|R_{IN}(0, \omega)| > R_L(\omega)$$

The oscillations will continue to build up as long as the loop resistance is negative.

The amplitude of the current must eventually reach a steady-state value (i.e. at $A=A_0$ and $\omega=\omega_0$), which occurs when the loop resistance is zero.

$Z_{IN}(A, \omega)$ is amplitude and frequency dependent. It is necessary to find another condition to guarantee a stable oscillation.

If the frequency dependence of $Z_{IN}(A, \omega)$ can be neglected for small variations around ω_0 .

Kurokawa has shown that a stable oscillation is obtained when (16) & (17) are satisfied and the following condition is also satisfied.

$$\left. \frac{\partial R_{IN}(A)}{\partial A} \right|_{A=A_0} \cdot \left. \frac{\partial X_L(\omega)}{\partial \omega} \right|_{\omega=\omega_0} - \left. \frac{\partial X_{IN}}{\partial A} \right|_{A=A_0} \left. \frac{\partial R_L(\omega)}{\partial \omega} \right|_{\omega=\omega_0} > 0$$

In many cases

$$\frac{\partial R_L(\omega)}{\partial \omega} = 0$$

In a given oscillator design, the i/p impedance of the active device is known for small-signal conditions.

A practical way of designing R_L is to select the value of R_L for maximum oscillator power.

If magnitude of -ve resistance is a linearly decreasing function of A , we can express $R_{IN}(A)$ in the form

$$R_{IN}(A) = -R_0 \left[1 - \frac{A}{A_m} \right]$$

$-R_0$ is value of $R_{IN}(A)$ at $A=0$ & A_m is maximum value of A .

The power delivered to R_L by R_{IN} (for $A < A_m$)

$$P = \frac{1}{2} \operatorname{Re} [VI^*] = \frac{1}{2} |I|^2 R_{IN}(A) = \frac{1}{2} A^2 R_0 \left[1 - \frac{A}{A_m} \right]$$

Value of A that maximizes the oscillation power is found from

$$\frac{dP}{dA} = \frac{1}{2} R_0 \left[2A - \frac{3A^2}{A_m} \right] = 0$$

which gives desired value of A , denoted by $A_{0,max}$, that maximizes the power

$$A_{0,max} = \frac{2}{3} A_m$$

At $A_{0,max}$ the value of $R_{in}(A_{0,max})$ is:

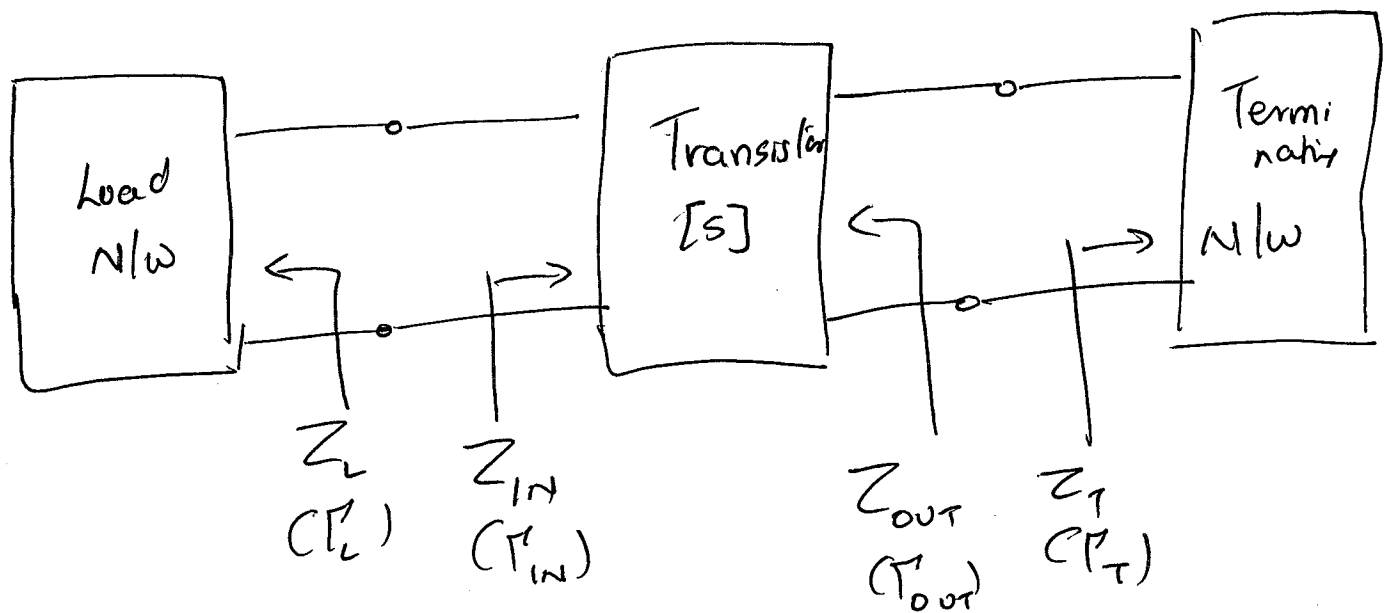
$$R_{in}(A_{0,max}) = -\frac{R_0}{3}$$

A convenient value of R_L , which maximizes the oscillator power

$$R_L = \frac{R_0}{3}$$

Two-part Negative-Resistance Oscillators.

→ The general block diagrams for two-part negative resistance oscillators are shown in figure (a) & (b).



- Either part can be used as the terminating part
- Once terminating part is selected, the other part is referred to as the i/p part.
- The load-matching N/w is connected to the i/p part in agreement with the one part notation.
- When the two-part is potentially unstable, an appropriate Z_T permits the two-part to be represented as a one-part negative-resistance device with input impedance Z_w .
- The conditions for a stable oscillation are.

$$R_{IN}(A_0, \omega_0) + R_L(\omega_0) = 0$$

$$X_{IN}(A_0, \omega_0) + X_L(\omega_0) = 0$$

$$\& \frac{\partial R_{IN}(A)}{\partial A} \bigg|_{A=A_0} \frac{\partial X_L(\omega)}{\partial \omega} \bigg|_{\omega=\omega_0} - \frac{\partial X_{IN}(A)}{\partial A} \bigg|_{A=A_0} \frac{\partial R_L(\omega)}{\partial \omega} \bigg|_{\omega=\omega_0} > 0$$

→ To start the oscillation, the value of R_L is selected according to

$$R_L = \frac{R_0}{3}, \text{ in general } R_L = \frac{|R_{IN}(0, \omega)|}{3}$$

When i/p port is made to oscillate, the terminating port also oscillates. The fact that both ports are oscillating can be proved as follows.

→ i/p port is oscillating when

$$\Gamma_{IN} \Gamma_L = 1$$

$$\Gamma_L = \frac{1}{\Gamma_{IN}} = \frac{1 - S_{22} \Gamma_T}{S_{11} - \Delta \Gamma_T}$$

$$\Gamma_T = \frac{1 - S_{11} \Gamma_L}{S_{22} - \Delta \Gamma_L}$$

$$\Gamma_{OUT} = \frac{S_{22} - \Delta \Gamma_L}{1 - S_{11} \Gamma_L}$$

$$\Gamma_{OUT} \Gamma_T = 1$$

which shows that the terminating port is also oscillating

A design procedure for a two-port oscillator is as follows.

- 1) Use a potentially unstable transistor at the frequency of oscillation ω_0 .
- 2) Design the terminating Nlw to make $|\Gamma_{IN}| > 1$.
Series or shunt feedback can be used to increase $|\Gamma_{IN}|$.
- 3) Design the load Nlw to resonate Z_{IN} , and to satisfy the start of oscillation condition, i.e.

$$X_L(\omega_0) = -X_{IN}(\omega_0)$$

2

$$R_L = \frac{R_0}{3}$$

5.3.1 Design 8 GHz GaAs FET

$$S_{11} = 0.98 \angle 163^\circ \quad S_{12} = 0.39 \angle -54^\circ$$

$$S_{21} = 0.675 \angle -161^\circ \quad S_{22} = 0.465 \angle 120^\circ$$

Soln

① $K = ?$ $K = 0.529$

② Draw Terminating stability circle

a) Calculate

$$D_S = \left| \frac{S_{12} S_{21}}{|S_{11}|^2 - |\Delta|^2} \right| \quad C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}$$

③ Choose a point in the unstable region

in the example we choose point A ($\Gamma_T = 1 \angle -163^\circ$)

The associated impedance $Z_T = -j7.5 \Omega$

④ With Z_T connected the i/p reflection coefficient is found:

$$\Gamma_{IN} = \frac{S_{11} - \Delta \Gamma_T}{1 - S_{22} \Gamma_T} \quad \Gamma_{IN} = S_{11} + \frac{S_{12} S_{21} \Gamma_T}{1 - S_{22} \Gamma_T}$$

⑤ Calculate $Z_{IN} = -58 - j2.6 \Omega$

⑥ Load matching N/w can be modeled as:

$$X_L(\omega_0) = -X_{IN}(\omega_0)$$

&

$$R_L = \frac{R_0}{3}$$

$$Z_L = 19 + j 2.6 \Omega$$

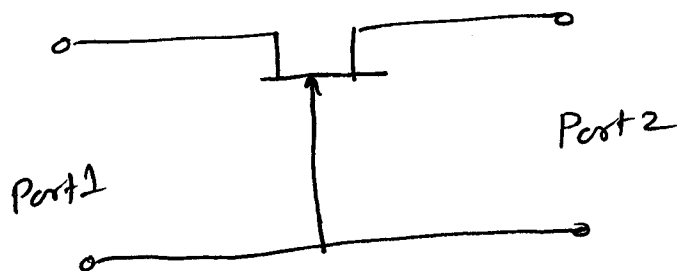
Example 2

$$S_{11} = 0.5 \angle -95^\circ$$

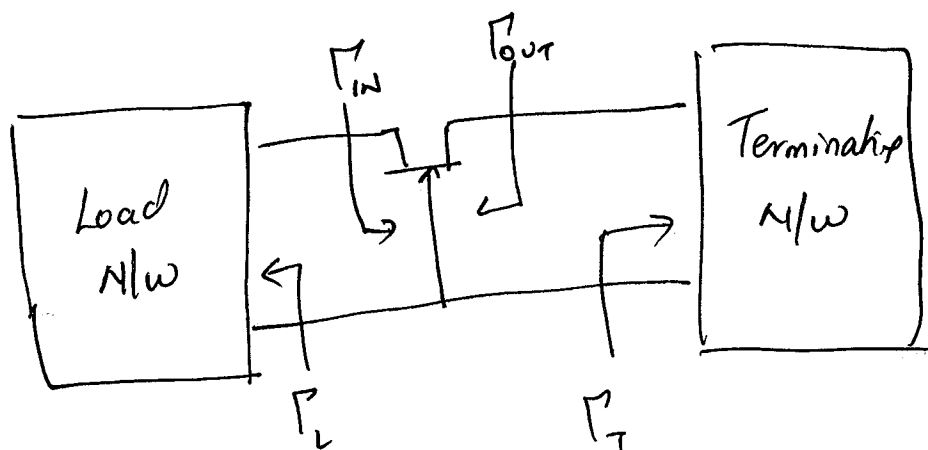
$$S_{21} = 1.1 \angle 44^\circ$$

$$S_{12} = 0.35 \angle -31^\circ$$

$$S_{22} = 0.8 \angle 46^\circ$$



$$f = 9 \text{ GHz}$$



Soln

① Calculate K & $|\Delta|$

$$K = 0.355$$

$$|\Delta| = 0.405$$

Since $K < 1$ the transistor is potentially unstable

(2) We use port 2 as the terminating port

(3) Calculate r_L & C_L

$$r_L = \left| \frac{S_{12} S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$$

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}$$

We get r_T & C_T

$$r_T = 0.808 \quad C_T = 1.493 \angle -59.8^\circ$$

(4) Any Γ_T in the shaded region will produce a negative resistance.

The reflection coefficient Γ_T is selected at a point for eq ($1 \angle -67.38^\circ$)

$$Z_T = -j75 \Omega$$

$$(5) \Gamma_{IN} = S_{11} + \frac{S_{12} S_{21} \Gamma_T}{1 - S_{22} \Gamma_T}$$

$$\Gamma_{IN} = 1.49 \angle -100.43^\circ$$

$$Z_{IN} = 16.25 - j38.95$$

$$X_{L'} - X_{IN} = 38.95 \Omega$$

$$R_{L'} = \frac{|R_{IN}|}{3} = \frac{16.25}{3} = 5.42 \Omega$$

For -ve resistance oscillators at microwave frequencies using BJTs the CB (Common base) configuration is normally used. For FETs, the CG configuration is commonly used.

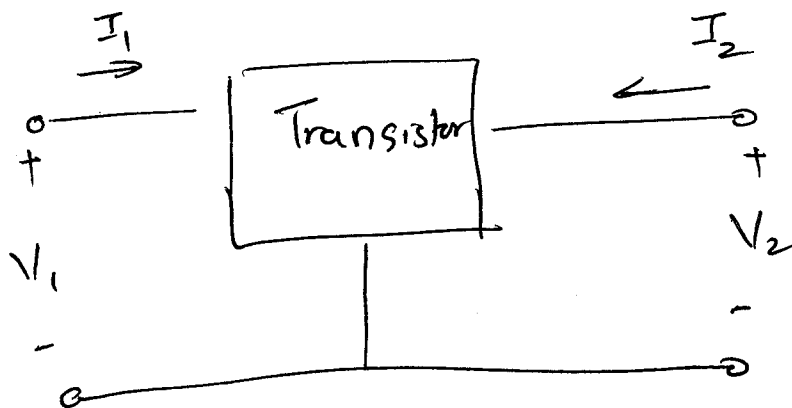
In these configurations the transistors are usually potentially unstable, and this is a very desirable condition in oscillator design.

- In -ve resistance oscillator, the capacitances of the transistor provide some or all of the feedback needed for oscillation.
- A properly designed series feedback N/w can significantly increase the -ve resistance presented by the two-port network.

The unstable region of the two-port N/w can be enhanced by the use of series feedback.

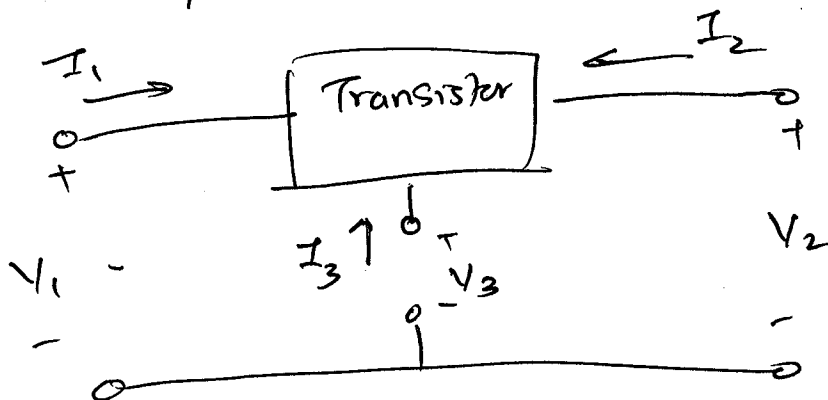
- For BJTs a series feedback inductor of a few nanohenres enhances the unstable region. For FETs either a series-feedback inductor or a series-feedback capacitor of a few picofarads enhances the unstable region.
- The purpose of a series-feedback element is to provide positive feedback making the configuration more unstable.

Method for designing series feedback N/w



[2 port N/w]

Let $[\hat{S}]$ be the 3-port scattering matrix & $[S]$ be the two-port-scattering matrix



The relation between 2-port & 3-port S-parameters can be written as:

$$\hat{S}_{11} = S_{11} + \frac{\sigma_{11}\sigma_{12}}{4-\sigma}$$

$$\hat{S}_{12} = S_{12} + \frac{\sigma_{11}\sigma_{21}}{4-\sigma}$$

$$\hat{S}_{13} = \frac{2\sigma_{11}}{4-\sigma}$$

$$\hat{S}_{21} = S_{21} + \frac{\sigma_{22}\sigma_{12}}{4-\sigma}$$

$$\hat{S}_{22} = S_{22} + \frac{\sigma_{22}\sigma_{21}}{4-\sigma}$$

$$\hat{S}_{23} = \frac{2\sigma_{22}}{4-\sigma}$$

$$\hat{S}_{31} = \frac{2\sigma_{12}}{4-\sigma}$$

$$\hat{S}_{32} = \frac{2\sigma_{21}}{4-\sigma}$$

$$\hat{S}_{33} = \frac{\sigma}{4-\sigma}$$

$$\sigma_{11} = 1 - S_{11} - S_{12}$$

$$\sigma_{12} = 1 - S_{11} - S_{21}$$

$$\sigma_{21} = 1 - S_{22} - S_{12}$$

$$\sigma_{22} = 1 - S_{22} - S_{21}$$

$$\sigma = S_{11} + S_{12} + S_{21} + S_{22}$$

&

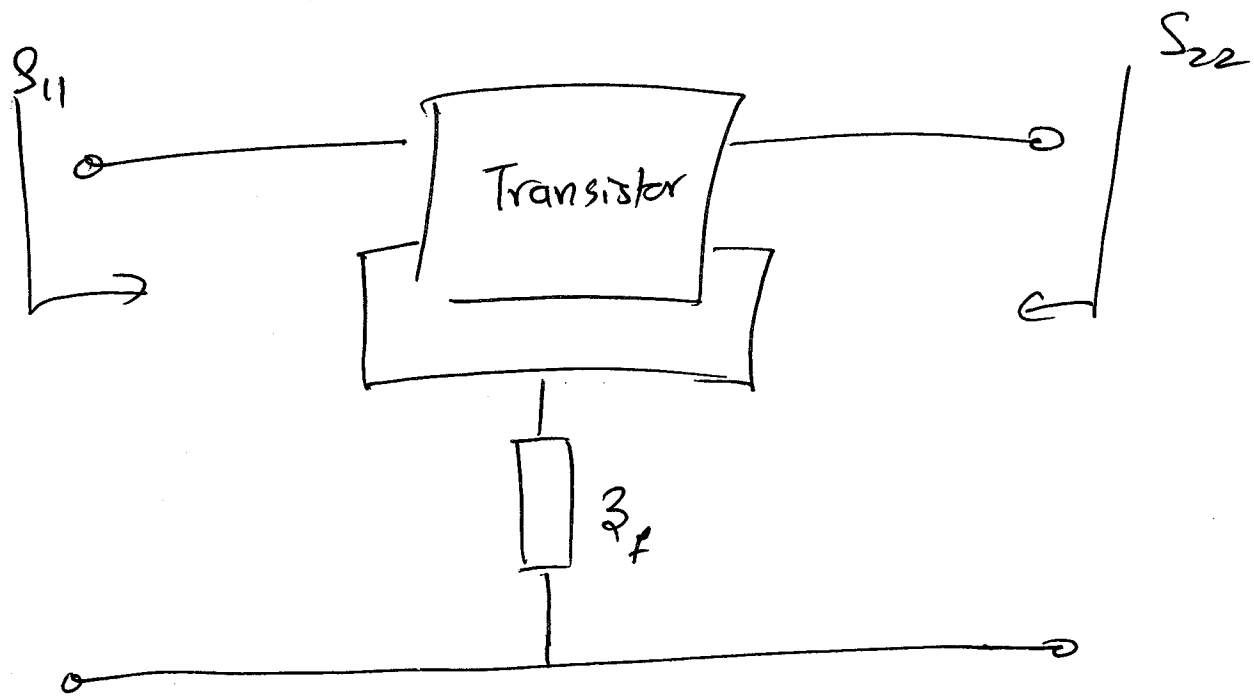
$$S_{11} = \hat{S}_{11} - \frac{\hat{S}_{13}\hat{S}_{31}}{1+\hat{S}_{33}}$$

$$S_{12} = \hat{S}_{12} - \frac{\hat{S}_{13}\hat{S}_{32}}{1+\hat{S}_{33}}$$

$$S_{21} = \hat{S}_{21} - \frac{\hat{S}_{23}\hat{S}_{31}}{1+\hat{S}_{33}}$$

$$S_{22} = \hat{S}_{22} - \frac{\hat{S}_{23}\hat{S}_{32}}{1+\hat{S}_{33}}$$

If a series feedback impedance is used, as shown in figure below



$$a_3 = \Gamma_f b_3$$

$1 + \hat{S}_{33}$ is replaced by $\hat{S}_{33} - \left(\frac{1}{\Gamma_f}\right)$

$$S_{11} = \frac{\Delta_1 \Gamma_f - \hat{S}_{11}}{\hat{S}_{33} \Gamma_f - 1}$$

$$S_{22} = \frac{\Delta_2 \Gamma_f - \hat{S}_{22}}{\hat{S}_{33} \Gamma_f - 1}$$

$$\Delta_1 = \hat{S}_{11} \hat{S}_{33} - \hat{S}_{13} \hat{S}_{31}$$

$$\Delta_2 = \hat{S}_{22} \hat{S}_{33} - \hat{S}_{23} \hat{S}_{32}$$

The mapping of $|\Gamma_f| \geq 1$ onto the S_{11} and S_{22} planes have centers & radii given by

$$C_1' = \frac{\hat{S}_{11} - \Delta_1 \hat{S}_{33}^*}{1 - |\hat{S}_{33}|^2}$$

$$r_1' = \frac{|\hat{S}_{13} \hat{S}_{31}|}{|1 - |\hat{S}_{33}|^2|}$$

&

$$C_2' = \frac{\hat{S}_{22} - \Delta_2 \hat{S}_{33}^*}{1 - |\hat{S}_{33}|^2}$$

$$r_2' = \frac{|\hat{S}_{23} \hat{S}_{32}|}{|1 - |\hat{S}_{33}|^2|}$$

for the S_{11} and S_{22} planes

The maximum value of $|S_{11}|$ occurs at a point where

$$S_{11}(\max) = (|C_1'| + r_1') L_{C_1}'$$

$r_{1}(\max)$ is given by

$$z_f = jx_{1(\max)} = \frac{\Delta_1 + \hat{S}_{11} - S_{11(\max)}(1 + \hat{S}_{33})}{\Delta_1 - \hat{S}_{11} + S_{11(\max)}(1 - \hat{S}_{33})}$$

To properly orient the mapping, the $z_f = \infty$ point maps onto the point

$$S_{11} = \frac{\Delta_1 - \hat{S}_{11}}{\hat{S}_{33} - 1}$$

& $z_f = 0$ maps onto the point

$$S_{11} = \frac{\Delta_1 + \hat{S}_{11}}{\hat{S}_{33} + 1}$$

|||^{ly} we obtain

$$S_{22(\max)} = (|C_2'| + r_2') L_{C_2}'$$

$$z_f = jx_{2(\max)} = \frac{\Delta_2 + \hat{S}_{22} - S_{22(\max)}(1 + \hat{S}_{33})}{\Delta_2 - \hat{S}_{22} + S_{22(\max)}(1 - \hat{S}_{33})}$$

& $z_f = \infty$ point maps onto

$$S_{22} = \frac{\Delta_2 - \hat{S}_{22}}{\hat{S}_{33} - 1}$$

and $\Gamma_f = 0$ point maps to

$$S_{22} = \frac{\Delta_2 + \hat{S}_{22}}{\hat{S}_{33} + 1}$$

Design Procedure

1. Convert 2 port S parameter to 3-port

2. Calculate C_1' , r_1' , C_2' & r_2'

3. Calculate $S_{11(\max)}$, $\Gamma_{1(\max)}$, $S_{22(\max)}$ & $\Gamma_{2(\max)}$

4. Plot mapping of the Γ_f plane onto the S_{11} & S_{22} planes.

5. Select appropriate Γ_f value.

eg:-

Design a 2.75 GHz oscillator using a BJT in common-base configuration. The S-parameters at 2.75 GHz, at a given Q point are.

$$S_{11} = 0.9 \angle 150^\circ$$

$$S_{21} = 1.7 \angle -80^\circ$$

$$S_{12} = 0.07 \angle 120^\circ$$

$$S_{22} = 1.08 \angle -56^\circ$$

Soln

1) Calculate K ($K = -0.64$)

We can use feedback to increase the region of instability with $L = 1.45 \text{ nH}$

The new S-parameters are

$$S_{11} = 1.72 \angle 100^\circ$$

$$S_{21} = 2.08 \angle -136^\circ$$

$$S_{12} = 0.712 \angle 94^\circ$$

$$S_{22} = 1.16 \angle -102^\circ$$

Either port can be used as the terminating port.

The emitter-to-ground port was selected for this example for the load matching N/W; and the collector to ground port for the terminating N/W.

- From the Smith chart we observe that the center of the Smith chart is unstable. Hence a 50Ω termination will ensure $|\Gamma_{in}| > 1$.

Although this is simple there are a couple of issues:

1) For associated value of Γ_{in} the required value of Γ_L for the oscillator might be difficult to implement.

2) Some tuning capabilities are needed in the matching N/W to attain the desired frequency of oscillation.

The values of Γ_{in} and Γ_T are related by:

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_T}{1 - S_{22}\Gamma_T}$$

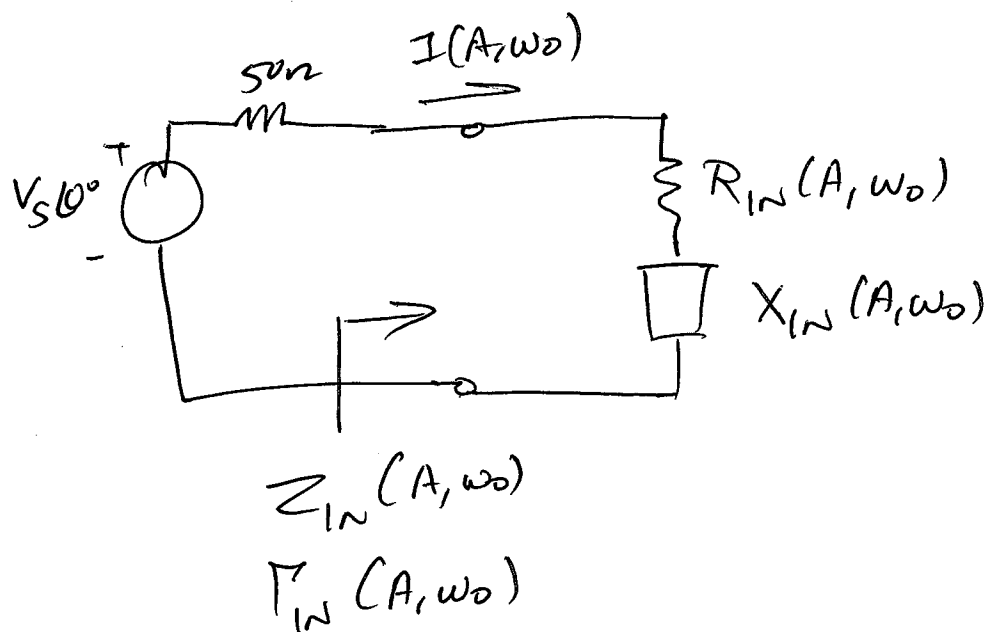
One way ^{is} of choosing Γ_T in the unstable region & calculating the associated value of Γ_{in}

Oscillator Design Using Large-Signal Measurements

- The previous design methods provide good practical results. However there is no assurance that the oscillator power is optimum.
- Here we develop a method that optimizes the oscillator power, based on large signal measurements.
- The method consists of designing the terminating N/w so that the two port presents a negative resistance at the i/p port
- The resulting one port negative resistance N/w can be placed in a nonoscillating circuit and the optimum load impedance as a function of power can be measured.
- The terminating N/w is designed so that Z_{in} presents -ve impedance at the load port that has a real part with magnitude smaller than 50Ω
- This is necessary because the equipment used to measure the large signal characteristics of the circuit has a 50Ω source impedance, and the total loop resistance must be positive in order to avoid oscillations during measurements

The large signal characterization is achieved, by measuring in the circuit shown below, the current amplitude and the impedance $Z_{IN}(A, \omega_0)$, as V_s is varied. The measurements are made at the desired frequency of oscillation ω_0 , and the source resistance is typically 50Ω .

With $|R_{IN}(A, \omega_0)| < 50 \Omega$ the ckt is stable.



$$I(A, \omega_0) = \frac{V_s}{R_s + R_{IN}(A, \omega_0) + jX_{IN}(A, \omega_0)}$$

$$P_D(A, \omega_0) = \frac{1}{2} |I(A, \omega_0)|^2 |R_{IN}(A, \omega_0)|$$

measurement of $P_D(A, \omega_0)$ versus $Z_{IN}(A, \omega_0)$
 generates the large signal characteristics of the one port n/w

If the one port is now terminated in the load impedance

$$Z_L(\omega_0) = -Z_{in}(A, \omega_0)$$

The power delivered to Z_L is given by $P_L(A, \omega_0) = P_D(A, \omega_0)$

Measuring $I(A, \omega_0)$ at microwave frequencies is difficult

The reflection coefficient $\Gamma_{in}(A, \omega_0)$ as a function of the available power from the source is measured. The available power from the source is given by

$$P_{avs} = \frac{V_s^2}{8R_s}$$

The power added P_{ADD} (ie reflected power - available i/p power) is given by

$$P_{ADD} = P_{avs} (|\Gamma_{in}|^2 - 1)$$

$$P_{ADD}(A, \omega_0) = \frac{V_s^2 |R_{in}(A, \omega_0)|}{2 [(R_{in}(A, \omega_0) + R_s)^2 + X_{in}^2]}$$

$$\text{ie } P_{ADD}(A, \omega_0) = \frac{1}{2} |I(A, \omega_0)|^2 |R_{in}(A, \omega_0)|$$

which shows that the added power is the power that the one-port N/w will deliver to load $Z_L(\omega_0) = -Z_{IN}(\omega_0)$

The design procedure uses the small-signal S parameters to establish the terminating impedance that results in a -ve resistance one-port N/w.

Then the one-port oscillator performance is described by the measured large-signal characteristics

$$\Gamma_{IN} = \frac{S_{11} - \Delta \Gamma_T}{1 - S_{22} \Gamma_T}$$

describes the mapping of Γ_T plane into Γ_{IN} plane.

The mapping of the \odot $|\Gamma_T| = 1$ into the Γ_{IN} plane gives information about the passive impedance at the terminating port (i.e. $|\Gamma_T| < 1$) that will make

$$|\Gamma_{IN}| > 1$$

$$C_{IN} = \frac{S_{11} - \Delta S_{22}^{\dagger}}{1 - |S_{22}|^2}$$

$$g_{IN} = \frac{|S_{12} S_{21}|}{|1 - |S_{22}|^2|}$$

Largest (P_{IN}) is obtained when we select value of P_{IN} on $|P_T| = 1 \odot$ that has the same phase as C_{IN} .
The value of P_T associated with $P_{IN, \max}$ produces the largest negative resistance for Z_{IN} , which can be realized using a passive terminating impedance.

The magnitude of $P_{IN, \max}$ at 'a' is

$$|P_{IN}| = |C_{IN}| + r_{IN}$$

phase

$$\angle P_{IN} = \angle C_{IN}$$

$$P_{IN, \max} = (|C_{IN}| + r_{IN}) \angle C_{IN}$$

value of P_T at point a, .

$$P_{T,0} = \frac{P_{IN, \max} - S_{11}}{P_{IN, \max} S_{22} - \Delta}$$