ECE 3600 Lumped-Parameter Transmission Line models

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Long-length Lines: over 200 miles (320 km)

Need: line length: len m or km

Resistance per unit length: R $\frac{\Omega}{m}$ or $\frac{\Omega}{km}$ Usually Assume: $G := 0 \cdot \frac{S}{km}$ $S := \text{siemens}$

Inductance per unit length: L $\frac{H}{m}$ or $\frac{H}{km}$

Capacitance per unit length: C $\frac{F}{m}$ or $\frac{F}{km}$

OR

Inductive reactance per unit length: ωL $\frac{\Omega}{m}$ or $\frac{\Omega}{km}$

Capacitance admittance per unit length: ωC $\frac{S}{m}$ or $\frac{S}{km}$

stick to the same unit length fo all parameters

A calculator with hyperbolic trig functions

Find:

Surge impedance: $\mathbf{Z_c} = \sqrt{\frac{j\cdot\omega\cdot L + R}{j\cdot\omega\cdot C + G}}$ Ω

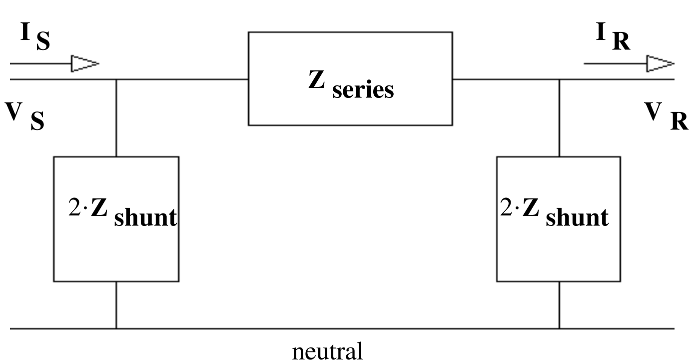
Propagation constant: $\gamma = \sqrt{(j\cdot\omega\cdot L + R)\cdot(j\cdot\omega\cdot C + G)}$ $\frac{1}{m}$ or $\frac{1}{km}$

Series impedance $\mathbf{Z_{series}} = \mathbf{Z_c}\cdot \sinh(\gamma\cdot len)$ Ω

Shunt admittance: $\frac{\mathbf{Y_{shunt}}}{2} = \frac{\tanh\left(\frac{\gamma\cdot len}{2}\right)}{\mathbf{Z_c}}$ S or $\frac{1}{\Omega}$

OR

Shunt impedance: $2\cdot\mathbf{Z_{shunt}} = \frac{\mathbf{Z_c}}{\tanh\left(\frac{\gamma\cdot len}{2}\right)}$ Ω



Model

Medium-length Lines: 100 to 200 miles (160 - 320 km)

Need: line length: len $\quad m$ or km

Resistance per unit length: $R \quad \frac{\Omega}{m}$ or $\frac{\Omega}{km}$ $\qquad$ Usually Assume: $G := 0 \cdot \frac{S}{km}$ $\qquad S := \text{siemens}$

Inductance per unit length: $L \quad \frac{H}{m}$ or $\frac{H}{km}$

Capacitance per unit length: $C \quad \frac{F}{m}$ or $\frac{F}{km}$

OR

Inductive reactance per unit length: $\omega L \quad \frac{\Omega}{m}$ or $\frac{\Omega}{km}$

Capacitance admittance per unit length: $\omega C \quad \frac{S}{m}$ or $\frac{S}{km}$ $\qquad$ stick to the same unit length fo all parameters

Find:

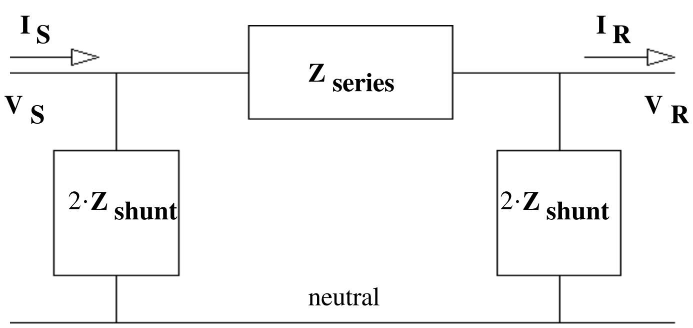
Series Resistance: $R_{line} = R \cdot len \qquad \Omega$

Series impedance $\mathbf{Z}_{\mathbf{series}} = (R + j \cdot \omega \cdot L) \cdot len \qquad \Omega$

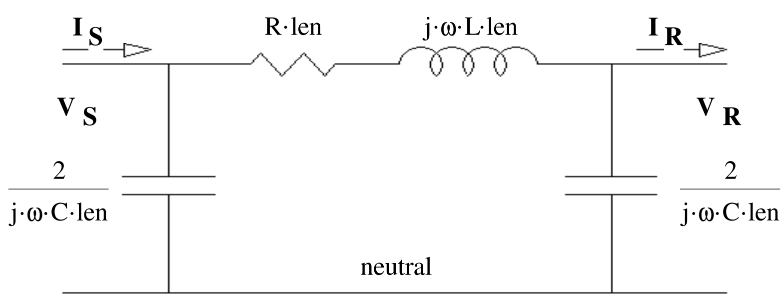
Shunt admittance: $\frac{\mathbf{Y}_{\mathbf{shunt}}}{2} = j \cdot \omega \cdot C \cdot \frac{len}{2} \qquad$ S or $\frac{1}{\Omega}$

OR

Shunt impedance: $2 \cdot \mathbf{Z}_{\mathbf{shunt}} = \frac{2}{j \cdot \omega \cdot C \cdot len} \qquad \Omega$

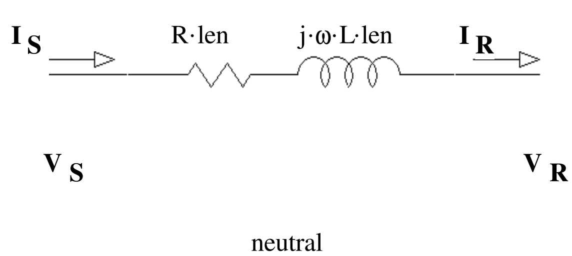


OR:



Short-length Lines: less than 100 miles (160km)

Same as above but without the capacitors



Ex1. A 500 kV transmission line is 500 km long and has line parameters shown in Table 4.1, p.4-10. Use the long-length model to find $\mathbf{V_S}$ and $\mathbf{I_S}$ if the line is loaded to 1.4 SIL and $|\mathbf{V_{RLL}}|$ is 490 kV. Assume the phase angle of $\mathbf{V_R}$ is 0° and assume load pf = 1.

$$len := 500\cdot km \qquad V_{RLL} := 490\cdot kV \qquad \mathbf{V_R} := \frac{V_{RLL}}{\sqrt{3}}$$

From Table 4-1, p.4-10

$$R := 0.029\cdot\frac{\Omega}{km} \qquad \text{Assume:} \quad G := 0\cdot\frac{S}{km}$$

$$\omega L := 0.326\cdot\frac{\Omega}{km} \qquad \omega C := 5.220\cdot 10^{-6}\cdot\frac{S}{km}$$

Long-length line model:

Surge Impedance: $\mathbf{Z_c} := \sqrt{\dfrac{j\cdot\omega L + R}{j\cdot\omega C + G}}$ $\qquad \mathbf{Z_c} = 250.151 - 11.104j\ \cdot\Omega$

Propagation constant: $\gamma := \sqrt{(j\cdot\omega L + R)\cdot(j\cdot\omega C + G)}$ $\qquad \gamma = 5.797\cdot10^{-5} + 1.306\cdot10^{-3}j\ \cdot\dfrac{1}{km}$

Not used in my solution

$\beta := \mathrm{Im}(\gamma)$ $\qquad \beta = 1.306\cdot10^{-6}\ \cdot m^{-1}$

$\alpha := \mathrm{Re}(\gamma)$ $\qquad \alpha = 5.797\cdot10^{-8}\ \cdot m^{-1}$

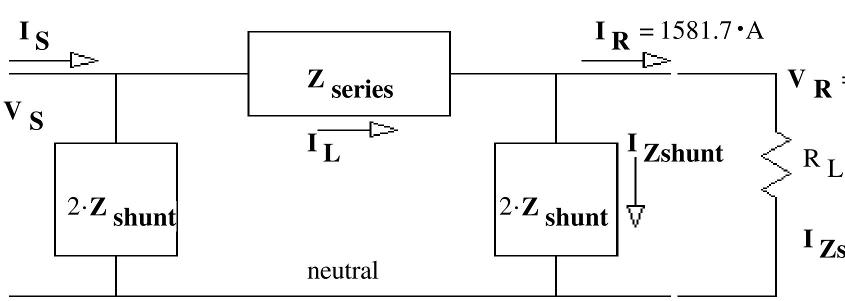
Series impedance: $\mathbf{Z_{series}} := \mathbf{Z_c}\cdot\sinh(\gamma\cdot len)$ $\qquad \mathbf{Z_{series}} = 12.508 + 151.772j\ \cdot\Omega$

Shunt admittance: Not used in my solution $\quad \mathbf{Y_{shunt}} := 2\cdot\dfrac{\tanh\left(\dfrac{\gamma\cdot len}{2}\right)}{\mathbf{Z_c}}$ $\qquad \dfrac{\mathbf{Y_{shunt}}}{2} = 4.49\cdot10^{-6} + 1.353\cdot10^{-3}j\ \cdot S$

Shunt impedance: $\mathbf{Z_{shunt}} := \dfrac{\mathbf{Z_c}}{2\cdot\tanh\left(\dfrac{\gamma\cdot len}{2}\right)}$ $\qquad 2\cdot\mathbf{Z_{shunt}} = 2.451 - 738.924j\ \cdot\Omega$

load resistance for SIL $\quad R_L := |\mathbf{Z_c}|$

$$SIL := \frac{{V_{RLL}}^2}{R_L} \qquad SIL = 959\cdot MVA \qquad \mathbf{I_R} := \frac{\left(\dfrac{1.4\cdot SIL}{3}\right)}{\left(\dfrac{V_{RLL}}{\sqrt{3}}\right)} \qquad \mathbf{I_R} = 1581.7\cdot A$$



$\mathbf{V_R} = 282.902\cdot kV$

$$\mathbf{I_{Zshunt}} := \frac{\mathbf{V_R}}{2\cdot\mathbf{Z_{shunt}}}$$

$\mathbf{I_{Zshunt}} = 1.27 + 382.852j\ \cdot A$

$\mathbf{I_L} := \mathbf{I_{Zshunt}} + \mathbf{I_R}$ $\qquad \mathbf{I_L} = 1.583\cdot10^{3} + 382.852j\ \cdot A$

$\mathbf{V_S} := \mathbf{V_R} + \mathbf{I_L}\cdot\mathbf{Z_{series}}$ $\qquad \mathbf{V_S} = 2.446\cdot10^{5} + 2.45\cdot10^{5}j\ \cdot V$ $\qquad |\mathbf{V_S}| = 346.228\cdot kV$ $\qquad \arg(\mathbf{V_S}) = 45.052\cdot deg$

$\mathbf{I_{ZshuntS}} := \dfrac{\mathbf{V_S}}{2\cdot\mathbf{Z_{shunt}}}$ $\qquad \mathbf{I_{ZshuntS}} = -330.521 + 332.113j\ \cdot A$ $\qquad \left|\sqrt{3}\cdot\mathbf{V_S}\right| = 599.685\cdot kV$

$\mathbf{I_S} := \mathbf{I_{ZshuntS}} + \mathbf{I_L}$ $\qquad \mathbf{I_S} = 1.252\cdot10^{3} + 714.966j\ \cdot A$ $\qquad |\mathbf{I_S}| = 1442\cdot A$ $\qquad \arg(\mathbf{I_S}) = 29.719\cdot deg$

Ex 2. A 500 kV transmission line is 220 km long and has line parameters shown in Table 4.1, p.4-10. Use the medium-length model to find $\mathbf{V_S}$ and $\mathbf{I_S}$ if the line is loaded to 0.9 SIL and $|\mathbf{V_{RLL}}|$ is 510 kV. Assume the phase angle of $\mathbf{V_R}$ is 0^o and and assume load pf = 1

$len := 220 \cdot km \qquad V_{RLL} := 510 \cdot kV \qquad \mathbf{V_R} := \frac{V_{RLL}}{\sqrt{3}}$

From Table 4-1, p.4-10 $\quad R := 0.029 \cdot \frac{\Omega}{km}$ $\qquad$ Assume: $G := 0 \cdot \frac{S}{km}$

$\omega L := 0.326 \cdot \frac{\Omega}{km} \qquad \omega C := 5.220 \cdot 10^{-6} \cdot \frac{S}{km}$

Medium-length line model:

Surge Impedance: $\mathbf{Z_c} := \sqrt{\frac{\omega L}{\omega C}}$ $\qquad \mathbf{Z_c} = 249.9 \cdot \Omega$

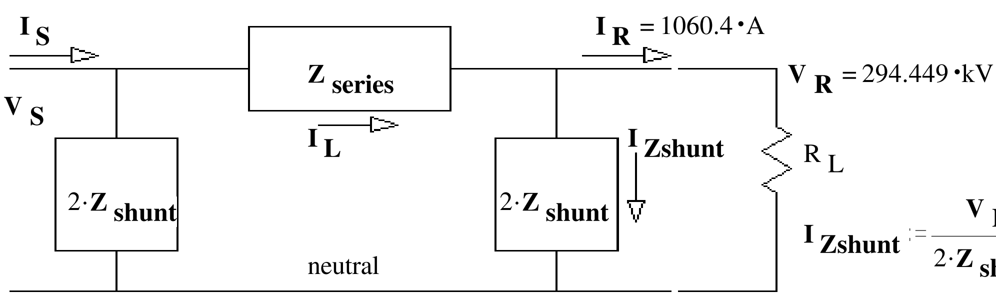
Series impedance: $\mathbf{Z_{series}} := (R + j \cdot \omega L) \cdot len$ $\qquad \mathbf{Z_{series}} = 6.38 + 71.72j \cdot \Omega$

Shunt admittance: $\mathbf{Y_{shunt}} := j \cdot \omega C \cdot len$ $\qquad \frac{\mathbf{Y_{shunt}}}{2} = 5.742 \cdot 10^{-4} j \cdot S$
Not used in my solution

Shunt impedance: $\mathbf{Z_{shunt}} := \frac{1}{j \cdot \omega C \cdot len}$ $\qquad 2 \cdot \mathbf{Z_{shunt}} = -1.742 \cdot 10^{3} j \cdot \Omega$

load resistance for SIL $\qquad R_L := |\mathbf{Z_c}|$

$SIL := \frac{V_{RLL}^2}{R_L} \qquad SIL = 1041 \cdot MVA \qquad \mathbf{I_R} := \frac{\left(\frac{0.9 \cdot SIL}{3}\right)}{\left(\frac{V_{RLL}}{\sqrt{3}}\right)} \qquad \mathbf{I_R} = 1060.4 \cdot A$



$\mathbf{I_{Zshunt}} := \frac{\mathbf{V_R}}{2 \cdot \mathbf{Z_{shunt}}}$

$\mathbf{I_{Zshunt}} = 169.072j \cdot A$

$\mathbf{I_L} := \mathbf{I_{Zshunt}} + \mathbf{I_R} \qquad \mathbf{I_L} = 1.06 \cdot 10^{3} + 169.072j \cdot A$

$\mathbf{V_S} := \mathbf{V_R} + \mathbf{I_L} \cdot \mathbf{Z_{series}} \qquad \mathbf{V_S} = 2.891 \cdot 10^{5} + 7.713 \cdot 10^{4} j \cdot V \qquad |\mathbf{V_S}| = 299.201 \cdot kV \qquad \arg(\mathbf{V_S}) = 14.939 \cdot deg$

$|\sqrt{3} \cdot \mathbf{V_S}| = 518.232 \cdot kV$

$\mathbf{I_{ZshuntS}} := \frac{\mathbf{V_S}}{2 \cdot \mathbf{Z_{shunt}}} \qquad \mathbf{I_{ZshuntS}} = -44.289 + 165.994j \cdot A$

$\mathbf{I_S} := \mathbf{I_{ZshuntS}} + \mathbf{I_L} \qquad \mathbf{I_S} = 1.016 \cdot 10^{3} + 335.067j \cdot A \qquad |\mathbf{I_S}| = 1070 \cdot A \qquad \arg(\mathbf{I_S}) = 18.25 \cdot deg$