

Transformer Examples

b

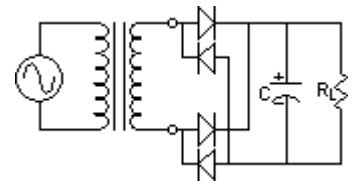
Ex.1 a) An ideal transformer has 360 turns on the primary winding and 36 turns on the secondary. If the primary is connected across a 120 V (rms) generator, what is the rms output voltage?

$$120 \cdot \text{volt} \cdot \frac{36}{360} = 12 \cdot \text{volt}$$

b) If you used a full-wave rectifier and a capacitor to make a DC power supply with this transformer, what DC voltage should you get?

$$12 \cdot \text{V} \cdot \sqrt{2} - 2 \cdot 0.7 \cdot \text{V} = 15.6 \cdot \text{V} \quad \text{less under load}$$

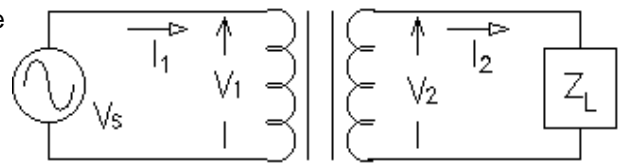
peak 2 diodes



Ex.2 A transformer has $N_1 = 320$ turns and $N_2 = 1000$ turns. If the input voltage is $v(t) = (255 \text{ V})\cos(\omega t)$, what rms voltage is developed across the secondary coil?

$$\frac{255 \cdot \text{volt}}{\sqrt{2}} \cdot \frac{1000}{320} = 563 \cdot \text{volt}$$

Ex.3 A transformer is rated at 480V / 120V, 1.2kVA. Assume the transformer is ideal and all voltages and currents are RMS.



a) What is the current rating of the primary?

$$\frac{1.2 \cdot \text{kVA}}{480 \cdot \text{V}} = 2.5 \cdot \text{A}$$

$$\begin{aligned} |Z_L| &= 20 \cdot \Omega \\ \text{pf} &:= 75\% \quad \text{lagging} \\ V_L &:= 110 \cdot \text{V} \end{aligned}$$

b) What is the current rating of the secondary?

$$\frac{1.2 \cdot \text{kVA}}{120 \cdot \text{V}} = 10 \cdot \text{A}$$

c) The secondary has 100 turns of wire. How many turns does the primary have?

$$N_2 := 100 \quad N_1 := \frac{480 \cdot \text{V}}{120 \cdot \text{V}} \cdot N_2 \quad N_1 = 400 \text{ turns}$$

d) $V_L := 110 \cdot \text{V}$ How big is the source voltage ($|V_S|$)?

$$V_S := \frac{N_1}{N_2} \cdot V_L \quad V_S = 440 \cdot \text{V}$$

e) The secondary load (Z_L) has a magnitude of 20Ω at a power factor of 75%. Find the secondary current, I_2 (magnitude **and angle**).

$$\text{pf} := 75\%$$

$$I_2 = \frac{V_L}{20 \cdot \Omega} = 5.5 \cdot \text{A} \quad \text{pf} = 0.75 \quad \arccos(\text{pf}) = 41.4 \cdot \text{deg} \quad I_2 = 5.5 \text{ A } \underline{-41.4^\circ}$$

f) Find the primary current, I_1 (magnitude **and angle**).

$$I_1 = \frac{100}{400} \cdot 5.5 \cdot \text{A} = 1.375 \cdot \text{A} \quad \arccos(\text{pf}) = 41.4 \cdot \text{deg} \quad I_1 = 1.375 \text{ A } \underline{-41.4^\circ}$$

Transformer is ideal, so angle is exactly the same as the load.

g) How much average power does the load dissipate?

$$P_L = |V_2| \cdot |I_2| \cdot \text{pf} = 110 \cdot \text{V} \cdot 5.5 \cdot \text{A} \cdot 75\% = 453.8 \cdot \text{watt}$$

h) How much average power does the power source (V_S) supply?

$$P_S = P_L = 454 \cdot \text{watt}$$

i) What is the load as seen by V_S ? (magnitude **and angle**)

$$\left(\frac{400}{100}\right)^2 \cdot 20 \cdot \Omega = 320 \cdot \Omega \quad \arccos(\text{pf}) = 41.4 \cdot \text{deg} \quad Z_{eq} = 320 \Omega \underline{41.4^\circ}$$

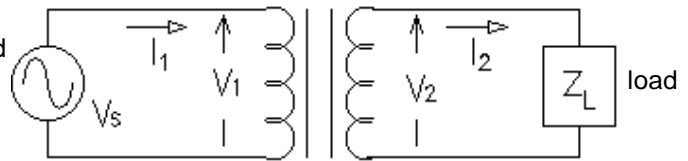
$$\text{OR: } \frac{440 \cdot \text{V}}{1.375 \cdot \text{A}} = 320 \cdot \Omega \underline{0 - -41.4^\circ}$$

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Ex.4 A transformer is rated at 480V/240V, 1.2kVA.

Assume the transformer is ideal and all voltages and currents are RMS.

How much power does the load consume?



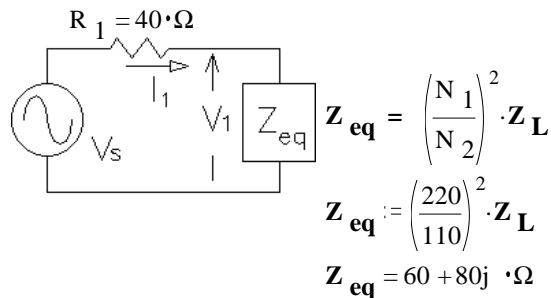
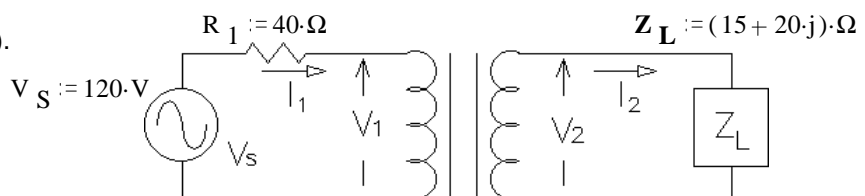
$$\begin{aligned} \mathbf{V}_L &:= \mathbf{V}_S \cdot \left(\frac{240}{480} \right) & |\mathbf{V}_L| &= 220 \cdot \text{V} & |\mathbf{V}_S| &= 440 \cdot \text{V} & |\mathbf{Z}_L| &= 16 \cdot \Omega \\ & & & & & & \text{pf} &:= 80\% \quad \text{lagging} \\ I_2 &:= \frac{|\mathbf{V}_L|}{|\mathbf{Z}_L|} & P_L &:= |\mathbf{V}_L| \cdot I_2 \cdot \text{pf} & P_L &= 2.42 \cdot \text{kW} \end{aligned}$$

Ex.5 The transformer shown in the circuit below is ideal. It is rated at 220/110 V, 200 VA, 60 Hz

Find the following:

a) The primary current (magnitude).

$$|\mathbf{I}_1| = ?$$



$$\mathbf{R}_1 + \mathbf{Z}_{eq} = 100 + 80j \cdot \Omega$$

$$\begin{aligned} I_1 &:= \frac{V_S}{\sqrt{100^2 + 80^2} \cdot \Omega} & I_1 &= 0.937 \cdot \text{A} \end{aligned}$$

b) The primary voltage (magnitude).

$$|\mathbf{V}_1| = ?$$

$$\begin{aligned} V_1 &:= I_1 \cdot \sqrt{60^2 + 80^2} \cdot \Omega & V_1 &= 93.7 \cdot \text{V} \end{aligned}$$

c) The secondary voltage (magnitude).

$$|\mathbf{V}_2| = ?$$

$$V_2 = \frac{110}{220} \cdot V_1 = 46.85 \cdot \text{V}$$

d) The power supplied by the source.

$$P_S = ?$$

$$P_S = I_1^2 \cdot 100 \cdot \Omega = 87.8 \cdot \text{W}$$

e) Is this transformer operating within its ratings? Show your evidence.

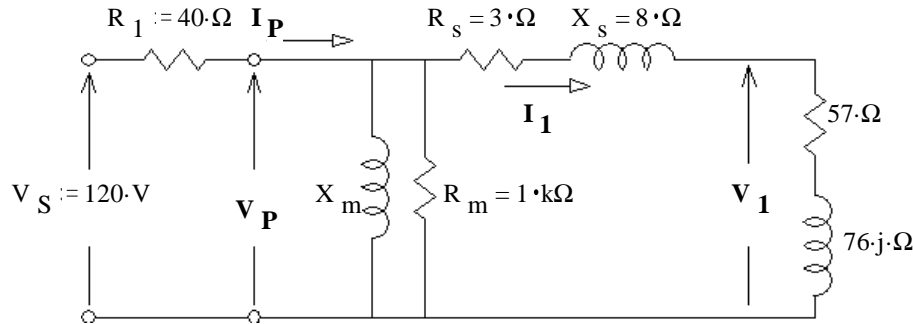
$$\begin{aligned} I_{1\max} &= \frac{200 \cdot \text{VA}}{220 \cdot \text{V}} = 0.909 \cdot \text{A} < I_1 = 0.937 \cdot \text{A} \\ &\text{NO} \end{aligned}$$

ALWAYS CHECK CURRENT

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Ex.6 Repeat Ex.5 with a non-ideal transformer whose characteristics are shown below.

$$R_m := 1 \cdot k\Omega \quad X_m := 400 \cdot \Omega \quad R_s := 3 \cdot \Omega \quad X_s := 8 \cdot \Omega \quad N := 1.95$$



$$Z_{eq} := (1.95)^2 \cdot (15 + 20j) \cdot \Omega$$

$$Z_{eq} = 57.037 + 76.05j \cdot \Omega$$

Find the following:

a) The primary current (magnitude).

$$|I_P| = ?$$

$$R_s + X_s \cdot j + Z_{eq} = 60.037 + 84.05j \cdot \Omega$$

$$\frac{1}{\frac{1}{X_m \cdot j} + \frac{1}{R_m} + \frac{1}{(60 + 84j) \cdot \Omega}} = 43.689 + 68.412j \cdot \Omega$$

$$R_1 + (43.689 + 68.412j) \cdot \Omega = 83.689 + 68.412j \cdot \Omega$$

$$I_P := \frac{120 \cdot V}{(83.689 + 68.412j) \cdot \Omega}$$

$$I_P = 0.86 - 0.703j \cdot A$$

$$|I_P| = 1.11 \cdot A$$

b) The primary voltage (magnitude).

$$|V_P| = ?$$

$$V_P := I_P \cdot (43.689 + 68.412j) \cdot \Omega$$

$$V_P = 85.619 + 28.105j \cdot V$$

$$|V_P| = 90.114 \cdot V$$

c) The secondary voltage (magnitude).

$$|V_2| = ?$$

$$I_1 := \frac{V_P}{(60 + 84j) \cdot \Omega}$$

$$I_1 = 0.704 - 0.517j \cdot A$$

$$V_1 := I_1 \cdot (57 + 76j) \cdot \Omega$$

$$V_1 = 79.375 + 24.026j \cdot V$$

$$|V_1| = 82.931 \cdot V$$

$$|V_2| = \frac{1}{N} \cdot 82.931 \cdot V = 42.529 \cdot V$$

OR, simply:

$$I_1 := \frac{90.114 \cdot V}{\sqrt{60^2 + 84^2} \cdot \Omega}$$

$$I_1 = 0.873 \cdot A$$

$$V_2 = \frac{I_1 \cdot \sqrt{57^2 + 76^2} \cdot \Omega}{1.95} = 42.529 \cdot V$$

d) The power supplied by the source.

$$P_S = ?$$

$$P_S = 120 \cdot V \cdot \text{Re}(I_P) = 103.14 \cdot W$$

e) Is this transformer operating within its ratings? Show your evidence.

$$I_{2\max} = \frac{200 \cdot VA}{110 \cdot V} = 1.818 \cdot A > |I_1| \cdot N = 1.702 \cdot A$$

YES

f) Find the efficiency, assuming that the only useful output is from Z_L .

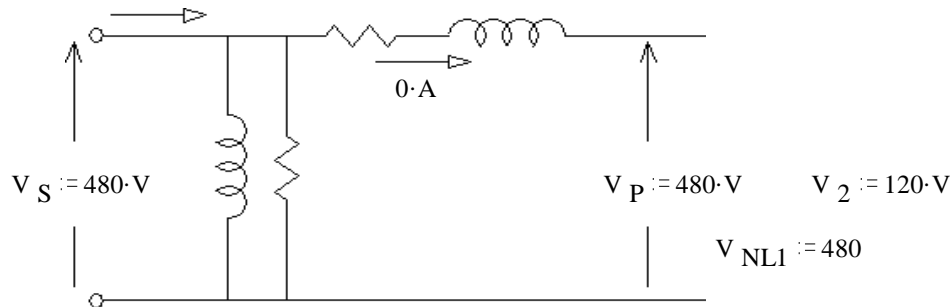
$$\eta = \frac{(|I_1|)^2 \cdot 57 \cdot \Omega}{103.14 \cdot W} \cdot 100\% = 42.12\%$$

Ex.7 Find the voltage regulation and full-load efficiency of the transformer with the following ratings and characteristics.

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Rated at 480/120 V, 2 kVA, 60 Hz $R_m := 8.4 \cdot \text{k}\Omega$ $X_m := 2 \cdot \text{k}\Omega$ $R_s := 5 \cdot \Omega$ $X_s := 15 \cdot \Omega$
 $V_{\text{Prated}} := 480 \cdot \text{V}$ $V_{\text{Srated}} := 120 \cdot \text{V}$ $S_{\text{rated}} := 2000 \cdot \text{VA}$

No load:

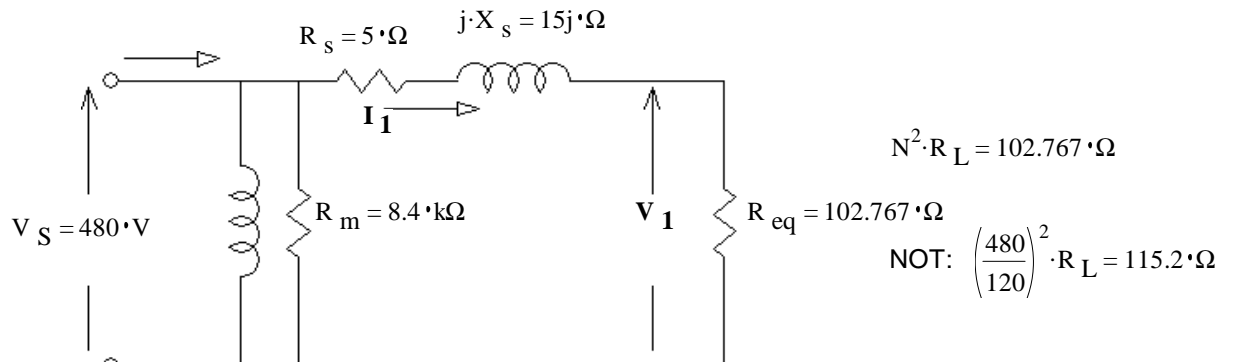


Full load, assuming $\text{pf} = 1$, which would be the manufacturer's assumption in order to get a consistent number for %VR:

$$R_L := \frac{(120 \cdot \text{V})^2}{2000 \cdot \text{W}} \quad R_L = 7.2 \cdot \Omega$$

Find the actual turns ratio: (See page 3 of notes)

$$R_x := \frac{V_{\text{Prated}}^2}{S_{\text{rated}}} - 2 \cdot R_s \quad R_{\text{eq}} := \frac{R_x + \sqrt{R_x^2 - 4 \cdot (R_s^2 + X_s^2)}}{2} \quad \text{Turns ratio } N := \sqrt{\frac{R_{\text{eq}}}{R_L}} \quad N = 3.778$$



$$N^2 \cdot R_L = 102.767 \cdot \Omega$$

$$\text{NOT: } \left(\frac{480}{120} \right)^2 \cdot R_L = 115.2 \cdot \Omega$$

OR, simply:

$$I_1 := \frac{V_S}{R_{\text{eq}} + R_s + 15j \cdot \Omega}$$

$$I_1 = 4.369 - 0.608j \cdot \text{A}$$

$$I_1 := \frac{V_S}{\sqrt{(R_{\text{eq}} + R_s)^2 + X_s^2}} \quad I_1 = 4.412 \cdot \text{A}$$

$$V_1 := I_1 \cdot R_{\text{eq}}$$

$$V_1 = 449.031 - 62.5j \cdot \text{V}$$

$$V_{\text{FL1}} := |V_1|$$

$$V_{\text{FL1}} = 453.359 \cdot \text{V}$$

$$V_{\text{FL1}} = I_1 \cdot R_{\text{eq}} = 453.359 \cdot \text{V}$$

$$\text{Voltage regulation: } \% \text{VR} = \frac{V_{\text{no_load}} - V_{\text{full_load}}}{V_{\text{full_load}}} \cdot 100\% = \frac{V_S - V_{\text{FL1}}}{V_{\text{FL1}}} \cdot 100\% = 5.876\%$$

$$\text{Efficiency } \eta = \frac{P_{\text{out}}}{P_{\text{in}}} \cdot 100\% = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{losses}}} \cdot 100\%$$

$$P_{\text{out}} := I_1^2 \cdot R_{\text{eq}}$$

$$P_{\text{out}} = 2 \cdot \text{kW}$$

$$P_{\text{losses}} := I_1^2 \cdot 5 \cdot \Omega + \frac{(480 \cdot \text{V})^2}{8400 \cdot \Omega}$$

$$P_{\text{losses}} = 0.125 \cdot \text{kW}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{losses}}} \cdot 100\% = 94.129\%$$

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Ex.8 A 500/100-V, 2.5-kVA transformer is subjected to an OC test and a SC test with the results below.

a) Draw a model of this transformer and find the values of all the elements of the model, including the turns ratio.

During the open-circuit test: $I_{OC} := 0.5 \cdot A$ $P_{OC} := 150 \cdot W$ $V_{sec} := 108 \cdot V$

During the short-circuit test: $V_{SC} := 22 \cdot V$ $P_{SC} := 70 \cdot W$

$S_{rated} := 2.5 \cdot kVA$ $V_{Prated} := 500 \cdot V$ $I_{Prated} := \frac{S_{rated}}{V_{Prated}}$ $I_{Prated} = 5 \cdot A$

Open-circuit test used to find R_m and X_m and turns ratio.

$V_{OC} := V_{Prated}$

$R_m := \frac{V_{OC}^2}{P_{OC}}$ $R_m = 1.667 \cdot k\Omega$

$Q_{OC} := \sqrt{(V_{OC} \cdot I_{OC})^2 - P_{OC}^2}$ $Q_{OC} = 200 \cdot VAR$

$X_m := \frac{V_{OC}^2}{Q_{OC}}$ $X_m = 1.25 \cdot k\Omega$

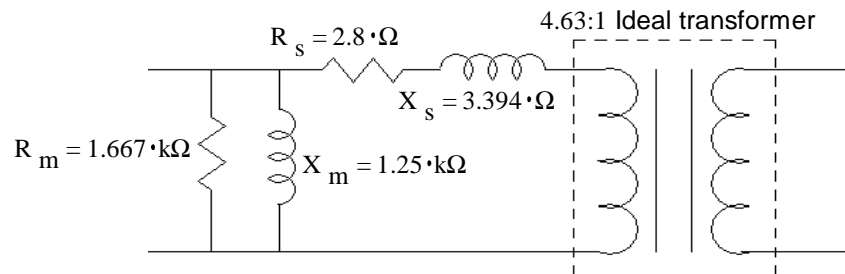
Turns ratio: $N := \frac{V_{Prated}}{V_{sec}}$ $N = 4.63$

Short-circuit test used to find R_s and X_s $I_{SC} := I_{Prated}$

$R_s := \frac{P_{SC}}{I_{SC}^2}$ $R_s = 2.8 \cdot \Omega$

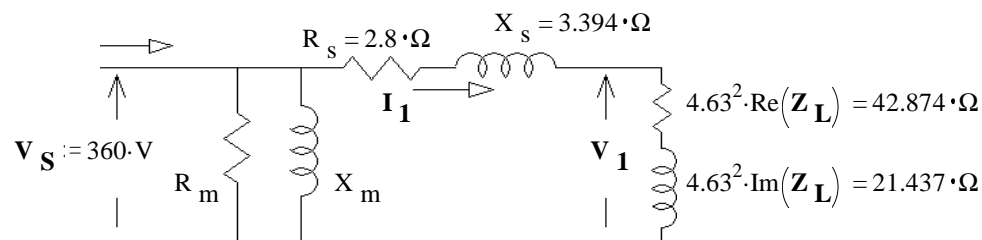
$Q_{SC} := \sqrt{(V_{SC} \cdot I_{SC})^2 - P_{SC}^2}$ $Q_{SC} = 0.085 \cdot kVAR$

$X_s := \frac{Q_{SC}}{I_{SC}^2}$ $X_s = 3.394 \cdot \Omega$



b) The transformer is connected to a primary source voltage of 360V and loaded with $Z_L := (2 + 1 \cdot j) \cdot \Omega$

Find the secondary voltage. Magnitude only. $|V_2| = ?$



$|V_1| = V_1 := V_S \cdot \frac{\sqrt{(42.874 \cdot \Omega)^2 + (21.437 \cdot \Omega)^2}}{\sqrt{(R_s + 42.874 \cdot \Omega)^2 + (X_s + 21.437 \cdot \Omega)^2}}$ $V_1 = 331.935 \cdot V$

$|V_2| = V_2 = \frac{V_1}{4.63} = 71.69 \cdot V$

c) Is this transformer operating within its ratings? Show all evidence and calculations needed to determine this.

$|I_2| = I_2 := \frac{V_S}{\sqrt{(R_s + 42.874 \cdot \Omega)^2 + (X_s + 21.437 \cdot \Omega)^2}} \cdot N$ $I_2 = 32.059 \cdot A > I_{Srated} = \frac{S_{rated}}{100 \cdot V} = 25 \cdot A$

NO !

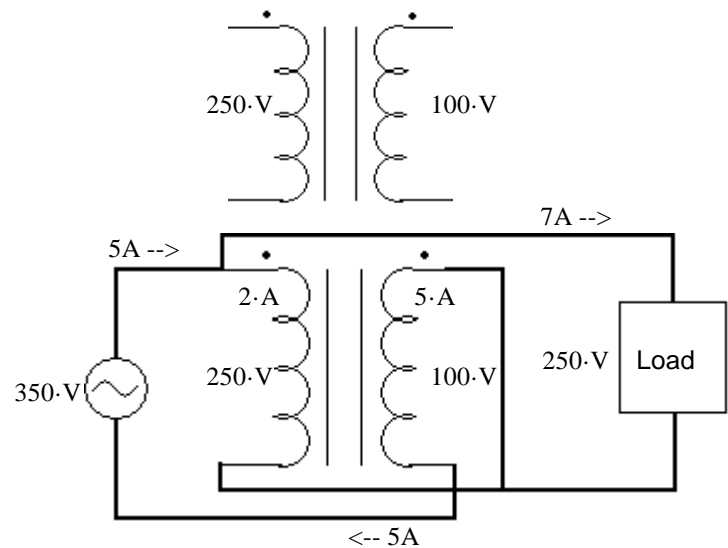
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Ex.9 You have a 250/100-V, 500-VA transformer.

- Show the necessary connections to use this transformer to transform 350 V to 250 V. Also show the 350-V source and the load.
- Connected this way, determine the maximum power that could be converted from 350 V to 250 V without overloading the transformer.

$$\text{ratings: } \frac{500 \cdot \text{VA}}{250 \cdot \text{V}} = 2 \cdot \text{A} \quad \frac{500 \cdot \text{VA}}{100 \cdot \text{V}} = 5 \cdot \text{A}$$

$$\begin{aligned} \text{new VA rating and} \\ \text{maximum power: } 5 \cdot \text{A} \cdot 350 \cdot \text{V} &= 1.75 \cdot \text{kVA} \\ \text{OR: } 7 \cdot \text{A} \cdot 250 \cdot \text{V} &= 1.75 \cdot \text{kVA} \\ &1.75 \cdot \text{kW} \end{aligned}$$



- Besides the right impedance magnitude, what other characteristic must the load possess in order to actually use this much power?

Load must be purely resistive (power factor is 1).

- Could this transformer also be used to transform 280 V to 200 V ? If yes, what is the maximum power that could be transformed?

$$\text{Same connections as above} \quad \text{Maximum power: } 5 \cdot \text{A} \cdot 280 \cdot \text{V} = 1.4 \cdot \text{kW}$$

Ex.10 A 345kV/138kV, 750-MVA transformer is shown.

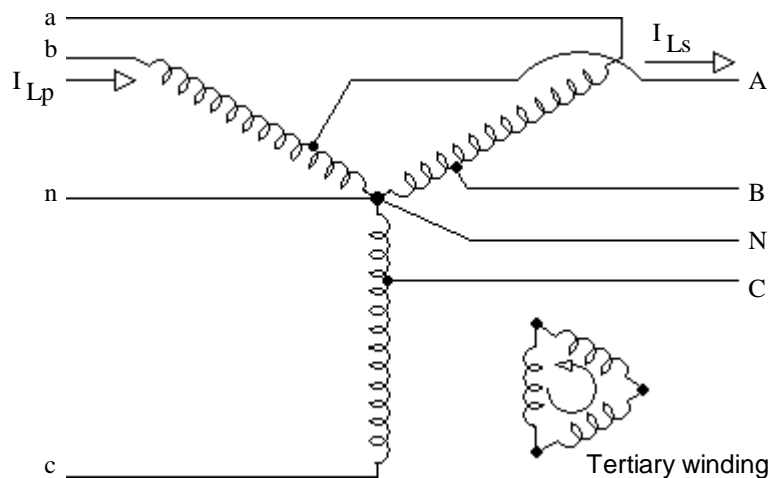
- What is the purpose of the tertiary winding?

To allow 3rd harmonic currents to flow without affecting currents outside the transformer.

- Find the maximum I_{Lp} and I_{Ls} .

$$I_{Lp.\text{rated}} := \frac{\left(\frac{750 \cdot \text{MVA}}{3} \right)}{\left(\frac{345 \cdot \text{kV}}{\sqrt{3}} \right)} \quad I_{Lp.\text{rated}} = 1255 \cdot \text{A}$$

$$I_{Ls.\text{rated}} := \frac{345}{138} \cdot I_{Lp.\text{rated}} \quad I_{Ls.\text{rated}} = 3138 \cdot \text{A}$$



- Find the currents flowing in the transformer when operated at rated capacity.

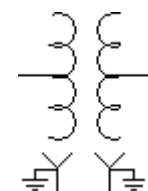
$$\text{Current from primary terminal to the tap: } I_p = I_{Lp.\text{rated}} = 1255 \cdot \text{A}$$

$$\text{Current from neutral to the tap: } I_p = I_{Ls.\text{rated}} - I_{Lp.\text{rated}} = 1883 \cdot \text{A}$$

$$\text{Current from tap to secondary output of the transformer: } I_s = I_{Ls.\text{rated}} = 3138 \cdot \text{A}$$

- At what fraction of the total turns is the tap located? $\frac{138}{345} = 0.4 = \frac{4}{10}$ OR at 40%

- What one-line symbol would be used for this transformer?



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