ECE 3600 3-Phase Examples

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9/9/09
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Ex. 1 A Y-connected load is connected to 208-V, 3-phase.
It draws 1.2kW of power at a power factor of 75%, leading. $P_{3\phi} := 1.2\cdot kW$ $\quad pf := 0.75$

a) Find the apparent power and the reactive power.

$S_{3\phi} := \dfrac{P_{3\phi}}{pf}$ $\quad S_{3\phi} = 1.6\cdot kVA$ $\quad Q_{3\phi} := -\sqrt{S_{3\phi}^2 - P_{3\phi}^2}$ $\quad Q_{3\phi} = -1.058\cdot kVAR$

Negative because the power factor is leading.

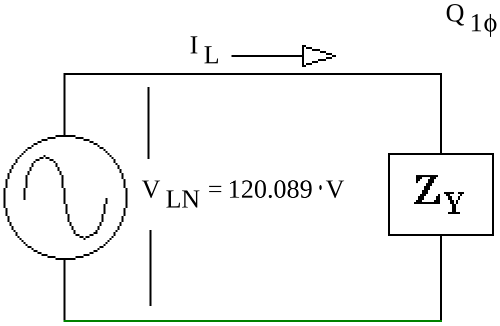
b) Find the line current.

Our Approach 1) Change all Δ-connected loads to equivalent Y-connected loads $\mathbf{Z_Y} = \dfrac{\mathbf{Z_\Delta}}{3}$ NOT NEEDED

2) Find all voltages as $V_{LN} := \dfrac{208\cdot V}{\sqrt{3}}$ $\quad V_{LN} = 120.089\cdot V$

3) Change all power numbers to 1ϕ. $P_{1\phi} := \dfrac{P_{3\phi}}{3}$ $\quad P_{1\phi} = 400\cdot W$ $\quad S_{1\phi} := \dfrac{S_{3\phi}}{3}$ $\quad S_{1\phi} = 533.333\cdot VA$

$Q_{1\phi} := \dfrac{Q_{3\phi}}{3}$ $\quad Q_{1\phi} = -352.767\cdot VAR$

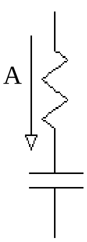


$V_{LN} = 120.089\cdot V$ $\quad S_{1\phi} = 533.333\cdot VA$

$I_L := \dfrac{S_{1\phi}}{V_{LN}}$ $\quad I_L = 4.441\cdot A$

c) Find the values of the load components, assuming they are connected in series.

The components must be a resistor and a capacitor because there is some real power and the power factor is leading.



$I_L = 4.441\cdot A$ $\quad R_L$ $\quad C_L$

$R_L := \dfrac{P_{1\phi}}{I_L^2} = \dfrac{P_{1\phi}}{I_L^2} = 20.28\cdot\Omega$ $\quad$ assume $\omega = 377\cdot\dfrac{rad}{sec}$

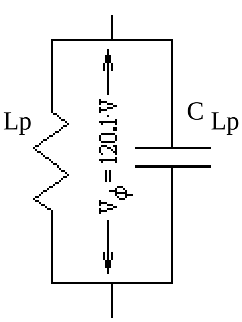
$X_C := \dfrac{Q_{1\phi}}{I_L^2}$ $\quad X_C = -17.885\cdot\Omega = -\dfrac{1}{\omega\cdot C_L}$ $\quad C_L := -\dfrac{1}{\omega\cdot X_C}$ $\quad C_L = 148.3\cdot\mu F$

d) Find the values of the load components, assuming they are connected in parallel.

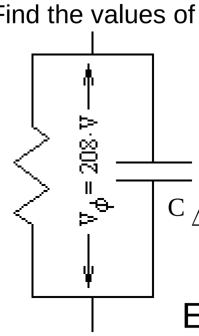
Still a resistor and a capacitor.

$R_{Lp} := \dfrac{V_{LN}^2}{P_{1\phi}}$ $\quad R_{Lp} = 36.053\cdot\Omega$ $\quad X_C := \dfrac{V_{LN}^2}{Q_{1\phi}}$ $\quad X_C = -40.881\cdot\Omega = -\dfrac{1}{\omega\cdot C_{Lp}}$

$C_{Lp} := -\dfrac{1}{\omega\cdot X_C}$ $\quad C_{Lp} = 64.9\cdot\mu F$



e) Find the values of the load components, assuming they are connected in parallel AND Δ.



$R_\Delta := \dfrac{(208\cdot V)^2}{P_{1\phi}}$ $\quad X_{C\Delta} := \dfrac{(208\cdot V)^2}{Q_{1\phi}}$ $\quad X_{C\Delta} = -122.642\cdot\Omega = -\dfrac{1}{\omega\cdot C_\Delta}$

$R_\Delta = 108.16\cdot\Omega$ $\quad C_\Delta := -\dfrac{1}{\omega\cdot X_{C\Delta}}$ $\quad C_\Delta = 21.6\cdot\mu F$

$3\cdot R_{Lp} = 108.16\cdot\Omega$ check !

$3\cdot C_{Lp} = 194.7\cdot\mu F$ huh?? $\quad \dfrac{C_{Lp}}{3} = 21.6\cdot\mu F$ OH!

$3\cdot X_C = -122.642\cdot\Omega$

f) Correct the power factor with Y-connected components.
Need inductors

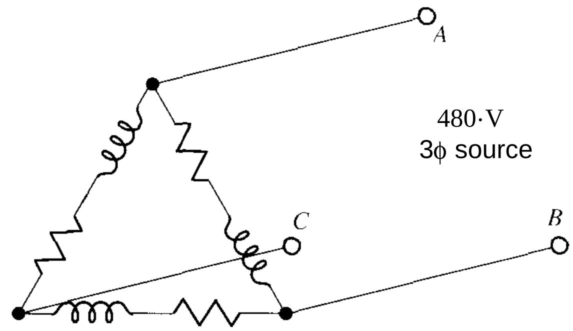
$$Q_{1\phi Ind} := -Q_{1\phi} = \frac{V_\phi^{\ 2}}{\omega \cdot L_Y} \qquad L_Y := \frac{V_\phi^{\ 2}}{\omega \cdot -Q_{1\phi}} \qquad L_Y = 325.3 \cdot mH$$

g) Correct the power factor with Δ-connected components.

$$L_\Delta := \frac{\left(\sqrt{3} \cdot V_\phi\right)^2}{\omega \cdot -Q_{1\phi}} \qquad L_\Delta = 975.9 \cdot mH$$

OR $\omega \cdot L_\Delta = \mathbf{Z}_\Delta = 3 \cdot \mathbf{Z}_\mathbf{y} = 3 \cdot \omega \cdot L_Y$ $\qquad 3 \cdot L_Y = 975.9 \cdot mH$ check !

Ex. 2 From F08, exam 1, Find the following:



All $\mathbf{Z} := (30 + 12 \cdot j) \cdot \Omega$

a) The line current that would be measured by an ammeter.

$$V_{LL} := 480 \cdot V \qquad \mathbf{Z}_\Delta := (30 + 12 \cdot j) \cdot \Omega$$

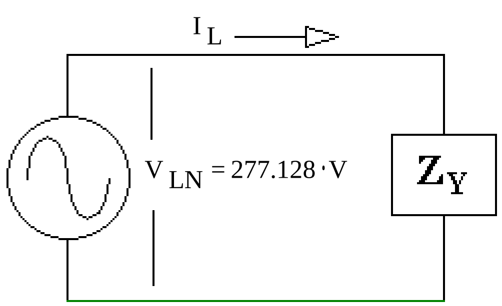
Our Approach

1) Change all Δ-connected loads to equivalent Y-connected loads

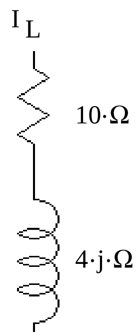
$$\mathbf{Z}_\mathbf{Y} := \frac{\mathbf{Z}_\Delta}{3} \qquad \mathbf{Z}_\mathbf{Y} = 10 + 4j \ \cdot \Omega$$

2) Find all voltages as V_{LN} $\qquad V_{LL} = 480 \cdot V \qquad V_{LN} := \frac{V_{LL}}{\sqrt{3}} \qquad V_{LN} = 277.128 \cdot V$

3) Change all power numbers to 1ϕ. NOT NEEDED

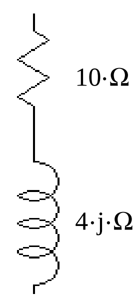


$\mathbf{Z}_\mathbf{Y} = 10 + 4j \ \cdot \Omega$ =



$$I_L := \frac{V_{LN}}{\left|\mathbf{Z}_\mathbf{Y}\right|} = \frac{277.128 \cdot V}{\sqrt{10^2 + 4^2} \cdot \Omega} = 25.731 \cdot A$$

$I_L = 25.731 \cdot A$



b) The power consumed by the three-phase load.

$$P_{1\phi} = I_L^{\ 2} \cdot 10 \cdot \Omega = 6.62 \cdot kW$$

$$P_{3\phi} = 3 \cdot \left(I_L^{\ 2} \cdot 10 \cdot \Omega\right) = 19.86 \cdot kW$$

c) The value of Y-connected impedances that would result in exactly the same line currents and same pf.

$$\mathbf{Z}_\mathbf{Y} = 10 + 4j \ \cdot \Omega$$

d) The value of Y-connected capacitors that would correct the pf.

$$Q_{1\phi} := \sqrt{S_{1\phi}^{\ 2} - P_{1\phi}^{\ 2}} \qquad Q_{1\phi} := \sqrt{\left(V_{LN} \cdot I_L\right)^2 - (6.62 \cdot kW)^2} \qquad Q_{1\phi} = 2.65 \cdot kVAR$$

so we need:

$$Q_C := -Q_{1\phi} \qquad Q_C = -2.65 \cdot kVAR = -\frac{V_{LN}^{\ 2}}{\left(\frac{1}{\omega C}\right)} = -V_{LN}^{\ 2} \cdot \omega \cdot C \qquad C := \frac{Q_C}{-V_{LN}^{\ 2} \cdot \omega} \qquad C = 91.5 \cdot \mu F$$

Ex. 3 For the three-phase delta-connected load in fig P1 .7,
The line-to-line voltage and line current are:

$\mathbf{V}_{\mathbf{AB}} := 480\cdot V\ \underline{/0^o}$ $\qquad \mathbf{I}_{\mathbf{A}} = 10A\underline{/-40^o}$

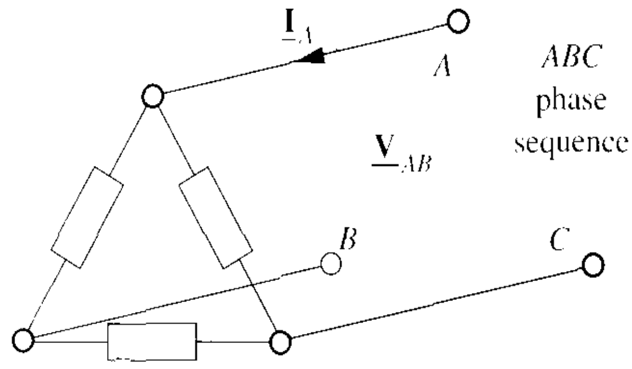
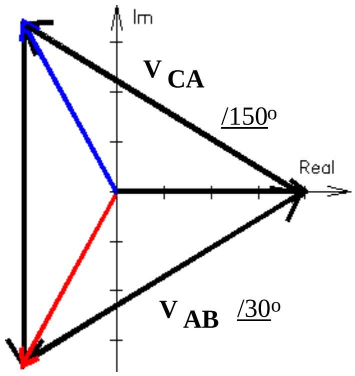


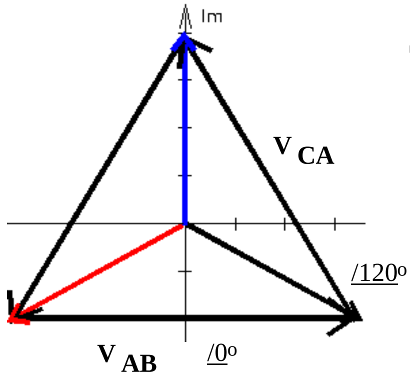
Figure P1.7

a) What is $\mathbf{V}_{\mathbf{CA}}$?

Normal phase angles



Rotate CW 30 deg



$\mathbf{V}_{\mathbf{CA}} := 480\cdot V\ \underline{/120^o}$

$= 480\cdot V\ \underline{/-240^o}$

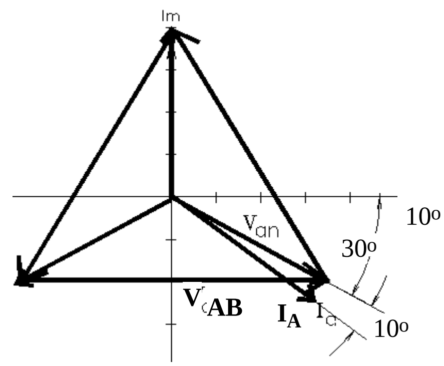
b) What is the phase current in the load?

$$I_{LL} = \frac{I_L}{\sqrt{3}} \qquad \frac{10\cdot A}{\sqrt{3}} = 5.774\cdot A$$

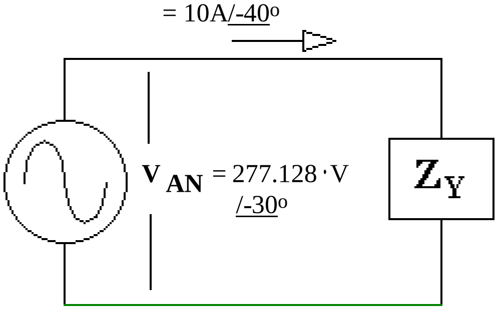
c) What is the time-average power into the load?

$\mathbf{V}_{\mathbf{AN}} := \dfrac{480\cdot V}{\sqrt{3}}\ \underline{/-30^o}$ $\qquad$ Since $\mathbf{I}_{\mathbf{A}} = 10A\underline{/-40^o}$ $\qquad$ **I** lags **V** by 10^o

$\theta := 10\cdot deg$



$= 10A\underline{/-40^o}$



$$P_{1\phi} = (277.128\cdot V\cdot 10\cdot A)\cdot\cos(\theta) = 2.729\cdot kW$$

$$P_{3\phi} = 3\cdot(277.128\cdot V\cdot 10\cdot A)\cdot\cos(\theta) = 8.188\cdot kW$$

d) What is the phase impedance?

$$\mathbf{Z}_{\mathbf{Y}} := \frac{277.128\cdot V}{10\cdot A}\ \underline{/ -30 - (-40)^o} \qquad \mathbf{Z}_{\mathbf{Y}} = 27.71\cdot\Omega\ \ \underline{/10^o}$$

$$\mathbf{Z}_{\boldsymbol{\Delta}} = 3\cdot\mathbf{Z}_{\mathbf{Y}} = 83.14\cdot\Omega\ \ \underline{/10^o}$$

Ex. 4 In the three-phase circuit shown in Fig. P1.9. find the following:

a) The line current that would be measured by an ammeter.

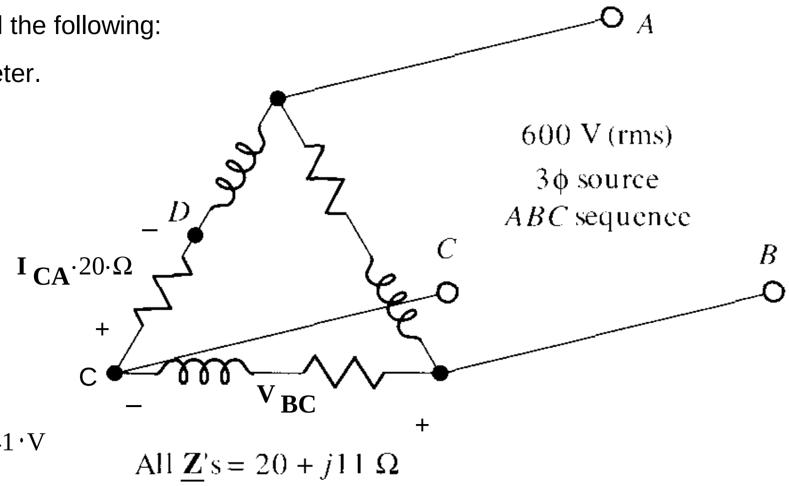


Figure P1.9

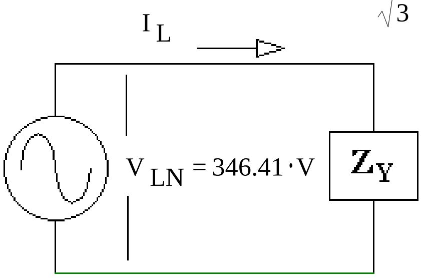
Direct way

$V_{LL} := 600\cdot V$ $\qquad$ $Z_{\Delta} := (20 + 11\cdot j)\cdot\Omega$

$I_{AB} := \left|\dfrac{V_{LL}}{Z_{\Delta}}\right|$ $\qquad$ $I_{AB} = 26.286\cdot A$

$I_A := \sqrt{3}\cdot I_{AB}$ $\qquad$ $I_A = 45.53\cdot A$

Our Approach $\qquad$ $V_{LN} := \dfrac{600\cdot V}{\sqrt{3}}$ $\qquad$ $V_{LN} = 346.41\cdot V$

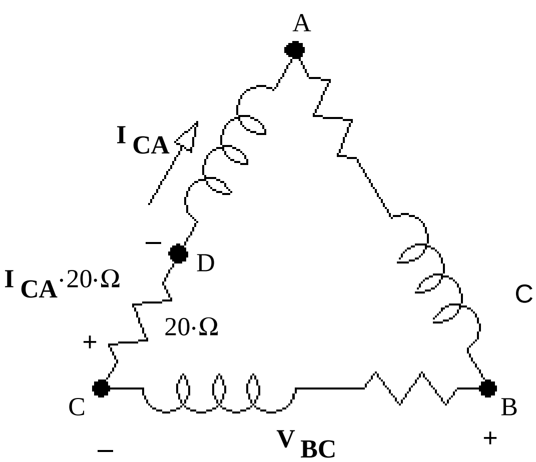


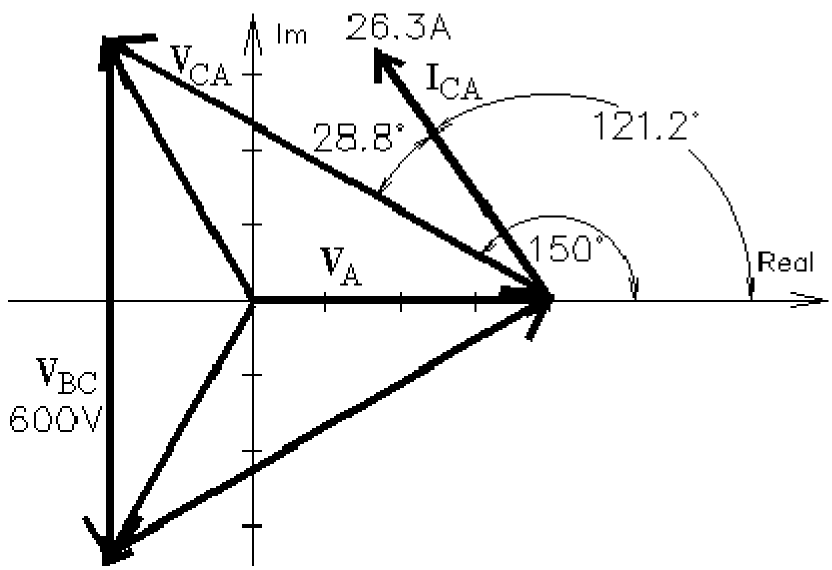
$Z_Y := \dfrac{Z_{\Delta}}{3}$ $\qquad$ $Z_Y = 6.667 + 3.667j \ \cdot\Omega$

$I_L := \dfrac{V_{LN}}{|Z_Y|} = \dfrac{346.41\cdot V}{\sqrt{6.667^2 + 3.667^2}}$ $\qquad$ $I_L = 45.53\cdot A$

b) The power factor of the three-phase load. $\qquad$ $\theta := \text{atan}\left(\dfrac{11}{20}\right)$ $\qquad$ $\theta = 28.811\cdot deg$ $\qquad$ pf $\ \cos(\theta) = 0.876$

c) The voltage that would be measured between B and D by a voltmeter.





Using $\mathbf{V_A}$ as reference (0º):

$V_{BC} := 600\cdot V\cdot e^{-j\cdot 90\cdot deg}$

$I_{CA} := 26.286\cdot A\cdot e^{j\cdot(150 - 28.811)\cdot deg}$

$V_{CD} := I_{CA}\cdot 20\cdot\Omega$

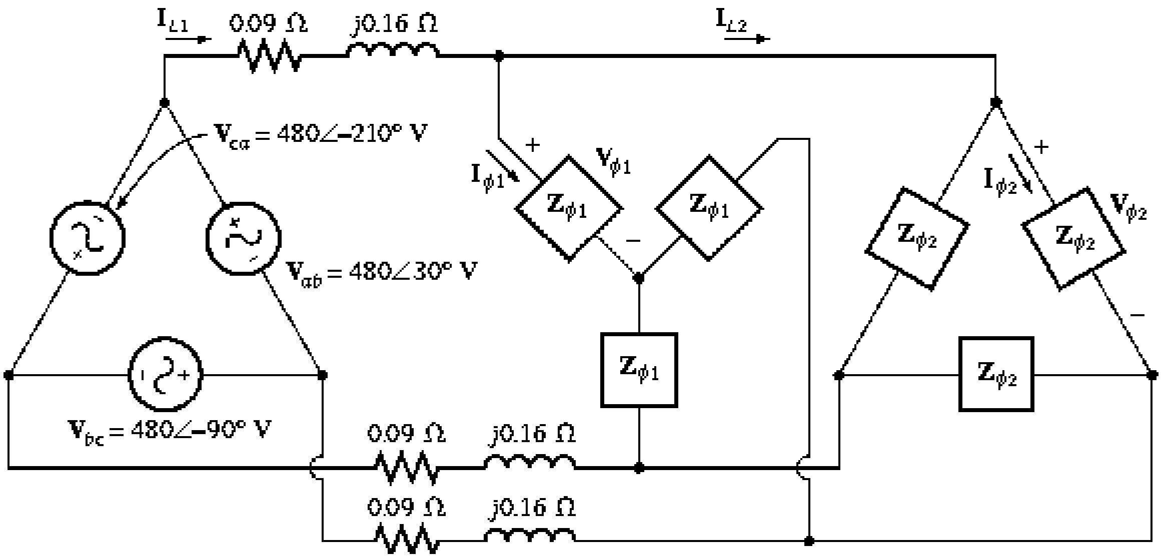
$V_{CD} = -272.251 + 449.734j \ \cdot V$

$V_{BD} := V_{BC} + V_{CD}$ $\qquad$ $V_{BD} = -272.251 - 150.266j \ \cdot V$ $\qquad$ $|V_{BD}| = 311\cdot V$

(must be the sum, NOT the difference, see the + and - signs on the drawing.)

Ex. 5 When all you have is impedances and an input voltage, it gets messy & luckily, it's not a common problem.

Textbook problem 2-2. Figure P2-1 shows a three-phase power system with two loads. The Δ-connected generator is producing a line voltage of 480 V, and the line impedance is 0.09 + j0.16 Ω. Load 1 is Y-connected, with a phase impedance of 2.5Ω $\underline{/36.87}^o$ and load 2 is Δ-connected, with a phase impedance of 5Ω $\underline{/-20}^o$.



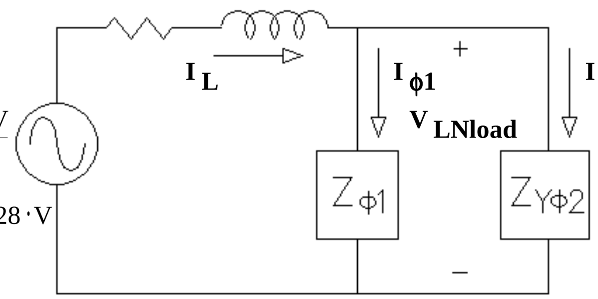
$Z_{\phi1} := 2.5\cdot e^{j\cdot 36.87\cdot deg}\cdot\Omega$ $\qquad Z_{\phi2} := 5\cdot e^{-j\cdot 20\cdot deg}\cdot\Omega$

a) What is the line voltage at the two loads?

Find an equivalent Y-only circuit: $\qquad Z_{line} := (0.09 + 0.16\cdot j)\cdot\Omega$

$V_Y := \dfrac{480\cdot V}{\sqrt{3}}$

$V_Y = 277.128\cdot V$



$Z_{Y\phi2} := \dfrac{Z_{\phi2}}{3}$

$Z_{Y\phi2} = 1.566 - 0.57j \;\cdot\Omega$

$Z_{Yloads} := \dfrac{1}{\dfrac{1}{Z_{\phi1}} + \dfrac{1}{Z_{Y\phi2}}}$ $\qquad Z_{Yloads} = 1.13 + 0.044j \;\cdot\Omega$ $\qquad |Z_{Yloads}| = 1.131\cdot\Omega$ $\quad \arg(Z_{Yloads}) = 2.254\cdot deg$

$Z_{Ytot} := Z_{line} + Z_{Yloads}$ $\qquad Z_{Ytot} = 1.22 + 0.204j \;\cdot\Omega$ $\qquad |Z_{Ytot}| = 1.237\cdot\Omega$ $\quad \arg(Z_{Ytot}) = 9.516\cdot deg$

$I_L := \dfrac{V_Y}{Z_{Ytot}}$ $\qquad I_L = 220.998 - 37.047j \;\cdot A$ $\qquad |I_L| = 224.082\cdot A$ $\quad \arg(I_L) = -9.516\cdot deg$

$V_{LNload} := I_L\cdot Z_{Yloads}$ $\qquad V_{LNload} = 251.311 - 32.025j \;\cdot V$ $\qquad |V_{LNload}| = 253.343\cdot V$ $\quad \arg(V_{LNload}) = -7.262\cdot deg$

$V_{Lload} := V_{LNload}\cdot\sqrt{3}$ $\qquad$ NO! $V_{Lload} = 435.283 - 55.47j \;\cdot V$ Angle makes no sense $\qquad |V_{Lload}| = 438.803\cdot V$

But, the magnitude DOES make sense

b) What is the voltage drop on the transmission lines?

$\mathbf{V}_{\mathbf{linedrop}} := \mathbf{I}_{\mathbf{L}} \cdot \mathbf{Z}_{\mathbf{line}}$ $\quad \mathbf{V}_{\mathbf{linedrop}} = 25.817 + 32.025j \cdot V$ $\quad \left|\mathbf{V}_{\mathbf{linedrop}}\right| = 41.136 \cdot V$

$\arg\left(\mathbf{V}_{\mathbf{linedrop}}\right) = 51.126 \cdot deg$

Check: $\mathbf{V}_{\mathbf{Y}} - \mathbf{V}_{\mathbf{LNload}} = 25.817 + 32.025j \cdot V$

c) Find the real and reactive powers supplied to each load.

$I_{\phi 1} := \dfrac{\left|\mathbf{V}_{\mathbf{LNload}}\right|}{\left|\mathbf{Z}_{\boldsymbol{\phi}\mathbf{1}}\right|}$ $\quad I_{\phi 1} = 101.337 \cdot A$ $\quad I_{L2} := \dfrac{\left|\mathbf{V}_{\mathbf{LNload}}\right|}{\left|\mathbf{Z}_{\mathbf{Y}\boldsymbol{\phi}\mathbf{2}}\right|}$ $\quad I_{L2} = 152.006 \cdot A$

$P_{3\phi 1} := 3 \cdot I_{\phi 1}^{\,2} \cdot \mathrm{Re}\left(\mathbf{Z}_{\boldsymbol{\phi}\mathbf{1}}\right)$ $\quad P_{3\phi 1} = 61.615 \cdot kW$ $\quad P_{3\phi 2} := 3 \cdot I_{L2}^{\,2} \cdot \mathrm{Re}\left(\mathbf{Z}_{\mathbf{Y}\boldsymbol{\phi}\mathbf{2}}\right)$ $\quad P_{3\phi 2} = 108.562 \cdot kW$

$Q_{3\phi 1} := 3 \cdot I_{\phi 1}^{\,2} \cdot \mathrm{Im}\left(\mathbf{Z}_{\boldsymbol{\phi}\mathbf{1}}\right)$ $\quad Q_{3\phi 1} = 46.212 \cdot kVAR$ $\quad Q_{3\phi 2} := 3 \cdot I_{L2}^{\,2} \cdot \mathrm{Im}\left(\mathbf{Z}_{\mathbf{Y}\boldsymbol{\phi}\mathbf{2}}\right)$ $\quad Q_{3\phi 2} = -39.513 \cdot kVAR$

d) Find the real and reactive power losses in the transmission line.

$P_{3\phi L} := 3 \cdot \left(\left|\mathbf{I}_{\mathbf{L}}\right|\right)^2 \cdot \mathrm{Re}\left(\mathbf{Z}_{\mathbf{line}}\right)$ $\quad P_{3\phi L} = 13.557 \cdot kW$

$Q_{3\phi L} := 3 \cdot \left(\left|\mathbf{I}_{\mathbf{L}}\right|\right)^2 \cdot \mathrm{Im}\left(\mathbf{Z}_{\mathbf{line}}\right)$ $\quad Q_{3\phi L} = 24.102 \cdot kVAR$

e) Find the real power, reactive power, and power factor supplied by the generator.

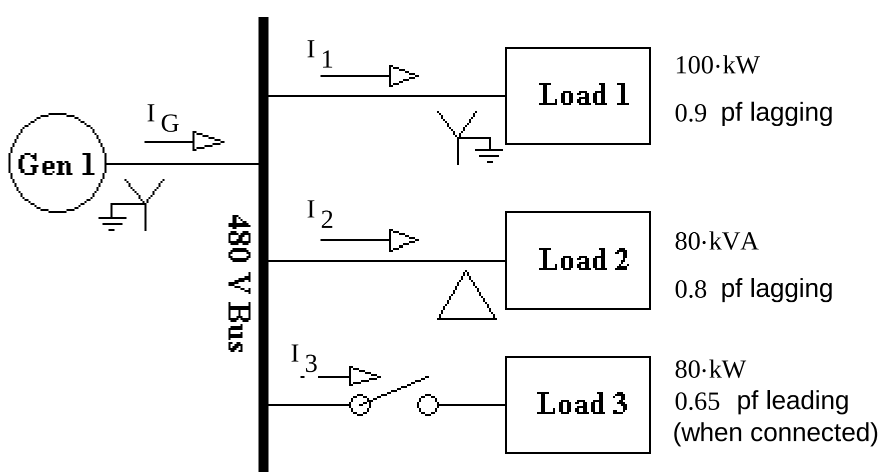
$P_{3\phi gen} := P_{3\phi L} + P_{3\phi 1} + P_{3\phi 2}$ $\quad P_{3\phi gen} = 183.734 \cdot kW$

$Q_{3\phi gen} := Q_{3\phi L} + Q_{3\phi 1} + Q_{3\phi 2}$ $\quad Q_{3\phi gen} = 30.801 \cdot kVAR$ $\quad pf = \dfrac{P_{3\phi gen}}{3 \cdot \left|\mathbf{V}_{\mathbf{Y}}\right| \cdot \left|\mathbf{I}_{\mathbf{L}}\right|} = 0.986$ lagging

f) What is the efficiency of this system? $\quad \eta = \dfrac{P_{3\phi 1} + P_{3\phi 2}}{P_{3\phi gen}} = 92.621 \cdot \%$

The next example uses a "one-line diagram" to show how a generator is connected to 3 loads. In these diagrams, one line represents all 3 phases and neutral. Because the individual lines are not shown, there may be notes or symbols to indicate Y or Δ connections. All powers given will be 3-phase values, all voltages will be line voltages (that is line-to-line) and all currents will line currents. The term "bus" refers common connection area.

Ex. 6 The one-line diagram below shows a single, Y-connected generator and 3 loads. Assume all lines are lossless.



Find:

a) The phase voltage and currents in Load 1.

$V_{LL} := 480 \cdot V \qquad V_{LN} := \frac{V_{LL}}{\sqrt{3}} \qquad V_{LN} = 277.128 \cdot V = V_{L1\phi}$

$pf_{L1} := 0.9 \qquad S_{L1.1\phi} := \frac{100 \cdot kW}{3 \cdot pf_{L1}} \qquad I_1 := \frac{S_{L1.1\phi}}{V_{LN}} \qquad I_1 = 133.646 \cdot A = I_{L1\phi}$

b) The phase voltage and currents in Load 2.

$V_{LL} := 480 \cdot V = V_{L2\phi} \qquad pf_{L2} := 0.8 \qquad S_{L2.1\phi} := \frac{80 \cdot kVA}{3}$

$I_2 := \frac{S_{L2.1\phi}}{V_{LN}} \qquad I_2 = 96.225 \cdot A = \sqrt{3} \cdot I_{L2\phi} \qquad I_{L2\phi} = \frac{I_2}{\sqrt{3}} = 55.556 \cdot A$

c) The real, reactive and apparent power supplied by the generator with the switch to load 3 open.

$P_1 := 100 \cdot kW \qquad P_2 := 80 \cdot kVA \cdot pf_{L2} \qquad P_2 = 64 \cdot kW \qquad P_G := P_1 + P_2 \qquad P_G = 164 \cdot kW$

$Q_1 := \sqrt{\left(\frac{100 \cdot kW}{pf_{L1}}\right)^2 - (100 \cdot kW)^2} \qquad Q_1 = 48.432 \cdot kVAR \qquad Q_2 := \sqrt{(80 \cdot kVA)^2 - (64 \cdot kW)^2} \qquad Q_1 = 48.432 \cdot kVAR$

$Q_G := Q_1 + Q_2 \qquad Q_G = 96.432 \cdot kVAR$

$S_G := \sqrt{{P_G}^2 + {Q_G}^2} \qquad S_G = 190.25 \cdot kVAR$

d) The total line current from the generator, I_G, with the switch to load 3 open.

$I_G = \frac{\left(\frac{S_G}{3}\right)}{V_{LN}} = 228.836 \cdot A$

e) The real, reactive and apparent power supplied by the generator with the switch to load 3 closed.

$pf_{L3} := 0.65 \qquad S_{L3.1\phi} := \frac{80 \cdot kW}{3 \cdot pf_{L3}} \qquad Q_3 := -\sqrt{\left(\frac{80 \cdot kW}{pf_{L3}}\right)^2 - (80 \cdot kW)^2} \qquad Q_3 = -93.53 \cdot kVAR$

$P_G := P_1 + P_2 + 80 \cdot kW \qquad P_G = 244 \cdot kW \qquad Q_G := Q_1 + Q_2 + Q_3 \qquad Q_G = 2.902 \cdot kVAR$

$S_G := \sqrt{{P_G}^2 + {Q_G}^2} \qquad S_G = 244.017 \cdot kVAR$

f) How does the total line apparent power from the generator, S_G, compare to the sum of the three individual apparent powers, $S_1 + S_2 + S_3$? If they aren't equal, why not? (Switch closed)

$3 \cdot S_{L1.1\phi} + 80 \cdot kVAR + 3 \cdot S_{L3.1\phi} = 314.188 \cdot kVAR \neq S_G = 244.017 \cdot kVAR$

Can't Add Magnitudes

g) The total line current from the generator, I_G, with the switch to load 3 closed.

$I_G := \frac{\left(\frac{S_G}{3}\right)}{V_{LN}} \qquad I_G = 293.507 \cdot A$

h) How does the total line current from the generator, I_G, compare to the sum of the three individual currents, $I_1 + I_2 + I_3$? If they aren't equal, why not? (Switch closed)

$I_3 := \frac{S_{L3.1\phi}}{V_{LN}} \qquad I_3 = 148.039 \cdot A$

$I_1 + I_2 + I_3 = 377.909 \cdot A \neq I_G = 293.507 \cdot A$

Can't Add Magnitudes