ECE 3600 AC Power Examples

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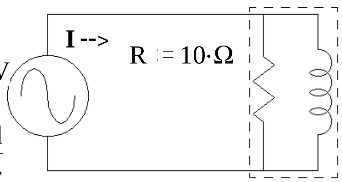
Ex. 1 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

$\omega := 2\cdot\pi\cdot 60\cdot Hz$ $\quad V_S := 110\cdot V$ $\quad \omega = 376.991\cdot\frac{rad}{sec}$



$$\mathbf{Z} := \frac{1}{\left(\frac{1}{R}+\frac{1}{j\cdot\omega\cdot L}\right)} = \frac{1}{0.1458\cdot\frac{1}{\Omega}\cdot e^{-j\cdot 46.7\cdot deg}}$$

$\mathbf{Z} = 4.704 + 4.991j\ \cdot\Omega$ $\quad |\mathbf{Z}| = 6.859\cdot\Omega$ $\quad \theta := \arg(\mathbf{Z})$ $\quad \theta = 46.7\cdot deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.686$

$\mathbf{I} := \frac{V_S}{\mathbf{Z}}$ $\quad \mathbf{I} = 11 - 11.671j\ \cdot A$ $\quad I = |\mathbf{I}| = 16.038\cdot A$ $\quad \arg(\mathbf{I}) = -46.7\cdot deg$

$P := V_S\cdot|\mathbf{I}|\cdot pf$ $\quad P = 1.21\cdot kW$

$Q := V_S\cdot|\mathbf{I}|\cdot\sin(\theta)$ $\quad Q = 1.284\cdot kVAR$ $\quad$ OR... $\quad Q := V_S\cdot|\mathbf{I}|\cdot\sqrt{1^2 - pf^2}$ $\quad Q = 1.284\cdot kVAR$

$\mathbf{S} := V_S\cdot\overline{\mathbf{I}}$ OR.. $\mathbf{S} := P + j\cdot Q$ $\quad \mathbf{S} = 1.21 + 1.284j\ \cdot kVA$ $\quad S := \sqrt{\mathrm{Re}(\mathbf{S})^2 + \mathrm{Im}(\mathbf{S})^2} = |\mathbf{S}| = 1.764\cdot kVA$

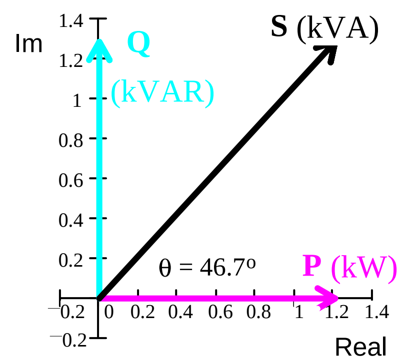
$\mathrm{atan}\left(\frac{\mathrm{Im}(\mathbf{S})}{\mathrm{Re}(\mathbf{S})}\right) = 46.696\cdot deg$ same as θ $\quad \mathbf{S} = 1.764kVA\ \underline{/46.7^o}$

OR, since we know that the voltage across each element of the load is V_S & real power is dissipated only by resistors...

$P := \frac{{V_S}^2}{R}$ $\quad P = 1.21\cdot kW$ $\quad$ & $\quad Q := \frac{{V_S}^2}{\omega\cdot L}$ $\quad Q = 1.284\cdot kVAR$

$\mathbf{S} := P + j\cdot Q$ $\quad S = |\mathbf{S}| = \sqrt{P^2 + Q^2} = 1.764\cdot kVA$ $\quad pf = \frac{P}{|\mathbf{S}|} = 0.686$

$I = \frac{|\mathbf{S}|}{|\mathbf{V_S}|} = \frac{S}{110\cdot V} = \frac{1764\cdot VA}{110\cdot V} = 16.04\cdot A$



What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

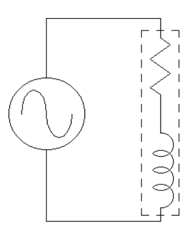
$Q_C = -Q = -1284\cdot VAR = -\frac{(110\cdot V)^2}{X_C} = -(110\cdot V)^2\cdot\omega\cdot C$

$C = \frac{-1284\cdot VAR}{-(110\cdot V)^2\cdot\omega} = 281.5\cdot\mu F$ $\quad$ Now, at the source: $\quad S = P$ & $I = \frac{1210\cdot W}{110\cdot V} = 11\cdot A$

Ex. 2 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

Series R & L

$V_S := 110\cdot V$ $\quad \omega := 377\cdot\frac{rad}{sec}$



$\mathbf{Z} := R + j\cdot\omega\cdot L$

$\mathbf{Z} = 10 + 9.425j\ \cdot\Omega$ $\quad |\mathbf{Z}| = 13.741\cdot\Omega$

$\theta := \arg(\mathbf{Z})$ $\quad \theta = 43.304\cdot deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.728$

$\mathbf{I} := \frac{V_S}{\mathbf{Z}}$ $\quad \mathbf{I} = 5.825 - 5.49j\ \cdot A$

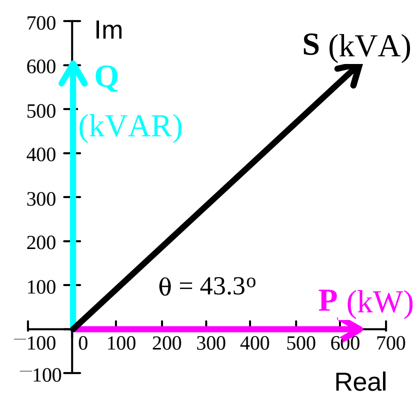
$|\mathbf{I}| = 8.005\cdot A$ $\quad \arg(\mathbf{I}) = -43.304\cdot deg$

$P := V_S\cdot|\mathbf{I}|\cdot pf$ $\quad P = 640.8\cdot W$

$Q := V_S\cdot|\mathbf{I}|\cdot\sin(\theta)$ $\quad Q = 603.94\cdot VAR$

$\mathbf{S} := V_S\cdot\overline{\mathbf{I}}$ $\quad \mathbf{S} = 640.8 + 603.94j\ \cdot VA$

$|\mathbf{S}| = 880.55\cdot VA$ $\quad \arg(\mathbf{S}) = 43.304\cdot deg$ $\quad \mathbf{S} = 881VA\underline{/43.3^o}$



OR, if we first find the magnitude of the current which flows through each element of the load...

$|\mathbf{I}| = I := \dfrac{V_S}{\sqrt{R^2 + (\omega \cdot L)^2}}$ $\qquad I = 8.005 \cdot A$

$P := I^2 \cdot R$ $\qquad P = 640.786 \cdot W$ $\qquad$ & $\qquad Q := I^2 \cdot (\omega \cdot L)$ $\qquad Q = 0.604 \cdot kVAR$

$\mathbf{S} := P + j \cdot Q$ $\qquad |\mathbf{S}| = \sqrt{P^2 + Q^2} = 880.54 \cdot VA$ $\qquad pf = \dfrac{P}{|\mathbf{S}|} = 0.728$

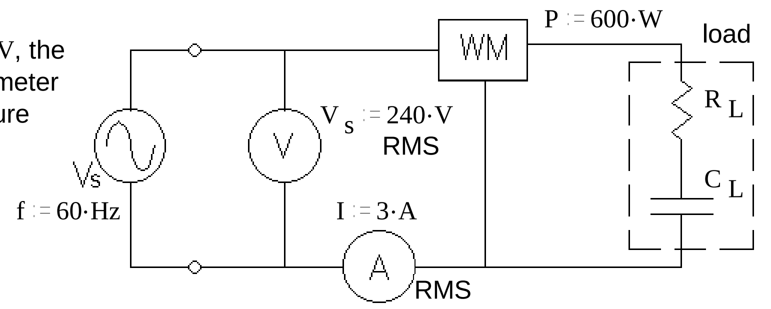
What value of C in parallel with R & L would make $pf = 1$ $(Q = 0)$?

$Q = 603.9 \cdot VAR$ $\qquad$ so we need: $\quad Q_C := -Q$ $\qquad Q_C = -603.9 \cdot VAR = \dfrac{{V_{in}}^2}{X_C}$

$X_C := \dfrac{{V_S}^2}{Q_C}$ $\qquad X_C = -20.035 \cdot \Omega = \dfrac{-1}{\omega \cdot C}$ $\qquad C := \dfrac{1}{|X_C| \cdot \omega}$ $\qquad C = 132 \cdot \mu F$

Check: $\dfrac{1}{\dfrac{1}{R + j \cdot \omega \cdot L} + j \cdot \omega \cdot C} = 18.883 \cdot \Omega$ $\qquad$ No j term, so $\theta = 0^o$

Ex. 3 R_L, & C_L together are the load in the circuit shown. The RMS voltmeter measures 240 V, the RMS ammeter measures 3 A, and the wattmeter measures 600 W. Find the following: Be sure to show the correct units for each value.



a) The value of the load resistor. $R_L = ?$

$P = I^2 \cdot R_L$

$R_L := \dfrac{P}{I^2}$ $\qquad R_L = 66.7 \cdot \Omega$

b) The apparent power. $|\mathbf{S}| = ?$ $\qquad S := V_s \cdot I$ $\qquad S = 720 \cdot VA$

c) The reactive power. Q = ? $\qquad Q := -\sqrt{S^2 - P^2}$ $\qquad Q = -398 \cdot VAR$

Remember that the square root is $\pm$, **-** in this case because the load is capacitive.

d) The complex power. $\mathbf{S} = ?$ $\qquad \mathbf{S} := P + j \cdot Q$ $\qquad \mathbf{S} = (600 - 398i) \cdot VA$

e) The power factor. $pf = ?$ $\qquad pf := \dfrac{P}{V_s \cdot I}$ $\qquad pf = 0.833$

f) The power factor is leading or lagging? $\qquad$ leading (load is capacitive, Q is negative)

g) The two components of the load are in a box which cannot be opened. Add (draw it) another component to the circuit above which can correct the power factor (make $pf = 1$). Show the correct component in the correct place and <u>find its value</u>. This component should not affect the real power consumption of the load.

$f = 60 \cdot Hz$ $\qquad \omega := 2 \cdot \pi \cdot f$ $\qquad \omega = 376.991 \cdot \dfrac{rad}{sec}$

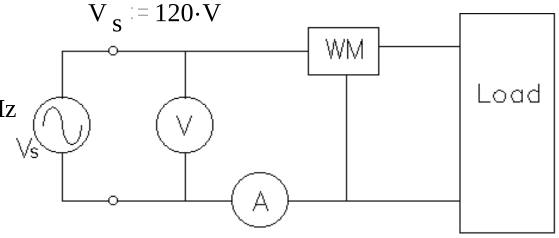
Add an inductor in parallel with load

$Q = -398 \cdot VAR$ $\qquad$ so we need: $\quad Q_L := -Q$ $\qquad Q_L = 398 \cdot VAR = \dfrac{{V_s}^2}{X_L}$

$X_L := \dfrac{{V_s}^2}{Q_L}$ $\qquad X_L = 144.725 \cdot \Omega = \omega \cdot L$ $\qquad L := \dfrac{|X_L|}{\omega}$ $\qquad L = 384 \cdot mH$

Ex. 4 For the 60 Hz load shown in the figure, the RMS voltmeter measures 120 V. The phasor diagram for the power is also shown. Find the following:

$V_s := 120 \cdot V$



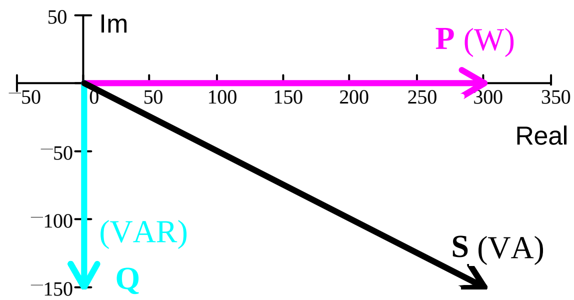
a) The complex power. **S** = ?

$P := 300 \cdot W$ $\quad Q := -150 \cdot VAR$

$\mathbf{S} := P + j \cdot Q$ $\quad \mathbf{S} = (300 - 150j) \cdot VA$

b) The apparent power. |**S**| = ? $\quad |\mathbf{S}| = \sqrt{P^2 + Q^2} = 335.4 \cdot VA$

c) The power factor. pf = ? $\quad pf := \dfrac{P}{|\mathbf{S}|}$ $\quad pf = 0.894$



d) The item marked "WM" in the figure is a wattmeter, what does it read? (give a number) $\quad P = 300 \cdot W$

e) The item marked "A" in the figure is an RMS ammeter, what does it read? (give a number)

$I := \dfrac{|\mathbf{S}|}{V_s}$ $\quad I = 2.795 \cdot A$ $\quad I = 2.8 \cdot A$

f) The power factor is leading or lagging? $\quad$ leading (Q is negative)

g) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and <u>find its value</u>. This component should not affect the real power consumption of the load.

$\omega = 377 \cdot \dfrac{rad}{sec}$

Add an inductor in parallel with load

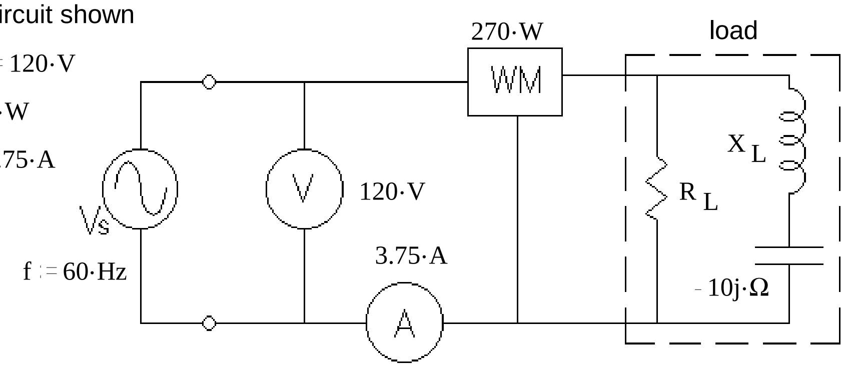
$Q = -150 \cdot VAR$ $\quad$ need: $Q_L := -Q$ $\quad Q_L = 150 \cdot VAR$ $\quad = \dfrac{V_s^{\,2}}{\omega \cdot L}$ $\quad L := \dfrac{V_s^{\,2}}{\omega \cdot Q_L}$ $\quad L = 255 \cdot mH$

Ex. 5 R, L, & C together are the load in the circuit shown

The RMS voltmeter measures 120 V. $\quad V_s := 120 \cdot V$

The wattmeter measures 270 W. $\quad P := 270 \cdot W$

The RMS ammeter measures 3.75 A. $\quad I := 3.75 \cdot A$



Find the following: Be sure to show the correct units for each value.

a) The value of the load resistor. R_L = ?

$P = \dfrac{V_s^{\,2}}{R_L}$ $\quad R_L := \dfrac{V_s^{\,2}}{P}$ $\quad R_L = 53.3 \cdot \Omega$

b) The magnitude of the impedance of the load inductor (reactance) . $|\mathbf{Z_L}| = X_L$ = ?

$I_R := \dfrac{V_s}{R_L}$ $\quad I_R = 2.25 \cdot A$ $\quad I_L := \sqrt{I^2 - I_R^{\,2}}$ $\quad I_L = 3 \cdot A$ $\quad X := \dfrac{V_s}{I_L}$ $\quad X = 40 \cdot \Omega$

$X_C := -10 \cdot \Omega$ $\quad X_L := X - X_C$ $\quad X_L = 50 \cdot \Omega$

c) The reactive power. Q = ? $\quad Q := \sqrt{(V_s \cdot I)^2 - P^2}$ $\quad Q = 360 \cdot VAR$ $\quad$ positive, because the load is primarily inductive

d) The power factor is leading or lagging? $\quad$ lagging (load is inductive, Q is positive)

e) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and find its value. This component should not affect the real power consumption of the load.

Add a capacitor in parallel with load $\qquad f = 60\cdot Hz \qquad \omega := 2\cdot\pi\cdot f \qquad \omega = 376.991\cdot\frac{rad}{sec}$

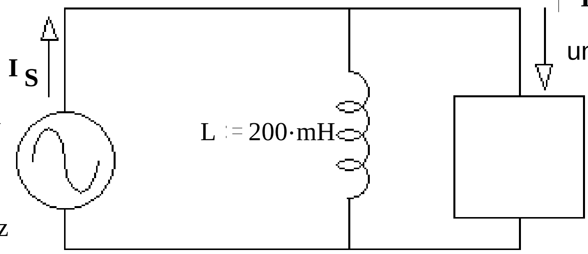
$Q = 360\cdot VAR$ so we need: $Q_C := -Q \qquad Q_C = -360\cdot VAR = -\dfrac{V_s^2}{\frac{1}{\omega\cdot C}} = -\omega\cdot C\cdot V_s^2$

$$C := \frac{Q_C}{-\omega\cdot V_s^2} \qquad C = 66.3\cdot\mu F$$

Ex. 6 An inductor is used to completely correct the power factor of a load.

Find the following:

a) The power consumed by the load. $P_L = ?$



$\mathbf{I_S}$ $\qquad |\mathbf{I_L}| = 4\cdot A$ unknown phase

$\mathbf{V_S} := 120\cdot V$ /0° 60·Hz $\qquad L := 200\cdot mH$

$I_L := 4\cdot A \qquad \omega = 376.99\cdot\frac{rad}{sec}$

$Q_L := \dfrac{(|\mathbf{V_S}|)^2}{\omega\cdot L} \qquad Q_L = 190.986\cdot VAR \qquad Q_{load} := -Q_L$

$S_L := |\mathbf{V_S}|\cdot I_L \qquad S_L = 480\cdot VA \qquad P_L := \sqrt{S_L^2 - Q_{load}^2} \qquad P_L = 440.4\cdot W$

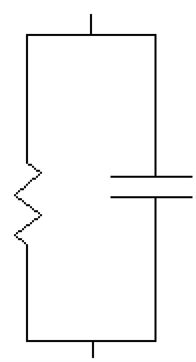
b) The power supplied by the source. $P_S = P_L = 440\cdot W$

c) The source current (magnitude and phase). $\mathbf{I_S} := \dfrac{P_L}{\mathbf{V_S}} \qquad \mathbf{I_S} = 3.67\cdot A$ /0°

because the source sees a pf = 1

d) The load can be modeled as 2 parts in parallel. Draw the model and find the values of the parts.

$P = \dfrac{V^2}{R}$



$Q_C = V^2\cdot(\omega\cdot C)$

$R_L := \dfrac{(|\mathbf{V_S}|)^2}{P_L} \qquad R_L = 32.7\cdot\Omega$

$C_L := \dfrac{-Q_{load}}{\omega\cdot(|\mathbf{V_S}|)^2}$

$C_L = 35.181\cdot\mu F$

e) The inductor, L, is replaced with a 50 mH inductor.

circle one

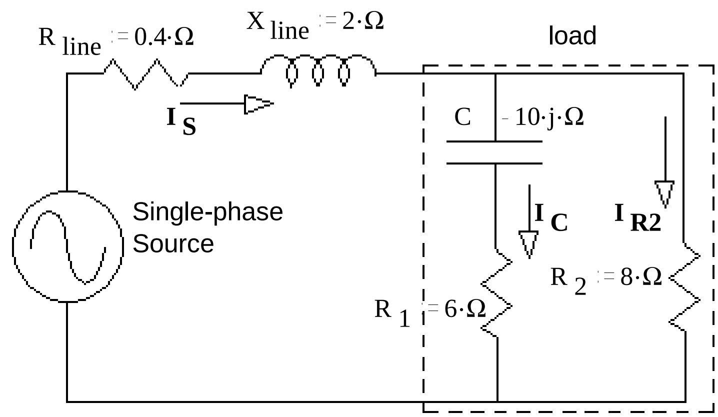
i) The **new** source current $|\mathbf{I_S}|$ is **greater** than that calculated in part c). <-- Answer

ii) The **new** source current $|\mathbf{I_S}|$ is **the same** as that calculated in part c).

iii) The **new** source current $|\mathbf{I_S}|$ is **less** than that calculated in part c).

Ex. 7 C, R_1, & R_2 together are the load (in dotted box). The reactive power used by the load is

$Q_{load} := -600 \cdot VAR$ Find:



a) The real power used by the load. $P_{load} = ?$

$X_C := -10 \cdot \Omega$

$|\mathbf{I_C}| = I_C := \sqrt{\frac{Q_{load}}{X_C}}$ $\qquad I_C = 7.746 \cdot A$

$V_{load} := I_C \cdot \sqrt{R_1^2 + X_C^2}$ $\qquad V_{load} = 90.333 \cdot V$

$P_{load} := I_C^2 \cdot R_1 + \frac{V_{load}^2}{R_2}$ $\qquad P_{load} = 1.38 \cdot kW$

b) The apparent power of the load. $|\mathbf{S}| = S := \sqrt{P_{load}^2 + Q_{load}^2}$ $\qquad S = 1.505 \cdot kVA$

c) The power factor of the load. $pf := \frac{P_{load}}{S}$ $\qquad pf = 0.917$

d) This power factor is: i) leading ii) lagging Leading, capacitor

e) The voltage at the load (magnitude). $V_{load} = 90.333 \cdot V$ found above

f) The magnitudes of the three currents. $|\mathbf{I_C}| = ?$ $\quad |\mathbf{I_{R2}}| = ?$ $\quad |\mathbf{I_S}| = ?$

$|\mathbf{I_C}| = I_C = 7.746 \cdot A$ found above

$|\mathbf{I_{R2}}| = I_{R2} = \frac{V_{load}}{R_2} = 11.292 \cdot A$

$|\mathbf{I_S}| = I_S := \frac{S}{V_{load}}$ $\qquad I_S = 16.658 \cdot A$

g) The source voltage (magnitude). $V_S = ?$

$P_{Line} := I_S^2 \cdot R_{line}$ $\qquad P_{Line} = 111 \cdot W$

$Q_{Line} := I_S^2 \cdot X_{line}$ $\qquad Q_{Line} = 555 \cdot VAR$

$|\mathbf{S_S}| = S_S := \sqrt{(P_{load} + P_{Line})^2 + (Q_{load} + Q_{Line})^2}$ $\qquad S_S = 1.492 \cdot kVA$

$V_S := \frac{S_S}{I_S}$ $\qquad V_S = 89.546 \cdot V$

h) Is there something weird about this voltage? If so, what? V_S is less than V_{Load}

Why? Because the Q of the line partially cancels the Q of the load

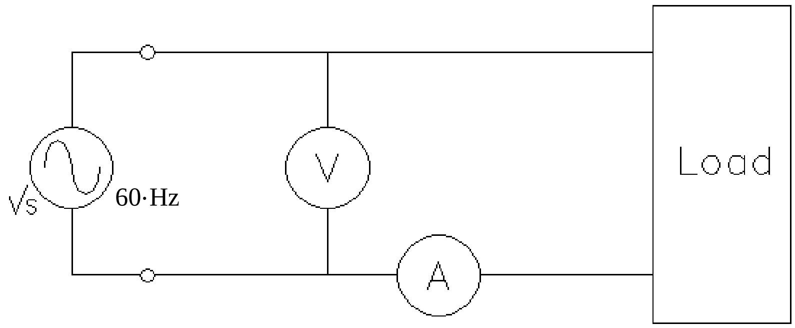
OR Partial resonance between the inductance in the line and the capacitance of the load.

i) The efficiency. $\eta = ?$

When asked for efficiency, assume the power used by R_{line} is a loss and P_{load} is the output power.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} = \frac{P_{load}}{P_{load} + P_{Line}} = 92.56 \cdot \%$$

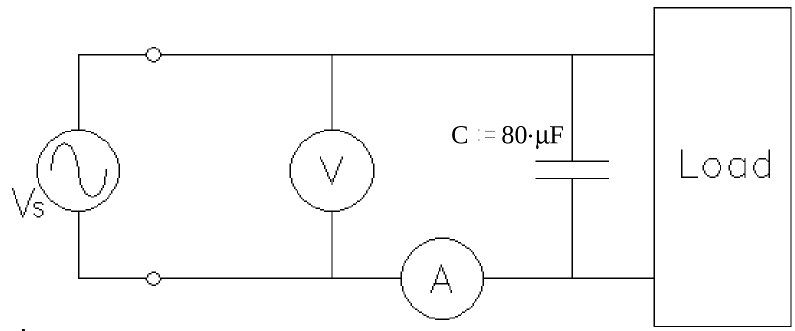
Ex. 8 In the circuit shown, the ideal voltmeter, V, reads 120V and ideal ammeter, A, reads 5A.



$S_{load} := 120 \cdot V \cdot 5 \cdot A \qquad S_{load} = 600 \cdot VA$

a) You add a capacitor, C, and the ammeter reading changes to 5.3A. Find the following:

$P_{load} = ? \qquad Q_{load} = ?$



I_C is **NOT** 0.3A, That's <u>subtracting magnitudes</u>

$$S_{load} := 120 \cdot V \cdot 5 \cdot A \qquad S_{load} = 600 \cdot VA = \sqrt{P_{load}^2 + Q_{load}^2}$$

$$\text{OR} \qquad (600 \cdot VA)^2 = P_{load}^2 + Q_{load}^2$$

$$P_{load}^2 = (600 \cdot VA)^2 - Q_{load}^2$$

$$Q_C := \frac{(120 \cdot V)^2}{\left(-\frac{1}{\omega \cdot C}\right)} = -(120 \cdot V)^2 \cdot \omega \cdot C \qquad Q_C = -434.294 \cdot VAR$$

With Capacitor:

$$S_S := 120 \cdot V \cdot 5.3 \cdot A \qquad S_S = 636 \cdot VA = \sqrt{P_{load}^2 + \left(Q_{load} + Q_C\right)^2}$$

$$\text{OR} \qquad (636 \cdot VA)^2 = P_{load}^2 + \left(Q_{load} + Q_C\right)^2$$

$$\text{Substitute in} \qquad (636 \cdot VA)^2 = \left[(600 \cdot VA)^2 - Q_{load}^2\right] + \left(Q_{load} + Q_C\right)^2$$

$$= \left[(600 \cdot VA)^2 - Q_{load}^2\right] + \left(Q_{load}^2 + 2 \cdot Q_C \cdot Q_{load} + Q_C^2\right)$$

$$= (600 \cdot VA)^2 + 2 \cdot Q_C \cdot Q_{load} + Q_C^2$$

$$Q_{load} := \frac{(636 \cdot VA)^2 - (600 \cdot VA)^2 - Q_C^2}{2 \cdot Q_C} \qquad Q_{load} = 165.919 \cdot VAR$$

$$P_{load} := \sqrt{S_{load}^2 - Q_{load}^2} \qquad P_{load} = 576.603 \cdot W$$

Double Check: $S_S = \sqrt{P_{load}^2 + \left(Q_{load} + Q_C\right)^2} = 636 \cdot VA$

The power factor was way over corrected by $C = 80 \cdot \mu F$